Decomposing the Rise in Top Wealth Shares*

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Abstract

The growth of the wealth share of a top percentile can be decomposed into three terms: a *within* term, which is the difference between the growth of individuals in the top percentile and the growth of the economy, a *displacement* term, which accounts for the flow of individuals in and out of the top percentile, and a *demography* term, which accounts for death and population growth. After applying this framework to the data, I find that displacement accounts for more than half the rise in top wealth inequality in the U.S. I examine the implications of this finding for the relationship between wealth inequality and mobility.

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1 Introduction

What drives the recent rise in top wealth inequality? One common view is that households in top percentiles are growing faster than the economy, i.e. that the "rich are getting richer" (Piketty (2014)). This view implicitly assumes that the composition of households in top percentiles remains constant over time. Yet less than 10% of the households in the 1983 Forbes list of the 400 richest households in the United States were still on the list in 2017. These large composition changes naturally drive a wedge between the growth of top wealth shares and the average wealth growth of households in the top percentiles. This paper examines the role of these composition changes for the rise in top wealth shares.

I propose an accounting framework that decomposes the growth of the wealth share of a top percentile into three terms: a *within* term, a *displacement* term, and a *demography* term. The within term corresponds to the growth of the top share, holding constant the composition of households in the top percentile. It is measured as the difference between the average growth of individuals in the top percentile and the aggregate growth per capita. It captures the extent to which individuals at the top grow faster than the economy, i.e. that the "rich are getting richer".

The displacement term accounts for the flow of existing households in and out of top percentiles. It is measured as the difference between the wealth of households entering the top percentile and the wealth of households exiting it. It captures the extent to which top shares are driven by the creation of new fortunes, rather than by the growth of existing ones.

Finally, the demography term accounts for death, inheritance, and population growth. It is measured as the difference between the wealth of households that die, and the wealth of households that replace them.

I then examine the result of this decomposition in a wide range of random-growth models. I obtain closed-form formula for the within, displacement, and demography terms as the time period tends to zero. For instance, when wealth follows a diffusive process (log-normal shocks), the displacement term equals $1/2(\zeta - 1)\mathrm{sd}^2$, where sd denotes the standard deviation of wealth growth for top households and ζ denotes the local Pareto exponent of the wealth distribution around the percentile threshold. When wealth follows a jump-diffusion process (non log-normal shocks), the displacement term depends on all higher-order cumulants of wealth growth, such as skewness and kurtosis.

These analytical formula are helpful for two distinct reasons. First, they shed light on the statistical properties of wealth growth that are important in driving top wealth shares. Second, they can also aid measurement of the displacement term and demography term in absence of panel data, which is a common occurrence.

I then use this framework for a number of applications. I first decompose the rise of the Forbes 400 list. I find that displacement accounts for more than half of the increase in top wealth inequality since 1983. More precisely, the 3.9% annual growth of the top share can be decomposed into a within term equal to 3.0%, a displacement term equal to 2.5%, and a demography term equal to -1.5%. Put differently, without displacement, the yearly growth of top shares would have been 1.4% instead of 3.9%.

The magnitude of this displacement term is well explained by a simple diffusion model (i.e. log-normal shocks). Indeed, with a measured Pareto exponent $\zeta \approx 1.5$, and a measured standard deviation of wealth growth sd $\approx 27\%$, the simple diffusion model predicts a displacement term around $1/2(\zeta - 1)\text{sd}^2 \approx 2\%$ per year, which is close to the actual displacement term.

Moreover, displacement has steadily declined over time, from 3.3% in the 1980s to 1.5% in the 2010s. I decompose this decline using the formula $1/2(\zeta - 1)\text{sd}^2$. A third of the decline is driven by a decrease in the dispersion of wealth growth among households (i.e. a decrease in sd from 0.27 to 0.23). The remaining two-thirds of the decline is driven by a thickening of the wealth distribution over time (i.e. a decrease in ζ from 1.6 to 1.4). Intuitively, it became gradually harder for households with positive shocks to reach the top.

I also compare the results of this accounting decomposition in the U.S. with China and Russia, two countries in which wealth inequality increased in recent years. One striking finding is that displacement accounts for all of the rise in top wealth inequality in China. The simple diffusion model $1/2(\zeta - 1)\text{sd}^2$ provides a way to account for the large magnitude of the displacement term in China, which averages to 4.3%. First, the dispersion wealth growth is much more important in China than in the U.S. (sd $\approx 35\%$). Second, wealth inequality is much lower in China to begin with, which makes it easier for successful households to reach the top ($\zeta \approx 1.65$).

I then estimate the role of displacement in the wealth share of top percentiles in the 1%, 0.1%, and 0.01% over the 20th century, for which we lack panel data. To estimate the standard deviation of wealth growth at top percentiles, I take the product of the share of wealth invested in equity with the yearly cross-sectional standard deviation of firm-level returns. This allows me to obtain a model-implied displacement term without using panel data. Overall, displacement matches the inverted U-shape of top wealth inequality over the 20th century. It first peaked during the Great Depression, remained low during World War II and the postwar economic boom, before peaking again during the technological revolutions of the 1980s and 1990s. This suggests that displacement is a driving force behind the low-frequency fluctuations in wealth inequality observed during the 20th century.

Finally, I discuss the implications of my results for the relationship between wealth inequality and mobility. I define wealth mobility as the average time a household in a top percentile remains there. I show that the within term and the displacement term have opposite effects on wealth mobility: while a rise in the within term decreases mobility, a rise in the displacement term increases mobility. This is true even though wealth inequality increases in the long-run.

Related Literature. This paper is related to a recent empirical literature documenting the rise in top wealth shares in the U.S. in the past thirty years (Kopczuk and Saez (2004), Piketty (2014), Saez and Zucman (2016), Piketty and Zucman (2015), Garbinti et al. (2017), and Kuhn et al. (2017)). This literature tends to interpret the rise in top wealth shares as a rise in the wealth growth of households in top percentiles relative to the rest of the economy. In particular, Saez and Zucman (2016) defines a "synthetic saving rate" as the difference between the wealth growth of top wealth shares and the average wealth return of top households. My paper clarifies that this synthetic saving rate is actually the sum of three conceptually different terms: a household saving rate, a displacement term due to the dispersion of wealth growth, and a demography term due to the death of households in top percentiles and population growth. I develop a new accounting framework that allows one to differentiate between these terms using panel data.

My accounting decomposition relates to a literature in macroeconomics that measures the contribution of entry and exit to productivity growth (Baily et al. (1992), Foster et al. (2008), Melitz and Polanec (2015)). My focus is different since I decompose the aggregate growth of a subgroup of the population (the top percentile). This leads me to split the contribution of entry and exit into two distinct terms: a displacement term, which refers to the flow of individuals in and out of the top percentile, and a demography term, which refers to the flow of individuals in and out of the economy. My concept of displacement is related to Gârleanu et al. (2012), that define it as the part of aggregate growth that come from the arrival of new agents, rather than the growth of existing agents. In a related paper, Gârleanu and Panageas (2017) stresses the growth of self-made billionaires over the long-run compared to pre-existing billionaires.

This work also contributes to a more theoretical literature that studies inequality through the lens of random growth models (Wold and Whittle (1957), Jones (2015), Luttmer (2012), Gabaix et al. (2016) and Jones and Kim (2016)). A central equation in this literature is the Kolmogorov-Forward equation, which gives the dynamics of the wealth density in terms of the dynamics of individual wealth. A key contribution of my paper is to use this equation to obtain the dynamics of top shares in terms of the dynamics of individual wealth. Moreover, I show that this equation can be directly mapped to the data, in the form of an accounting decomposition. One related paper in the mathematics literature is Steinbrecher and Shaw (2008), that derives the dynamics of quantiles in terms of the dynamics of individual process. Recent empirical papers stress the importance of idiosyncratic shocks at the very top. Benhabib et al. (2011), Benhabib et al. (2015b), and Benhabib et al. (2015a) examine the stationary wealth distribution in an economy with idiosyncratic returns. Bach et al. (2015) and Bach et al. (2017) stress the dispersion of wealth growth across households, using administrative data from Sweden. Similarly, Fagereng et al. (2016) documents a large heterogeneity in asset returns at the top using administrative data from Norway. [1] Relative to this literature, my contribution is to identify, empirically and theoretically, the contribution of the dispersion of wealth growth to the growth of top wealth shares.

A related paper is Campbell et al. (2019), that proposes to decompose the change in the variance of log wealth into a term due to differences in expected wealth growth and a term due to difference in unexpected wealth shocks. This is similar in spirit to the difference between the within and displacement term stressed in this paper. The key difference is that I focus on decomposing the growth of top shares, which fits more directly with the existing literature on top wealth inequality.

Outline. The rest of the paper is organized as follows. In Section 2, I present the accounting framework. In Section 3, I derive analytical formula for the within, displacement and demography term in continuous-time. In Section 4, I apply this framework to decompose the growth of billionaires, in the U.S., as well as China and Russia. In Section 5, I use the analytical framework to examine the role of displacement for the top 1%, 0.1%, and 0.01% in the U.S. over the 20th century. In Section 6, I discuss the implications of my findings for wealth mobility. Section 7 concludes.

2 Accounting Framework

In this section, I present an accounting framework to decompose the growth of the top wealth share into three terms: a within term, a displacement term, and a demography term. To simplify the exposition, Section 2.1 first presents the accounting decomposition without demographic forces: in this case, the growth of the top share is only the sum of the within and the displacement terms. Section 2.2 adds population death and population growth.

[1] Existing theories to explain the concentrated portfolios of the rich include moral hazard, expertise, taste or asymetric information (see, e.g., Di Tella (2016), Eisfeldt et al. (2017), or Roussanov (2010)).

2.1 Baseline

Consider a top percentile $p \in (0,1)$, for instance, the top $p = 1\%$. We are interested decomposing the growth in the top share S_t, the share of total wealth owned by individuals in the top percentile p at time t, between $t = 1$ and $t = 2$.

Denote w_{it} the normalized wealth of individual i at time t, i.e. individual wealth divided by the average wealth in the economy. For a set of individuals Ω, denote $\overline{w}_{\Omega,t}$ the average normalized wealth at time t of individuals in set Ω at time t, i.e. $\overline{w}_{\Omega,t} = \frac{1}{|\Omega|}\sum_{i\in\Omega} w_{it}$. The growth rate of the top share can be written as the change in the average normalized wealth of individuals in the top percentile:

$$\frac{S_2 - S_1}{S_1} = \frac{\overline{w}_{T',2} - \overline{w}_{T,1}}{\overline{w}_{T,1}}, \tag{1}$$

where T denotes the set of individuals in the top percentile at time 1, and T' denotes the set of individuals in the the top percentile at time 2. Critically, we typically have $T \neq T'$: the set of individuals in a given top percentile changes over time. Denoting X the set of individuals that exit the top percentile between 1 and 2, and E the set of individuals that enter the top percentile between 1 and 2, we can write

$$\overline{w}_{T',2} = s_T \overline{w}_T + s_E(\overline{w}_{E,2} - \overline{w}_{X,2}). \tag{2}$$

Plugging this equation into (1) gives the accounting decomposition:

$$\frac{S_2 - S_1}{S_1} = \underbrace{\frac{\overline{w}_{T,2} - \overline{w}_{T,1}}{\overline{w}_{T,1}}}_{\text{Within}} + \underbrace{s_E \frac{\overline{w}_{E,2} - \overline{w}_{X,2}}{\overline{w}_{T,1}}}_{\text{Displacement}}, \tag{3}$$

where s_E denotes the intensity of entry, i.e. the ratio between the number of individuals that enters and the number of individuals in the top percentile at time 2.

The first term ("within") corresponds to the change in the normalized wealth of individuals in the top percentile at the start of the period, whether or not they remain in the top by the end of the period. Since normalized wealth is defined as wealth divided by per capita wealth, this term corresponds, at the first-order, to the difference between the wealth growth of individuals in the top percentile and the per capita wealth growth in the economy.

The second term ("displacement") corresponds to the difference between the wealth of entering individuals and the wealth of exiting individuals. Note that the displacement term is always positive: by definition, individuals entering the top percentile have a higher wealth than individuals exiting the top percentile.

It is useful to distinguish the role of entry and exit in the growth of the top share, by rewriting the displacement term as follows:

$$s_E \frac{\overline{w}_{E,2} - \overline{w}_{X,2}}{\overline{w}_{T,1}} = s_E \frac{\overline{w}_{E,2} - q_2}{\overline{w}_{T,1}} + s_E \frac{q_2 - \overline{w}_{X,2}}{\overline{w}_{T,1}},$$

where q_2 is the wealth of the last individual in the top percentile. Take an individual below the percentile threshold who, after a large positive wealth shock, enters the top percentile. Because the mass of individuals in a given top percentile must remain constant, she displaces the last individual in the top, with wealth q_2. This effect is captured by the entry term. Conversely, when, after a very low wealth growth, an individual exits the top percentile, she is replaced by the individual just below the percentile threshold, with wealth q_2. This effect is captured by the exit term.

2.2 Adding Demography

To simplify the exposition, the preceding analysis abstracted away from changes in the composition of individuals in the economy. In reality, individuals die, and new individuals are born, which generates additional composition changes in the top percentile. I now derive the accounting decomposition in this more general case.

Denote w_{it} the normalized wealth of individual i at time t, i.e. individual wealth divided by the average per capita wealth in the economy. As above, the growth of the top share can be written as the change in the average normalized wealth of individuals in the top percentile:

$$\frac{S_2 - S_1}{S_1} = \frac{\overline{w}_{T',2} - \overline{w}_{T,1}}{\overline{w}_{T,1}}. \tag{4}$$

As above, we typically have $T \neq T'$. Denote D the set of individuals in the top percentile that die during the time period, and B the set of individuals that are born in the top percentile (or enter the top percentile due to inheritance). X denotes the set of individuals that exit the top percentile for reasons other than death, and E denotes the set of individuals that enter the top percentile for reasons other than inheritance. We now have $T' = (T \setminus D) \cup B \cup E \setminus X$, which leads to the following equality:

$$\overline{w}_{T',2} = s_{T \setminus D} \overline{w}_{T \setminus D,2} + s_B \overline{w}_{B,2} + s_E \overline{w}_{E,2} - s_X \overline{w}_{X,2} \tag{5}$$

$$s_T \overline{w}_{T,1} = s_{T \setminus D} \overline{w}_{T \setminus D,1} + s_D \overline{w}_{D,1}, \tag{6}$$

where, for a set Ω, s_Ω denotes the ratio between the number of individuals in the set Ω and the number of individuals in the top percentile at the end of the period.[2] Combining these two equations

[2]That is, $s_\Omega = |\Omega|/|T'|$ where Ω denotes the mass of individuals in the set Ω

allows me to rewrite $\overline{w}_{T',2}$ in terms of $\overline{w}_{T,1}$:

$$\begin{aligned}\overline{w}_{T',2} &= \frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}(s_T\overline{w}_{T,1} - s_D\overline{w}_{D,1}) + s_B\overline{w}_{B,2} + s_E\overline{w}_{E,2} - s_X\overline{w}_{X,2} \\ &= \frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{T,1} - (1-s_T)\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{T,1} - s_D\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{D,1} + s_B\overline{w}_{B,2} + s_E\overline{w}_{E,2} - s_X\overline{w}_{X,2}.\end{aligned}$$

Plugging this equation into the growth of the top share (4) gives the following decomposition for the growth of the top share:

Proposition 1. *The growth of the top share can be decomposed as follows*

$$\begin{aligned}\frac{S_2 - S_1}{S_1} = &\underbrace{\frac{\overline{w}_{T\setminus D,2} - \overline{w}_{T\setminus D,1}}{\overline{w}_{T\setminus D,1}}}_{Within} + \underbrace{s_E\frac{\overline{w}_{E,2} - q_2}{\overline{w}_{T,1}} + s_X\frac{q_2 - \overline{w}_{X,2}}{\overline{w}_{T,1}}}_{Displacement} \\ &+ \underbrace{\underbrace{s_D\frac{q_2 - \frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{D,1}}{\overline{w}_{T,1}}}_{Death} + \underbrace{(1-s_T)\frac{q_2 - \frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{T,1}}{\overline{w}_{T,1}}}_{Population\ Growth} + \underbrace{s_B\frac{\overline{w}_{B,2} - q_2}{\overline{w}_{T,1}}}_{Birth}}_{Demography}. \end{aligned} \tag{7}$$

The within term corresponds to the growth in the normalized wealth of individuals in the top at the start of the period and that do not die by the end of that period. The displacement term is similar to the one derived above, except that, with demographic forces, the fraction of individuals that enter may differ from the fraction of individuals that exit, i.e. $s_E \neq s_X$.

There is a new demography term, which is itself the sum of three terms. The first corresponds to the effect of death. When an individual dies, she is replaced by an individual at the percentile threshold, with wealth q_2. Overall, death decreases wealth at the top by the difference between the average wealth of deceased individuals if they had not died, $\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{D,1}$, and the wealth of this last individual q_2.

The second term corresponds to the effect of population growth. Population growth increases the number of individuals in the economy, which increases total wealth in the economy relative to per capita wealth. At the same time, population growth increases the number of individuals in the top percentile. Overall, the effect of population growth exactly mirrors the effect of death, except that the death rate s_D is replaced by the rate of population growth $1 - s_T$.

The third term corresponds to the effect of birth (or inheritance). It corresponds exactly to the entry term, except that it captures individuals that enter the top percentile because they are born in the economy (or because they inherit the money of deceased individuals) rather than individuals that enter the top percentile after a high wealth shock.

3 Analytical Framework

I now study the result of this accounting decomposition in a wide range of random-growth models. I obtain closed-form formula for the within, displacement, and demography terms as the time period tends to zero. These formula clarify the mapping between the accounting framework and existing models of wealth inequality. They will also help to estimate the displacement and demography term in absence of panel data. Section 3.1 starts with the baseline case of a diffusion process. Section 3.2 extends the results to jump-diffusion process, while Section 3.3 adds demographic forces.

3.1 Diffusion

Wealth Dynamics. Consider an economy populated by a mass one of agents. Individuals are indexed by $i \in [0, 1]$, and $t \in [0, +\infty)$ is time. Denote w_{it} the wealth of individual i at time t. I assume that the law of motion of individual wealth follows a diffusion process:

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_t(w_{it})\,\mathrm{d}t + \nu_t(w_{it})\,\mathrm{d}B_{it}, \tag{8}$$

where $B = \{B_{it} : \mathcal{F}_t, t \geq 0\}$ is an idiosyncratic Brownian motion for individual i, in a probability space $(\Omega, P, \mathcal{F})$, equipped with a filtration $\mathcal{F} = \{\mathcal{F}_t, t \geq 0\}$ with the usual conditions.

I add the following regularity assumptions. First, the initial wealth density, $g_0 : \mathbb{R}^+ \to \mathbb{R}^+$ exists, and has finite mean. Second, $\mu_t(\exp(.))$ and $\nu_t(\exp(.))$ possess bounded continuous derivatives of all orders. Third, the diffusion term never vanishes, i.e. there exists $\epsilon > 0$ such that $\nu_t(.)^2 \geq \epsilon$. These conditions ensure that the density of wealth exists, is smooth, and has finite mean.

To simplify notations, I assume that the wealth process is centered, i.e. $\mathrm{E}[w_{it}] = 1$ for $t \geq 0$. This is equivalent to redefining the drift $\mu_t(w)$ to be the difference between the drift of individual wealth and the per capita growth rate of the economy.

Top Share. Consider a top percentile $p \in (0, 1)$. Denote q_t the wealth of the last person in the top percentile (i.e. the $1 - p$ quantile). The wealth share owned by the top percentile p, S_t, can be written as the total wealth of individuals above the percentile threshold:

$$S_t = \int_{q_t}^{+\infty} w g_t(w)\,\mathrm{d}w. \tag{9}$$

The following proposition characterizes the dynamics of the top share in terms of the dynamics of individual wealth.

Proposition 2. *Consider the wealth process (8). The top wealth share S_t follows the law of motion:*

$$\frac{\mathrm{d}S_t}{S_t} = \underbrace{\mathrm{E}_w[\mu_t(w_{it}) \mid w_{it} \geq q_t]\,\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t}_{\textit{Displacement}}, \tag{10}$$

where E_w denotes the wealth-weighted average along the wealth distribution.

The instantaneous growth of the top share is the sum of a within term and a displacement term. The within term $\mathrm{E}_w[\mu_t(w_{it}) \mid w_{it} \geq q_t]\,\mathrm{d}t$ is the instantaneous growth of individual in the top. The displacement term depends on the idiosyncratic volatility of individual at the percentile top $\nu_t(q_t)$, as well as on the shape of the wealth distribution $\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2$.

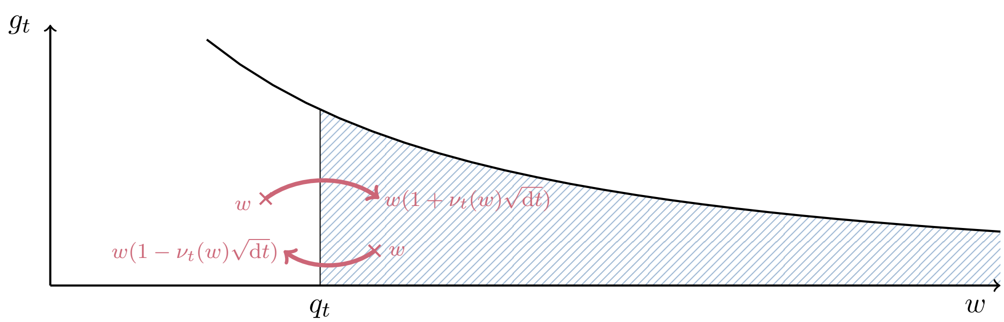
I now provide an heuristic derivation for the displacement term. For simplicity, I assume $\mu_t(w) = 0$. During a short period of time $\mathrm{d}t$, the wealth of an individual with wealth w is multiplied by $(1+\nu_t(w)\sqrt{\mathrm{d}t})w$ or $(1-\nu_t(w)\sqrt{\mathrm{d}t})w$ with equal probability.

Let us first compute the contribution of entry. As shown in Figure 1, some individuals just below the percentile threshold receive a positive wealth shock, and, therefore, they enter the top percentile. Because the mass of individual in the top percentile remains constant, these individuals displace individuals at the percentile threshold (with wealth q_t). Overall, the contribution of entry to the growth of the top share is $\frac{1}{S_t}\int_0^{q_t}((1+\nu_t(w)\sqrt{\mathrm{d}t})w - q_t)^+\frac{1}{2}g_t(w)\,\mathrm{d}w$.[3] This term can be approximated by $\frac{1}{2}\frac{g_t(q_t)q_t}{p}\nu_t(q_t)\sqrt{\mathrm{d}t} \times \frac{1}{2}\frac{q_t p}{S_t}\nu_t(q_t)\sqrt{\mathrm{d}t}$ using the trapezoid rule. In terms of the accounting framework presented above, the first term in this product corresponds to the fraction of individuals that enter the top percentile (i.e. $s_E \approx \frac{1}{2}\frac{g_t(q_t)q_t}{p}\nu_t(q_t)\sqrt{\mathrm{d}t}$), and the second term corresponds to the growth per entry (i.e. $\frac{\overline{w}_{E,2}-q_2}{\overline{w}_{T,1}} \approx \frac{1}{2}\frac{q_t p}{S_t}\nu_t(q_t)\sqrt{\mathrm{d}t}$). Taking the product gives $\frac{1}{4}\frac{q_t^2 g_t(q_t)}{S_t}\nu_t(q_t)^2\,\mathrm{d}t$, which corresponds to half of the displacement term.

Let us now compute the contribution of exit. Some individuals just above the percentile threshold receive a negative wealth shock, and, therefore, they exit the top percentile. They are replaced by individuals at the percentile threshold (with wealth q_t). Overall, the contribution of exit to the growth of the top share is $\frac{1}{S_t}\int_{q_t}^{+\infty}(q_t - (1-\nu_t(w)\sqrt{\mathrm{d}t})w)^+\frac{1}{2}g_t(w)\,\mathrm{d}w$. As above, it can be approximated by $\frac{1}{2}\frac{g_t(q_t)q_t}{p}\nu_t(q_t)\sqrt{\mathrm{d}t} \times \frac{1}{2}\frac{q_t p}{S_t}\nu_t(q_t)\sqrt{\mathrm{d}t}$. In terms of the accounting framework presented above, the first term in this product corresponds to the fraction of individuals that exit the top percentile (i.e. $s_X \approx \frac{1}{2}\frac{g_t(q_t)q_t}{p}\nu_t(q_t)\sqrt{\mathrm{d}t}$), and the second term corresponds to the growth per exit (i.e. $\frac{q_2-\overline{w}_{X,2}}{\overline{w}_{T,1}} \approx \frac{1}{2}\frac{q_t p}{S_t}\nu_t(q_t)\sqrt{\mathrm{d}t}$). Taking the product of these two terms gives $\frac{1}{4}\frac{q_t^2 g_t(q_t)}{S_t}\nu_t(q_t)^2\,\mathrm{d}t$, which corresponds to the remaining half of the displacement term.

[3] Here and in the rest of the paper, $x^+ = max(x, 0)$ denotes the positive part of x.

Figure 1: Displacement in the Diffusive Case



Long-run. One of the most ubiquitous regularities in economics and finance is that many distributions, including the wealth or income distribution, are well approximated by a power law. In this case, the decomposition takes a particularly simple form.

Formally, in this paragraph, assume that $\mu_t(w) \to \mu(w)$ and $\nu_t(w) \to \nu(w)$ as $t \to +\infty$, uniformly in w, with $\mu(w) \to \mu < 0$ and $\nu(w) \to \nu$ as $w \to +\infty$. Under these assumptions, the distribution converges to a stationary distribution with a Pareto tail, i.e. $g(w) \sim Cw^{-\zeta}$ as $w \to +\infty$.[4] ζ is called the Pareto exponent of the distribution. The lower ζ, the thicker the right tail of the density, and, therefore, the higher the level of wealth inequality. The assumption that $\mu < 0$ ensures that $\zeta > 1$, which ensures the distribution has a finite mean.

Plugging this expression for the stationary wealth distribution into Proposition 2 gives the following balance equation for the top share:

$$0 = \underbrace{\mu \,\mathrm{d}t}_{\text{Within}} + \underbrace{\frac{\zeta - 1}{2}\nu^2 \,\mathrm{d}t}_{\text{Displacement}}. \tag{11}$$

The displacement term only depends on two statistics: ζ, the Pareto exponent of the distribution, and ν, the idiosyncratic volatility of wealth growth at the top. The fact that the displacement term only depends on the Pareto exponent reflects the fact that the Pareto distribution is "scale-free". More precisely, for a distribution with a Pareto tail, the mass of households at a given relative distance of a threshold, relative to the mass of households above the threshold, is constant in the right tail, i.e. $\frac{g_t(q_t)q_t}{p} \to \zeta$. In our setup, this implies that the fraction of individuals that enter/exits the top percentile is also constant in the right tail, i.e. $\frac{1}{2}\frac{q_t p}{S_t}\nu_t(q_t)\sqrt{\mathrm{d}t} \to \frac{1}{2}\zeta\nu\sqrt{\mathrm{d}t}$.

Moreover, for a distribution with a Pareto tail, the wealth of households at a percentile threshold, relative to the mass of individuals above the percentile threshold is constant in the right tail, i.e.

[4] See, for instance, Gabaix et al. (2016).

$\frac{q_t p}{S_t} \to \frac{\zeta-1}{\zeta}$ (see, e.g., Saez (2001)). In our setup, this implies that the growth per entry/exit is also constant in the right tail, i.e. $\frac{q_t p}{S_t}\nu_t(q_t)\sqrt{\mathrm{d}t} \to \frac{\zeta-1}{\zeta}\nu\sqrt{\mathrm{d}t}$. Overall, these two properties give that the displacement term, which is the product, is constant in the right tail, i.e. $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t^2(q_t)\,\mathrm{d}t \to \frac{\zeta-1}{2}\nu^2\,\mathrm{d}t$.

The displacement term $\frac{\zeta-1}{2}\nu^2\,\mathrm{d}t$ increase with ζ (i.e. decreases with the level of wealth inequality) for two reasons. First, in a world in which inequality is high, the distribution is more spread out around the top percentile threshold, which means that the intensity of entry/exit $\frac{1}{2}\zeta\nu\sqrt{\mathrm{d}t}$ is low. Second, wealth at the percentile threshold is small relative to the average wealth above the top percentile, which means that the growth per entry/exit $\frac{1}{2}\frac{\zeta-1}{\zeta}\nu\sqrt{\mathrm{d}t}$ is small.

In Appendix A.2, I extend this formula to extend for type heterogeneity (i.e. drift and volatility that are heterogeneous across individuals with the same wealth level).

3.2 Jumps

Wealth Dynamics. The preceding analysis considers the case in which wealth follows a diffusion process (see (8)). This makes the expression for the displacement term particularly simple: during a short period of time $\mathrm{d}t$, only individuals close to percentile threshold flow in and out of the top percentile. This is why the displacement term only depends on the density and the idiosyncratic volatility at the percentile threshold.

I now extend the formula in the case in which wealth follows a jump-diffusion process, i.e. is discontinuous. For instance, a large literature in finance argues for the presence of jumps in asset prices (e.g. Aıt-Sahalia (2004), Barro and Ursúa (2012), Martin (2013), Backus et al. (2011)). Formally, I assume that wealth follows the law of motion:

$$\frac{\mathrm{d}w_{it}}{w_{it\text{-}}} = \mu_t(w_{it\text{-}})\,\mathrm{d}t + \nu_t(w_{it\text{-}})\,\mathrm{d}B_{it} + \left(e^{\phi_t(w_{it\text{-}})U_i} - 1\right)\mathrm{d}N_{it} - \mathrm{E}_U\left[e^{\phi_t(w_{it\text{-}})U_i} - 1\right]\lambda\,\mathrm{d}t, \tag{12}$$

where N_{it} is a compound Poisson process with intensity λ. The function $\phi_t(.)$ allows jump sizes to depend on the wealth level. We assume that the function $\phi_t(\exp(.))$ possess bounded derivatives of all orders, with $\mathrm{E}_U[e^{\sup_w ||\phi(w)||U_i}] < +\infty$. The compound Poisson process is compensated, which means that the jump term does not change the instantaneous expected growth rate of wealth, which is still given by $\mu_t(w)$.

The dynamics of the top wealth share is given by the following proposition.

Proposition 3. *Consider the wealth process (A18). Under some regularity conditions,*[5] *the top*

[5] I assume that the density is smooth enough, i.e. $g_t \in \mathcal{C}^{+\infty}$, and for each T, there exists $\overline{g}$ such that $g_t(w) \leq \overline{g}(w)$

wealth share S_t follows the law of motion:

$$\frac{\mathrm{d}S_t}{S_t} = \underbrace{\mathrm{E}^{wg_t}[\mu_t(w)|w \geq q_t]\,\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t}_{\textit{Displacement (due to diffusion)}}$$
$$+ \underbrace{\frac{1}{S_t}\mathrm{E}_U\left[\int_0^{q_t}(e^{\phi_t(w)U}w - q_t)^+ g_t(w)\,\mathrm{d}w + \int_{q_t}^{+\infty}(q_t - e^{\phi_t(w)U}w)^+ g_t(w)\,\mathrm{d}w\right]\lambda\,\mathrm{d}t}_{\textit{Displacement (due to jumps)}}. \tag{13}$$

The displacement term is now the sum of two terms: one is due to the diffusion part of the process, which is the same as above, and the other is due to jumps. The term due to jumps is itself the sum of an entry and an exit term. First, some individuals below the percentile threshold with positive jumps enter the top percentile, displacing individuals at the percentile threshold. The contribution of this entry is given by $\frac{1}{S_t}E_U\left[\int_0^{q_t}(we^{\phi_t(w)U} - q_t)^+ g_t(w)\,\mathrm{d}w\right]$. Conversely, some individuals above the percentile threshold with negative jumps exit the top percentile, and they are replaced by individuals at the the percentile threshold. The contribution of this exit is given by $\frac{1}{S_t}E_U\left[\int_{q_t}^{+\infty}(q_t - we^{\phi_t(w)U})^+ g_t(w)\,\mathrm{d}w\right]\lambda\,\mathrm{d}t$.

In contrast with the term due to the diffusion part of the process, the jump term depends on the wealth density and the distribution of jump sizes across the whole distribution, not just at the percentile threshold. This is because, due to jumps, individuals can enter the top percentile from any part of the wealth distribution, not just from the percentile threshold.

If the distribution of jumps is regular enough, however, we expect that most individuals who enter the top percentile do not do so from too far away. To make this point formally, I now rewrite the jump term as a Taylor expansion for jump sizes around zero, i.e. in terms of higher-order cumulants.

Higher-Order Cumulants. Denote $\kappa_{jt}(w)$ the instantaneous cumulant of log wealth growth, i.e. the j-th derivative of the cumulant generating function:

$$\kappa_{jt}(w) = \partial_\tau \partial_\theta^j \log E_t\left[e^{\theta\log\left(\frac{w_{it+\tau}}{w_{it\text{-}}}\right)}|w_{it\text{-}} = w\right]\Big|_{\theta=0,\tau=0}. \tag{14}$$

An application of Ito's lemma, combined with the law of motion(12), gives the following equations:

$$\kappa_{jt}(w) = \begin{cases} \nu_t(w)^2 + \lambda\phi_t(w)^2\mathrm{E}_U\left[U^2\right] & \text{for } j = 2 \\ \lambda\phi_t(w)^j\mathrm{E}_U\left[U^j\right] & \text{for } j > 2. \end{cases} \tag{15}$$

and $\int_{\mathbb{R}+}\overline{g}(w)w\,\mathrm{d}w < +\infty$ for $t \in (0,T)$. While I believe it to be the cas for the process defined in (12), I was not able to find a proof in the existing literature. The closest paper is Cass (2006), that examines the case of a time-homogeneous process.

The second cumulant, i.e. the variance, depends on the idiosyncratic volatility of wealth growth, as well as the second moment of jumps U. The cumulant of order j depends on the j-th moment of jumps. When the process follows a pure diffusion, all cumulants of order higher than three are zero. I now show that the displacement can be rewritten as a sum of all higher-order cumulants.

Corollary 1. *Consider the wealth process (12). Under some regularity conditions,*[6] *the top wealth share S_t follows the law of motion:*

$$\frac{\mathrm{d}S_t}{S_t} = \underbrace{\mathrm{E}_w\left[\mu_t(w) \mid w \geq q_t\right]\mathrm{d}t}_{\textit{Within}} + \underbrace{\sum_{k=2}^{+\infty} \frac{1}{j!}\frac{q_t}{S_t}\sum_{l=0}^{j-2}(-w\partial_w)^l (w g_t(w)\kappa_{jt}(w))\Big|_{w=q_t}\mathrm{d}t}_{\textit{Displacement}}. \tag{16}$$

The term for $j = 2$ is simply equal to $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\kappa_{2t}(q_t)\,\mathrm{d}t$, which is similar to the term obtained for a diffusion process. The only difference is that κ_{2t} now reflects the total variance of log wealth growth, which can come from the diffusion or the jump part.

The term for $j = 3$ depends on the level and derivative of the third cumulant of log wealth growth and the wealth density at the percentile threshold. Intuitively, the third cumulant (i.e. skewness) measures the asymmetry of the distribution of log wealth growth compared to a normal distribution. It is natural that its effect on displacement depends on the slope of the density around the percentile threshold.

The term for $j = 4$ depends on the level and first two derivatives of the fourth cumulant of log wealth growth and the wealth density. Intuitively, the fourth cumulant (i.e. kurtosis) measures the "tailedness" of the distribution log wealth growth compared to a normal distribution. It is natural that the effect of kurtosis depends on the curvature of the density around the percentile threshold.

More generally, the term of order j depends on level and first j-th derivatives of the j-th cumulant of log wealth growth and of wealth density. Intuitively, cumulants of higher-order reflect the contribution of large jump sizes to displacement. Their effects depend on the wealth density and the distribution of jump sizes far from the threshold.

Long-Run. Formally, in this paragraph, assume that $\mu_t(w) \to \mu(w)$ and $\nu_t(w) \to \nu(w)$ and $\phi_t(w) \to \phi(w)$ uniformly in w, with $\mu_t(w) \to \mu < 0$, $\nu(w) \to \nu$, and $\phi_t(w) \to \phi$ as $w \to +\infty$. Under these assumptions, the stationary distribution, if it exits, has a Pareto tail, i.e. $g(w) \sim Cw^{-\zeta}$, with a Pareto exponent $\zeta > 1$.

[6] More precisely, I assume that the function $v \to \int_{q_t e^{-v}}^{q_t}(e^v w - q_t)\frac{1}{\phi_t(\omega)}f_U(\frac{J}{\phi_t(\omega)})g_t(w)\,\mathrm{d}w$ is analytic, where f_U denotes the density of jump sizes.

Plugging this expression for the stationary distribution in Proposition 3 gives the following balance equation for the top share:

$$0 = \underbrace{\mu \, \mathrm{d}t}_{\text{Within}} + \underbrace{\sum_{k=2}^{+\infty} \frac{\zeta^{k-1} - 1}{k!} \kappa_k \, \mathrm{d}t}_{\text{Displacement}} . \tag{17}$$

In the limit, the displacement term has a particularly simple formula. In particular, it only depends on ζ, the Pareto exponent of the wealth distribution. Moreover, the displacement term increases with all higher-order cumulants. The first four terms can be rewritten as:

$$\frac{\zeta - 1}{2} \mathrm{sd}^2 \, \mathrm{d}t + \frac{\zeta^2 - 1}{6} \cdot \text{skewness} \cdot \mathrm{sd}^3 \, \mathrm{d}t + \frac{\zeta^3 - 1}{24} \cdot \text{excess kurtosis} \cdot \mathrm{sd}^4 \, \mathrm{d}t + \ldots$$

As in the case of the diffusion, the displacement term increases with ζ (i.e. decreases with the degree of wealth inequality). More interestingly, the level of ζ modulates the importance of higher-order cumulants relative to the variance term. A low level of wealth inequality (i.e. high ζ) is associated with a particularly high importance of higher-order cumulants. To take a simple example, going from a distribution with $\zeta = 1.5$ (the approximate Pareto exponent of the wealth distribution) to a distribution with $\zeta = 2.5$ (the approximate Pareto exponent of the labor income distribution), the term due to the variance of wealth shocks is multiplied by 4, the term due to skewness is multiplied by 6, while the term due to kurtosis is multiplied by 9. Intuitively, the lower the level of wealth inequality, the more entry and exit there is from individuals far from the percentile threshold, relative to individuals closer to it.

3.3 Demography

I now extend the analysis to incorporate death and population growth. I assume that individuals die with a hazard rate δ_t, and that population grows at rate η_t. Newborn agents enter the economy with an initial wealth density given by $g_{Bt}(w)$.

Individual wealth dynamic is given by the diffusion process (8), with the same regularity conditions. Moreover, I assume that $\delta_t, \eta_t, g_{Bt}(w)$ are continuous with respect to time, and that the average wealth of newborn agents is finite, i.e. $\int_{\mathbb{R}^+} w g_{Bt}(w) < +\infty$. As in the previous section, I assume, without loss of generality, that the wealth is centered, i.e. $\mathrm{E}[w_{it}] = 1$ for $t \geq 0$. This is equivalent to redefining the drift $\mu_t(w)$ to be the difference between the drift of individual wealth and the per capita growth rate of the economy.

Proposition 4. *Consider the wealth process (8), with death rate δ_t, population growth η_t, and wealth density of newborns g_{Bt}. The top wealth share S_t follows the law of motion:*

$$\begin{aligned}\frac{\mathrm{d}S_t}{S_t} =& \underbrace{\mathrm{E}^{wg_t}[\mu_t(w) \mid w \geq q_t]\,\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t}_{\textit{Displacement}} \\ &+ \underbrace{\underbrace{\left(\frac{q_t p}{S_t}-1\right)\delta_t\,\mathrm{d}t}_{\textit{Death}} + \underbrace{\left(\frac{q_t p}{S_t}-1\right)\eta_t\,\mathrm{d}t}_{\textit{Population Growth}} + \underbrace{\frac{1}{S_t}\left(\int_{q_t}^{+\infty}(w-q_t)g_{Bt}(w)\,\mathrm{d}w\right)(\delta_t+\eta_t)\,\mathrm{d}t}_{\textit{Birth}}}_{\textit{Demography}}. \qquad (18)\end{aligned}$$

As in the accounting decomposition in Section 2, the growth of the top share is now the sum of three terms: a within, a displacement, and a demography term.

The death term can be derived as follows. Between t and $t+\mathrm{d}t$, a mass $\delta_t p\,\mathrm{d}t$ of individuals in the top dies, which decreases total wealth in the top percentile by $\delta_t S_t\,\mathrm{d}t$. Because the mass of individuals in the top percentile must remain constant, they are replaced by individuals that enter at the percentile threshold. Overall, the total growth of S_t due to death is $(\frac{pq_t}{S_t}-1)\delta_t\,\mathrm{d}t$.

I now turn to the term due to population growth. Between t and $t+\mathrm{d}t$, total population increases by $\eta_t\,\mathrm{d}t$. This means that a new mass $\eta_t p\,\mathrm{d}t$ of individuals gets included in the top percentile, with wealth q_t. This increases total wealth in the top percentile by $\eta_t p q_t\,\mathrm{d}t$. At the same time, total wealth in the economy increases by $\eta_t\,\mathrm{d}t$ relative to per capita wealth. This decreases the top wealth share by η_t. Overall, the total growth of S_t due to population growth is $\left(\frac{pq_t}{S_t}-1\right)\eta_t\,\mathrm{d}t$.

I now turn to the term due to birth. Between t and $t+\mathrm{d}t$, the mass of individuals born with wealth w is $(\delta_t+\eta_t)g_B(t)(w)\,\mathrm{d}t$. Each newborn displaces the last individual in the top percentile. Overall, the total growth of S_t due to birth is $\frac{1}{S_t}\left(\int_{q_t}^{+\infty}(w-q_t)g_{Bt}(w)\,\mathrm{d}w\right)(\delta_t+\eta_t)\,\mathrm{d}t$.

The terms due to death and population growth are both negative, because wealth at the threshold is always lower or equal to the average wealth above the threshold. In contrast, the term due to birth is always positive. To make some progress on the relationship between the wealth of deceased individuals and the wealth of newborn individuals, I now specify a simple model of inheritance.

A Simple Model of Inheritance. In the previous section, I considered an arbitrary density of wealth among newborn agents, g_{Bt}. I now examine a simple model of inheritance, which allows one to relate the wealth density for newborn agents g_{Bt} to the wealth density of existing agents.

I assume that when individuals die they bequest a proportion $\chi \in [0,1]$ of their wealth to their offspring. This bequest is equally split between $k \in \mathbb{N}^*$ children.[7] To simplify the model, I assume

[7] Menchik (1980) provides evidence of equal sharing among children. Cowell (1998) also examines the effect of the

that children are born exactly when individuals die. Both χ and k are random variables. Note that the average number of children in the economy $\mathrm{E}[k]$ must be such that $\delta_t \mathrm{E}[k] = \eta_t + \delta_t$.[8]

This simple model of inheritance gives that the density of wealth for newborn agents obeys $(\delta_t + \eta_t) g_{Bt}(w) = \delta_t \mathrm{E}\left[k g_t\left(\frac{w}{\chi/k}\right)\frac{1}{\chi/k}\right]$. Plugging this expression into the birth term in (18) gives:

$$\frac{1}{S_t}\left(\int_{q_t}^{+\infty}(w-q_t)g_{Bt}(w)\,\mathrm{d}w\right)(\delta_t+\eta_t)\,\mathrm{d}t = \frac{1}{S_t}\mathrm{E}\left[\int_{\frac{q_t}{\chi/k}}^{+\infty} k\left(\frac{\chi}{k}w - q_t\right)g_t(w)\,\mathrm{d}w\right]\delta_t\,\mathrm{d}t, \tag{19}$$

where the expectation is taken with respect to the distribution of χ and k. The right hand side reflects the fact that, when an individual with wealth w, inheritance rate χ, and with a number of children k dies, her children enter the top percentile as long as $w \geq \frac{\chi}{k}q$.

The birth term increases in the inheritance rate χ and in the number of children k for top individuals (in the sense of first-order stochastic dominance). Of course, doubling the number of children does not decrease the birth term as much as halving the inheritance rate: when k increases, individual children have less wealth, but more children are born in the top.

The birth term increases after a mean preserving spread in the inheritance rate χ or in the distribution of children k. This convexity comes from the fact that only children that inherit enough enter the top percentile.[9]

Finally, with this particular inheritance structure, the birth term is always lower or equal than the death term. Intuitively, inheritance never completely compensates for death. In particular, this implies that the demography term systematically decreases with the death rate δ_t.

Long-run. As above, I now examine the decomposition of the top share in the long-run, when the wealth distribution is stationary. Assume that $\delta_t \to \delta$ and $\eta_t \to \eta$. As in the case of the diffusion process, assume that $\mu_t(w) \to \mu(w)$ and $\nu_t(w) \to \nu(w)$ as $t \to +\infty$, uniformly in w, with $\mu(w) \to \mu < \delta + \eta$, and $\nu(w) \to \nu$ as $w \to +\infty$. Under these assumptions, the wealth distribution converges to a stationary distribution with a Pareto tail, i.e. $g(w) \sim Cw^{-\zeta}$, with $\zeta > 1$, i.e. the stationary wealth distribution has finite mean.

Plugging this expression for the stationary distribution in (18) gives the following balance

number of children on the Pareto exponent of the wealth distribution.

[8] However, this average is taken with respect to the whole population, and it does not necessarily reflect the average number of children of individuals in top percentiles.

[9] The functions $\chi \to \int_{\frac{q_t}{\chi/k}}^{+\infty} k\left(\frac{\chi}{k}w - q_t\right)g_t(w)\,\mathrm{d}w$ and $k \to \int_{\frac{q_t}{\chi/k}}^{+\infty} k\left(\frac{\chi}{k}w - q_t\right)g_t(w)\,\mathrm{d}w$ are both convex.

equation for the top share:[10]

$$0 = \underbrace{\mu\,\mathrm{d}t}_{\text{Within}} + \underbrace{\frac{\zeta-1}{2}\nu^2\,\mathrm{d}t}_{\text{Displacement}} \underbrace{\underbrace{-\frac{1}{\zeta}\delta\,\mathrm{d}t}_{\text{Death}} \underbrace{-\frac{1}{\zeta}\eta\,\mathrm{d}t}_{\text{Pop. Growth}} \underbrace{+\frac{1}{\zeta}\mathrm{E}\left[k^{1-\zeta}\chi^{\zeta}\right]\delta\,\mathrm{d}t}_{\text{Birth}}}_{\text{Demography}}. \tag{20}$$

The demography term only depends on the Pareto exponent of the wealth distribution, ζ. This reflects the fact that the Pareto distribution is "scale-free". More precisely, as explained in the case of displacement, the wealth of households at a percentile threshold, relative to the mass of individuals above the percentile threshold is constant in the right tail, i.e. $\frac{q_t p}{S_t} \to \frac{\zeta-1}{\zeta}$. Therefore, this implies that the terms due to death and population growth are constant in the right tail, i.e. $\left(\frac{q_t p}{S_t} - 1\right)\delta_t \to -\frac{\delta}{\zeta}$ and $\left(\frac{q_t p}{S_t} - 1\right)\eta_t \to -\frac{\eta}{\zeta}$.

As explained earlier, in the case of displacement, Both the death and population growth terms increase with ζ (i.e. as wealth inequality decreases). Intuitively, when wealth inequality is high, the ratio between the wealth of individuals at the percentile threshold and the average wealth of individuals in the top percentile is low, which tends to decrease the death and population growth term.

On the other hand, the birth term decreases with ζ. Intuitively, in a world in which wealth inequality is high, individuals that die tend to be far away from the percentile threshold, and, therefore, their children are more likely to enter the top.

4 Decomposing the Growth of Billionaires' Shares

In this section, I apply the accounting framework to examine the growth of the wealth share of the Forbes 400 list. I present the Forbes 400 data in Section 4.1. I discuss the results of the decomposition in Section 4.2, and I discuss measurement error in Section 4.3. I also decompose the growth of billionaires in China and Russia in Section 4.4.

4.1 Data

I focus on the list of the wealthiest 400 American households constructed by Forbes Magazine annually since 1983. The list is created by a dedicated staff of the magazine, based on a mix

[10] Note that the demography term can be rewritten as $\frac{1}{\zeta}\mathrm{E}\left[\frac{k}{\mathrm{E}[k]}\left(\frac{\chi}{k}\right)^{\zeta}\right](\delta+\eta)\,\mathrm{d}t$, using the fact that $\mathrm{E}[k]\delta = \delta + \eta$, as mentioned above.

of public and private information.[11] Because Forbes nominatively identifies the 400 wealthiest individuals in the U.S, one can track the wealth of the same individuals over time, which is key for the accounting decomposition.[12] By contrast, other data sources used to track the level of wealth inequality in the U.S. rely on repeated cross-sections.[13]

The Forbes 400 list includes 1,518 distinct households between 1983 and 2017. I remove from the list any households that were themselves later removed due to methodological errors (73 households).

I focus on the percentile that includes the entirety of households in the Forbes 400 in 2017. Because a percentile includes a constant fraction of total population, this only includes 264 households in 1983. To obtain the wealth share of this percentile, I divide the wealth of households in the top percentile by the aggregate wealth of U.S. households from the Financial Accounts (Flow of Funds). While this top percentile accounts for a small percentage of the total U.S. population (3% of the top 0.01%), it accounts for a substantial share of total U.S. wealth (approximately 3% in 2017).

Figure 2 plots the cumulative growth of the share of wealth owned by this top percentile since 1983, as well as the cumulative growth of the wealth share of the top 0.01%, 0.01%, 1%, and 10% from Saez and Zucman (2016). As they noted, most of the increase of top wealth inequality during the period is concentrated in the top 0.01%. Moreover, the rise in the Forbes 400 wealth share tracks very well the rise in the wealth share of the top 0.01%. This suggests that understanding the wealth growth of the Forbes 400 can shed light on the overall rise in top wealth inequality in the U.S. during this period.

4.2 Results

Summary I apply the accounting decomposition defined in Proposition 1 every year from 1983 to 2017. I include as death any exit from the percentile due to wealth reallocation within the family,

[11] Forbes Magazine reports: "We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don't pretend to know what is listed on each billionaire's private balance sheet, although some candidates do provide paperwork to that effect."

[12] I extend the construction from Capehart (2014) for the last five years. In Appendix B.1, I describe how I obtain the wealth of individuals that exit the top percentile.

[13] The three main datasets on the wealth distribution in the U.S. are the Survey of Consumer Finances, Estate Tax Returns (see Kopczuk and Saez (2004)) and Income Tax Returns (see Saez and Zucman (2016)), which all correspond to repeated cross-sections.

Table 1: Decomposition

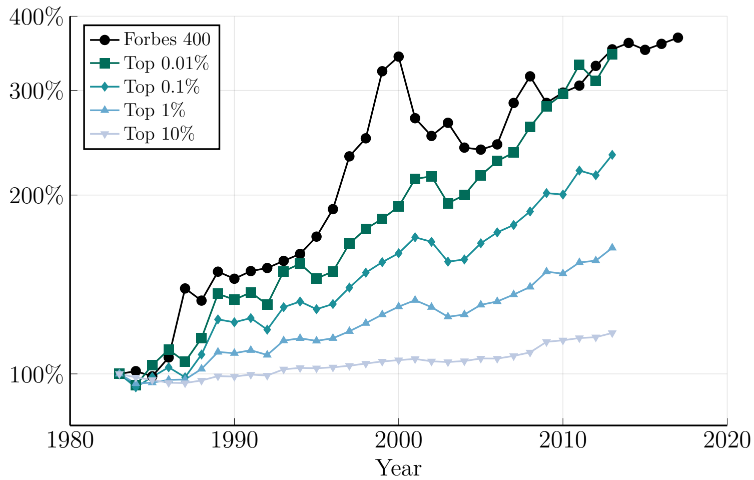
(a) Summary

	Total (%)	Within (%)			Displacement (%)			Demography (%)			
		Total	Top	-Per Capita	**Total**	Entry	Exit	**Total**	Death	Pop. Growth	Birth
All Years	3.9	3.0	5.6	−2.5	2.5	1.9	0.6	−1.5	−1.1	−0.8	0.4
1983-1994	4.3	2.5	4.7	−2.0	3.3	2.4	0.9	−1.4	−1.0	−0.7	0.4
1994-2005	3.7	2.9	7.2	−4.0	2.7	2.2	0.5	−1.8	−1.2	−1.0	0.4
2005-2017	3.7	3.6	5.1	−1.4	1.5	1.2	0.4	−1.4	−1.1	−0.7	0.4

(b) Details

	Displacement				Demography					
	Entry		Exit		Death		Pop. Growth		Birth	
	s_E (%)	$\frac{\overline{w}_{E,2}-q_2}{\overline{w}_{T,1}}$	s_X (%)	$\frac{q_2-\overline{w}_{X,2}}{\overline{w}_{T,1}}$	s_D (%)	$\frac{q_2-\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{D,1}}{\overline{w}_{T,1}}$	$1-s_T$ (%)	$\frac{q_2-\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{T,1}}{\overline{w}_{T,1}}$	s_B (%)	$\frac{\overline{w}_{B,2}-q_2}{\overline{w}_{T,1}}$
All Years	12.3	0.15	9.7	0.06	2.2	−0.51	1.2	−0.68	0.8	0.66
1983-1994	15.4	0.15	12.5	0.07	2.7	−0.38	1.3	−0.59	1.2	0.76
1994-2005	12.8	0.16	10.2	0.05	2.1	−0.57	1.4	−0.72	0.9	0.42
2005-2017	9.0	0.13	6.8	0.05	1.8	−0.59	0.9	−0.73	0.5	0.77

Figure 2: Growth of the Forbes 400 and Top 1%, 0.1%, 0.01% Wealth Share



Notes. The figure plots the cumulative growth of top wealth shares for groups defined in the top Forbes percentile, which includes 400 households in 2017. Data for the top 10%, 1%, 0.1%, 0.01% is from Saez and Zucman (2016).

and I classify as birth any entry in the top due to inheritance.

The first line of Table 1 reports each term geometrically averaged over the entire time period. I find that the 3.9% yearly growth of the top wealth share during the time period can be decomposed into a within term equal to 3.0%, a displacement term equal to 2.5%, and a demography term equal to -1.5%. The demography term is itself the sum of a death term (-1.1%), a population growth term (-0.8%), and a birth term (0.4%). Put differently, if there had not been any dispersion in the wealth growth among households, the growth of top shares would have been 1.4%, not 3.9%.

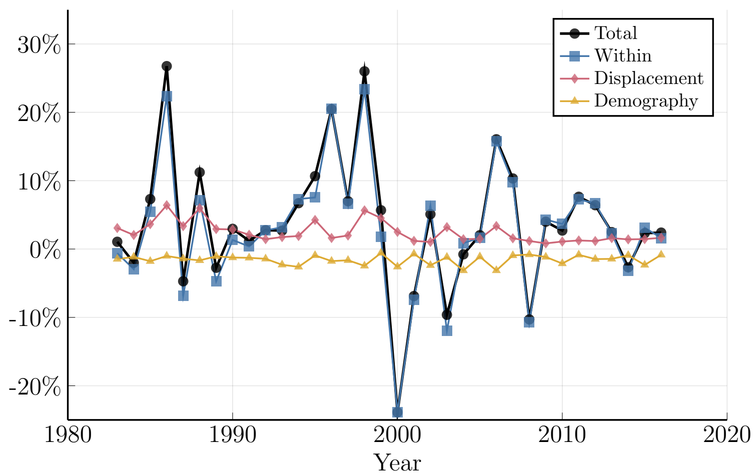
Figure 3 plots the result of the accounting decomposition every year. Yearly fluctuations in the top share are almost entirely driven by fluctuations in the within term. However, while the within term can be positive or negative, the displacement term is always positive. This is why the contribution of the displacement term becomes quantitatively more important at longer frequency.

To examine low-frequency changes in the decomposition since 1983, Table 1 also reports the terms averaged across three time periods of equal duration since 1983. The first period covers 1983-1994, which includes the 1990-1991 recession. The second period covers 1994-2005, which includes the 2001 recession. The third period covers 2005-2017, which includes the 2007-2009 recession.

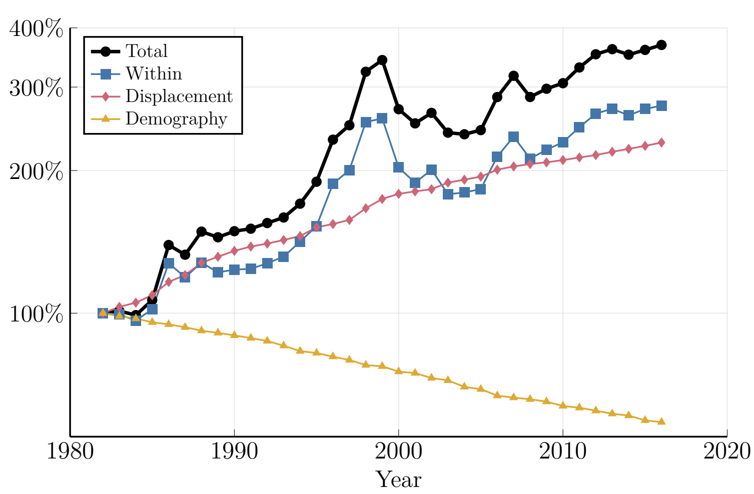
The displacement term has been gradually decreasing over time: it goes from 3.3% in the first part of the sample (1983-1994), to 2.7% in the second part of the sample (1994-2005), and finally to 1.5% in the third part of the sample (2005-2017). In the appendix, Table A2 formally shows that this decline is statistically significant. In the rest of the section, I focus on understanding the level

Figure 3: Decomposing the Growth of the Forbes 400 Wealth Share

(a) Yearly



(b) Cumulative



Notes. The figure plots the result of the accounting decomposition. Figure 3a plots the within, displacement, and demography terms every year, while Figure 3b plots the cumulative sum of of log terms over time. Data from Forbes.

and trend of displacement. I refer the reader to Appendix B for an analysis of the within term.

Effect of Variance. How does displacement relate the the statistical properties of individual wealth? In the case where wealth follows a diffusion, Section 3 suggests that the instantaneous displacement term should be close to $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$, where $\mathrm{sd}_t(q_t)$ denotes the yearly standard-deviation of log wealth growth at the percentile threshold. I now examine whether this simple formula approximate accurately the actual displacement term.

I estimate $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$ every year using local polynomial regression techniques. The ratio $g_t(q_t)q_t^2/S_t$ is estimated by regressing every year log top shares on log wealth around the percentile threshold. This estimation method relies on the fact that $\frac{g_t(q_t)q_t^2}{S_t} = \partial_{\ln w} \ln \left(\int_q^{+\infty} w g_t(w)\,\mathrm{d}w\right)|_{w=q_t}$.[14] Figure 4a plots the results. Overall, I obtain an average $\frac{g_t(q_t)q_t^2}{S_t} \approx 0.47$. If the distribution had a Pareto tail, this would correspond to a Pareto exponent $\zeta = 1 + \frac{g_t(q_t)q_t^2}{S_t} \approx 1.47$, which is consistent with existing studies.[15]

To estimate the yearly standard deviation of log wealth growth $\mathrm{sd}_t(q_t)$, I regress every year log wealth growth and its square on log wealth around the percentile threshold.[16] Combining the local estimates for the first and second moments gives an estimate for the yearly standard deviation at the percentile threshold. Figure 4a plots the results. Overall, I obtain an average standard deviation equal to 0.27. Figure 4b compares the model-implied term $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$ to the actual displacement term. The model-implied term tracks the dynamics of the displacement term well. Note that, even if individual wealth follows the simple diffusive model of (8), we should not expect the two terms to be equal, due to the fact that the decomposition is done in discrete time.[17] On average, the model-implied displacement term is equal to 1.7%, which tends to be lower than the the actual displacement term, 2.5%.

Effect of Skewness and Kurtosis. What is the role of higher-order cumulants for displacement? As shown in Section 3, when the process for wealth has jumps, the displacement term depends on all higher-order cumulants of wealth growth, as well as higher-order derivatives of wealth density and cumulants. To simplify the analysis, I only estimate the contributions of the third and fourth

[14] I thank an anonymous referee for the suggestion. I use a kernel with a bandwidth of one around the percentile threshold. Cattaneo et al. (2019) presents a similar method to estimate the density at a point using a local regression of the empirical CDF around the point.

[15] See, for instance, Klass et al. (2006)

[16] I use a local polynomial of degree one, with a triangular kernel with a bandwidth of one.

[17] Another reason is that there is only a finite number of agents and that the wealth density, as well as the standard deviation of wealth growth, are only estimated.

cumulants. I assume that their derivatives at the percentile threshold are zero, and that the wealth distribution is locally Pareto, i.e. that the displacement can be approximated by the following expression:

$$\frac{\zeta_t - 1}{2}\mathrm{sd}_t^2 \,\mathrm{d}t + \frac{\zeta_t^2 - 1}{3!} \cdot \text{skewness}_t \cdot \mathrm{sd}_t^3 \,\mathrm{d}t + \frac{\zeta_t^3 - 1}{4!} \cdot \text{excess kurtosis}_t \cdot \mathrm{sd}_t^4 \,\mathrm{d}t, \tag{21}$$

where sd_t is the standard deviation at the percentile threshold estimated above and ζ_t is one plus the estimated $\frac{g_t(q_t)q_t^2}{S_t}$, so that the first term coincides with the diffusive term estimated above.

I use local polynomial regressions to estimate the third and fourth moments of log wealth growth at the percentile threshold. As reported in Table 2a, the average estimated skewness is negative at -0.29 (i.e. more downward realizations compared to the log-normal distribution), while the average excess kurtosis is positive at 6.7 (i.e. more extreme realizations compared to the log-normal distribution). Plugging these estimates into Equation (21), this gives that skewness decreases displacement by -0.2%, while kurtosis decreases displacement by 0.4% annually (see Table 2b). The effect of higher-order cumulants on the displacement term tend to be small. As seen in Section 3.1, the intuition is that the wealth inequality is so high that most of the entry in the top percentile is driven by households already close to the percentile threshold, rather than by those far from the threshold with extreme wealth realization.

As shown in Figure 4b, adding skewness and kurtosis does not substantially change the dynamics of the model-implied displacement term. One interesting year, however, is 2000. In that year, the term due to variance drastically overestimates the actual displacement term, while the term incorporating skewness and kurtosis is much closer. This is because the burst of the dot-com bubble was associated with a large variance of wealth growth, but also a large negative skewness, which limited the effect of variance on the growth of the top share.

What Drives Displacement? I now use the analytical framework to understand better the drivers of the displacement term. I first use the analytical framework to examine the decline of the displacement term over time. The decline of the model-implied displacement between 1983-1994 and 2005-2017 can be written as the sum of a term due to the decline in the standard deviation of log wealth growth, and a term due to the spreading out of the wealth distribution:

$$\underbrace{\Delta\left(\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2\right)}_{-1.2\%} \approx \underbrace{\frac{1}{2}\overline{\frac{g_t(q_t)q_t^2}{S_t}}\Delta\left(\mathrm{sd}_t(q_t)^2\right)}_{-0.4\%} + \underbrace{\frac{1}{2}\overline{\mathrm{sd}_t(q_t)^2}\Delta\left(\frac{g_t(q_t)q_t^2}{S_t}\right)}_{-0.8\%}. \tag{22}$$

As reported in Table 2b, this decomposition says that a third of the decline is driven by the decrease in the dispersion of wealth shocks, while the remaining two-thirds is driven by an increase in top

Table 2: Displacement and Higher-Order Cumulants

(a) Statistics

	$\frac{g_t(q_t)q_t^2}{S_t}$	Standard Deviation	Skewness	Excess Kurtorsis
All Years	0.47	0.27	−0.29	6.66
1983-1994	0.64	0.27	−0.40	6.94
1994-2005	0.38	0.31	−0.38	8.28
2005-2017	0.39	0.23	−0.12	4.93

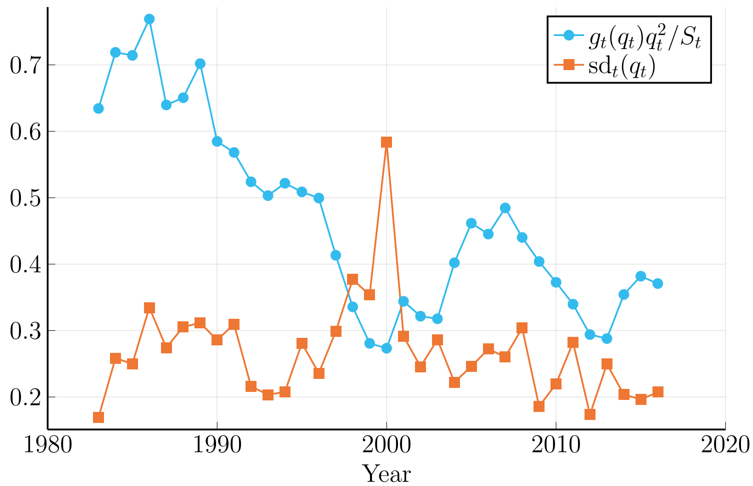
(b) Decomposition

	Displacement (%)	Model-Implied Term (%)			
		Total	Due to sd	Due to skew.	Due to kurt.
All Years	2.5	2.0	1.7	−0.2	0.4
1983-1994	3.3	2.8	2.4	−0.2	0.6
1994-2005	2.7	2.0	1.8	−0.5	0.6
2005-2017	1.5	1.3	1.1	0.0	0.1

Notes. Table 2a reports summary statistics on the shape of the wealth distribution as well as cumulants of log wealth growth during the time period. Table 2b reports the geometric average of the displacement term, as well as the average displacement term due to variance, skewness, and kurtosis.

Figure 4: Displacement and Higher-Order Cumulants

(a) Time series of $g_t(q_t)q_t^2/S_t$ and $\mathrm{sd}_t(q_t)$



(b) Effect of Variance, Skewness, and Kurtosis

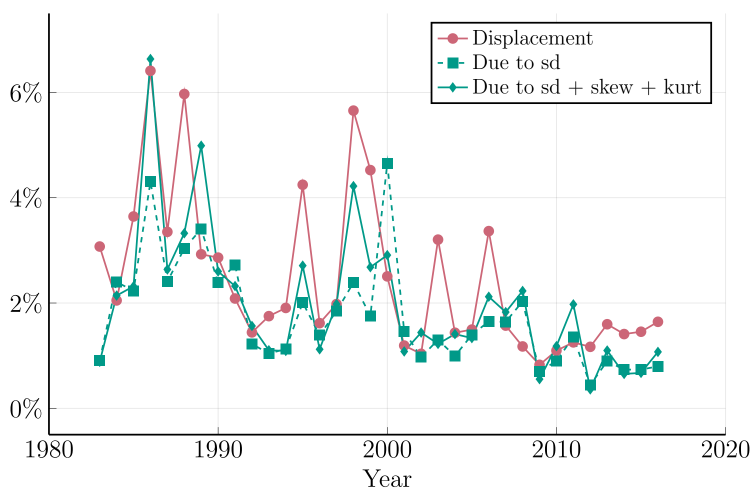


Figure 4a plots the time series of $g_t(q_t)q_t^2/S_t$ and the standard deviation of (log) wealth growth $\mathrm{sd}_t(q_t)$. The product of the term equals the model-implied term $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$.

Figure 4b plots the displacement term, as well as the model-implied term due to a diffusion $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$, as well as the model-implied term including jumps (21). Data from Forbes wealth inequality. Intuitively, the fact that the wealth distribution became more spread out made it harder for households with high growth rates to enter the top percentile.

I then examine what drives the standard deviation of log wealth growth. One simple idea is that it is driven by the standard deviation of their portfolio returns: most of households in the list tend to own the majority of their wealth in a few firms. To test this idea, I regress the variance of household-level wealth growth on the equal weighted variance of firm-level returns, computed using CRSp. The result is reported in Table 3. The $R^2 \approx 0.72$ is high, which suggests that the dynamics of the variance of log firm returns explains very well the dynamics of the variance of log wealth growth.

If each household in top percentiles invests in n uncorrelated firms, we expect the variance of their growth rate to be equal to the variance of firm-level returns divided by n. The estimate for the slope is 0.31, which implies an average number of distinct firms $n = 3$.[18] Finally, the estimate for the intercept is close to zero, which means that the number n, identified purely from time-series variation, also accounts for the level of the variance of wealth. This suggests that the variance of wealth growth is almost entirely driven by the variance of firm-level returns.

Table 3: Variance of Wealth Growth Correlates with Variance of Firm-Level Returns

	Variance of Log Wealth Growth
	(1)
Variance of Log Firm Returns	0.31***
	(0.05)
Constant	−0.02
	(0.01)
R^2	0.72
Period	1983-2016
N	34

Notes. The table reports the results of the regression of the cross-sectional variance of wealth growth for households at the top percentile ν on the cross-sectional variance of firm-level returns. Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,**,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively. Data from Forbes and CRSP.

[18] Alternatively, it could also reflect the fact that only a fraction of households in the top percentile have undiversified portfolios.

4.3 Robustness

The wealth of individuals at the top is inevitably measured with errors. This may lead to errors in the measure of the within and displacement term, as measured in the accounting decomposition Proposition 1.

The first concern is that Forbes magazine may systematically underestimate or overestimate the wealth of top 400 households. Along these lines, Atkinson (2008) argues the magazine may give inflated values of the wealth of top households, because debts are harder to track than assets. Empirically, Raub et al. (2010) document that the wealth of deceased households reported on estate tax returns is approximately half of the wealth estimated by Forbes. However, this measurement error in level does not impact the growth of top wealth shares.

A more serious concern is that Forbes measures the wealth of top households with noise. If the measurement error is completely persistent, as noted in Luttmer (2002), this leads Forbes to overestimate the level of top wealth shares, without affecting the growth of top wealth shares or the accounting decomposition. If, however, the measurement error has a transitory component, it may generate artificial entry and exit in the top percentile. While this would not change the growth of top wealth shares, this would lead me to underestimate the within term and overestimate the displacement term.

I deal with this potential bias in three ways. First, as explained above, I remove every household that was later removed by Forbes due to methodological error. Second, I check that wealth growth is close to a random walk in the remaining sample. Economic theory suggests that wealth should be close to a random walk at large levels of wealth (see, e.g., Achdou et al. (2016)). Indeed, Table 4 reports that the autocorrelation of wealth growth at the individual level is close to zero, which suggests that there is little mean-reversion in wealth growth.[19]

A final concern is that Forbes coverage may become more and more precise over time, and, therefore, that the magazine gradually discovers rich households that were not reported earlier. This would lead me to overestimate the displacement term as well as the growth of top wealth shares. First, to mitigate this effect, I start my decomposition in 1983, even though the Forbes 400 list started one year earlier, in 1982. Second, the fact that the model-implied displacement term, which only uses the distribution of wealth growth among existing households, is close to the actual

[19] In Appendix B.2, I show that the relative bias in the displacement term is well approximated by -2ρ, where ρ is the AR(1) coefficient of wealth growth. With an estimated $\rho \approx -0.01$, this suggests that transitory measurement error accounts for only 4 basis points in the displacement term.

displacement term, suggests that it is not an important driver of the top share.[20]

Table 4: Serial Correlation of Wealth Growth

	Log Wealth Growth
	(1)
Lagged Log Wealth Growth	−0.01
	(0.01)
Constant	0.05***
	(0.00)
R^2	0.20
Period	1983-2016
FE	Individual
N	11,392

Notes. The table reports the result of a regression of future wealth growth on current wealth growth, i.e. denoting w_{it} the wealth of household i at time t,

$$\log\left(\frac{w_{it+2}}{w_{it+1}}\right) = \alpha_i + \beta \log\left(\frac{w_{it+1}}{w_{it}}\right) + \epsilon.$$

Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,**,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively. Data from Forbes.

4.4 International Evidence

I now compare the accounting decomposition obtained for the U.S. with other countries. This is possible because Forbes also publishes a list of international billionaires starting in 1987.

One difficulty, however, is that the number of billionaires in each country tends to be much smaller than in the U.S. Therefore, I restrict myself to Russia and China, the two countries with the highest count of billionaires in 2010 outside the U.S.[21] I consider the wealth share of the percentile composed of 50 billionaires in 2010. I restrict myself to the 2010-2018 period, since China counts less than 50 billionaires before 2010. The evolution of wealth inequality in China and Russia has been discussed in Novokmet et al. (2018) and Piketty et al. (2019). Another concern is data quality may be lower for these two countries compared to the U.S. To handle this difficulty,

[20]Formally, suppose that some fortunes at the top are randomly hidden, and that Forbes discovers them with hazard rate θ. In this case, when wealth follows a diffusion (8), the measured displacement term would become $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \theta\,\mathrm{d}t$, which is higher than $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)_t^2\,\mathrm{d}t$.

[21]For the sake of the exercise, I group together China, Hong Kong, and Taiwan.

I manually check that each new Chinese or Russian billionaire can be traced back to a particular event, such as a successful IPO or a high stock market return, otherwise, I remove the household from the sample. Data on total household wealth in Russia and China is taken from the World Wealth and Income Database.

Table 5 gives the result of the accounting decomposition for these two countries, as well as the results of the U.S. for the same time period. Wealth inequality increased in both countries: the yearly growth rate of the top share is 4.2% in China, and 5.6% in Russia. Figure 5a plots the average within, displacement, and demography term in each country over time, as well as their standard errors. The average within term has a large standard error, which reflects the short time window for the decomposition, as well as the small number of households in each group. In contrast, the displacement term is much more precisely estimated. In the rest of the analysis, I focus on comparing the displacement term across countries.

The displacement term averages to 4.3% in China, which is much higher than the average in the U.S. during the time period, which is 1.4% (a difference of 2.9%). To understand why, I estimate a model-implied displacement term $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$ in China, using the same methodology as the U.S. (see Figure 5b), which averages to 3.7%. As shown in Figure 5b, the high level of displacement in China reflects the fact that the standard deviation of log wealth growth among top households is is much higher in China (34%), compared to the U.S (20%), during the time period. Another reason displacement is much higher in China is that wealth inequality there is much lower to begin with (i.e. the implied Pareto exponent is $\zeta = 1 + \frac{g_t(q_t)q_t^2}{S_t} \approx 1.6$ in China, compared to 1.35 in the U.S.). Therefore, it is much easier for households with high realized growth rates to displace existing fortunes at the top.

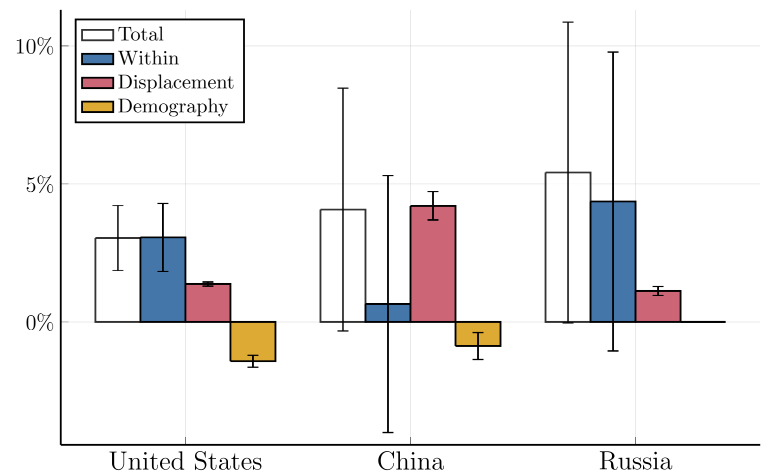
I now turn to Russia. The displacement term averages to 1.1% in Russia, which is a bit lower than the average displacement term in the U.S. during the time period, which is 1.4%. As shown in Figure 5b, the main reason is that the existing concentration of wealth in Russia is much higher than in the U.S., which makes it harder for households with high realized growth rates to displace existing fortunes.

5 Decomposing the Growth of Top Wealth Shares

The previous section showed that, in the case of the Forbes 400 list, the model-implied displacement term $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$ approximates well the actual displacement term. This suggests that one can potentially use a model-implied displacement term to estimate the actual displacement term for

Figure 5: Decomposing the Growth of Billionaires' Shares (2010-2018)

(a) Within, Displacement, and Demography



(b) Time series of $g_t(q_t)q_t^2/S_t$ and $\mathrm{sd}_t(q_t)$

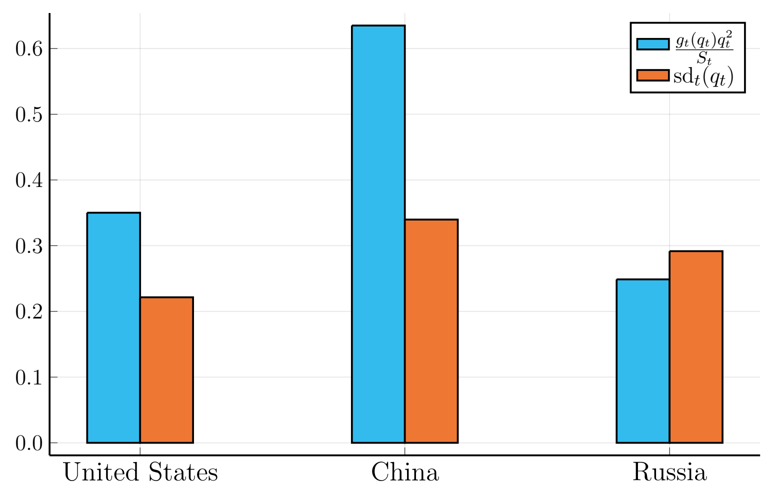


Table 5: Decomposing the Growth of Billionaires' Shares (2010-2018)

(a) Summary

	Total (%)	Within (%)			Displacement (%)			Demography (%)			
		Total	Top	-Per Capita	**Total**	Entry	Exit	**Total**	Death	Pop. Growth	Birth
United States	3.1	3.1	9.7	−6.0	1.4	1.1	0.2	−1.4	−0.9	−0.8	0.3
China	4.2	0.6	11.7	−9.9	4.3	2.9	1.4	−0.9	−0.7	−0.3	0.1
Russia	5.6	4.5	3.1	1.3	1.1	0.6	0.5	0.0	0.0	0.0	0.0

(b) Details

	Displacement				Demography					
	Entry		Exit		Death		Pop. Growth		Birth	
	s_E (%)	$\frac{\overline{w}_{E,2}-q_2}{\overline{w}_{T,1}}$	s_X (%)	$\frac{q_2-\overline{w}_{X,2}}{\overline{w}_{T,1}}$	s_D (%)	$\frac{q_2-\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{D,1}}{\overline{w}_{T,1}}$	$1-s_T$ (%)	$\frac{q_2-\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{T,1}}{\overline{w}_{T,1}}$	s_B (%)	$\frac{\overline{w}_{B,2}-q_2}{\overline{w}_{T,1}}$
United States	8.2	0.14	5.9	0.04	1.6	−0.56	1.0	−0.74	0.3	0.77
China	17.8	0.17	16.6	0.09	1.2	−0.54	0.5	−0.54	0.5	0.22
Russia	10.7	0.05	10.7	0.05	0.0		0.0	−0.78	0.0	

the Top 1%, 0.1%, 0.01% of the wealth distribution over the 20th century, for which we do not have panel data.

Methodology. In this section, I estimate the model-implied displacement term $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$ for the top $1\%, 0.1\%$, and 0.01% percentiles from 1916 to 2012.

I first estimate the shape of the wealth distribution $g_t(q_t)q_t^2/S_t$ at top percentiles $1\%, 0.1\%$, and 0.01% using cross-sectional data on the wealth distribution available from Kopczuk and Saez (2004) for 1916-1962, and Saez and Zucman (2016) for 1962-2012. More precisely, I estimate this ratio by comparing the change in the log top share with the change in the log top quantile between two percentiles.[22] Table 6 reports the estimated $g_t(q_t)q_t^2/S_t$ for top percentiles. The estimated $g_t(q_t)q_t^2/S_t$ does not change much across top percentiles, which reflects the fact that the wealth distribution is close to Pareto, at least above the top 1%.

I estimate the yearly standard deviation of wealth growth at each percentile threshold $\mathrm{sd}_t(q_t)$ by taking the product of the share of wealth invested in equity (using data from Kopczuk and Saez (2004) for 1916-1962, and Saez and Zucman (2016) for 1962-2012) and the cross sectional standard deviation of firm-level returns. This is motivated by the fact that, in the time series, the variance of log wealth growth correlates well with the variance of log returns, at least during the period 1983-2017 in the Forbes 400 (Table 3). I scale this product so that the standard deviation of the top 0.01% matches the standard deviation of the Forbes 400 in 1983-2012. Table 6 reports the estimated standard deviation for top percentiles. Over the time period, the standard deviation equals 14% for the Top 1%, and 21% for the top 0.01%. The fact that the standard deviation increases in the right tail of the distribution reflects the fact that top percentiles tend to invest more in equity.

Results. Figure 6a plots the model-implied displacement term $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2$ for the top 1%, the top 0.1%, and the top 0.01% percentiles from 1916 to 2012. The displacement term roughly follows a U-shape over time for all top percentiles. The displacement term for the top 0.01% peaked at 2% during the Great Depression, then steadily decreased, reaching its minimum in 1945. The displacement term again increased starting in 1960, and reached its maximum at the height of the dot-com bubble. Overall, the displacement term was roughly twice as high in 1983-2012 as it had been for the rest of the century.

To understand better what drives the displacement term over time, Figure 6b plots separately

[22] This is a discretized version of the local regression of log top shares on log quantiles done in the previous section.

the term due to the wealth distribution $g_t(q_t)q_t^2/S_t$ and the term due to the idiosyncratic variance of wealth $\mathrm{sd}_t(q_t)^2$ for the top 0.01%. Most fluctuations in the model-implied displacement term arise from fluctuations in the idiosyncratic variance of wealth rather than from fluctuations in the shape of the wealth distribution. These low-frequency fluctuations in the dispersion of stock market returns has been documented by Campbell et al. (2001) and Herskovic et al. (2016).[23] The fact that the wealth distribution moves more slowly is discussed in Gabaix et al. (2016).

According to Saez and Zucman (2016), the yearly growth rate of the wealth share of the top 0.01% in 1982-2012 averaged to 4.3%, while the yearly growth rate of the top 1% averaged to 1.9%, i.e. a difference of 2.4% per year. The results of Table 6 suggest that the differences in displacement between the two percentiles can explain a 1.3% differential. Moreover, this differential is driven by differences in the standard deviation of wealth growth across the two percentiles rather than differences in the shape of the wealth distribution.[24]

Table 6: Displacement over the 20th Century in the U.S.

	Top 1%	Top 0.1%	Top 0.01%	Top 400
Panel A: 1926-1982				
$\frac{g_t(q_t)q_t^2}{S_t}$	0.64	0.70	0.86	
$\mathrm{sd}_t(q_t)$	0.10	0.14	0.17	
Model-Implied Displacement (%)	0.4	0.7	1.3	
Panel B: 1983-2012				
$\frac{g_t(q_t)q_t^2}{S_t}$	0.54	0.56	0.58	0.47
$\mathrm{sd}_t(q_t)$	0.19	0.22	0.27	0.27
Model-Implied Displacement (%)	1.0	1.4	2.3	1.8

Notes. Data from from Kopczuk and Saez (2004) and Saez and Zucman (2016). The model-implied displacement term is $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t^2\,\mathrm{d}t$.

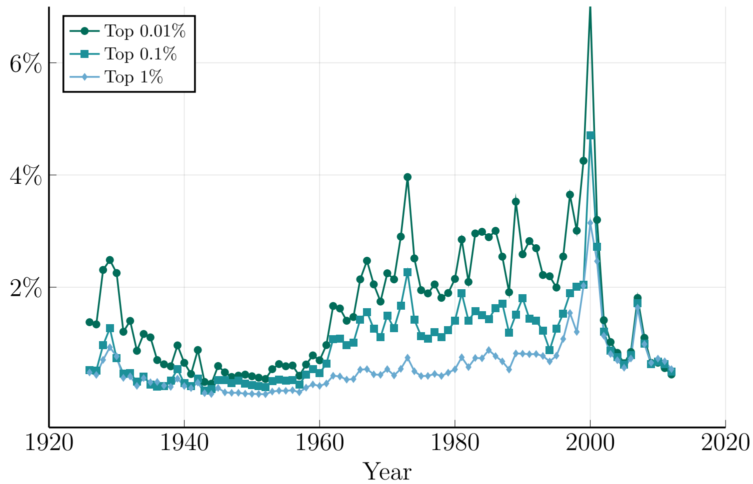
[23] Theories to explain these fluctuations have been discussed in Brandt et al. (2009), Fink et al. (2010), and Herskovic et al. (2018).

[24] Formally, we can write

$$\underbrace{\Delta\left(\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{sd}_t(q_t)^2\right)}_{1.3\%} \approx \underbrace{\frac{1}{2}\overline{\frac{g_t(q_t)q_t^2}{S_t}}\Delta\left(\mathrm{sd}_t(q_t)^2\right)}_{1.1\%} + \underbrace{\frac{1}{2}\overline{\mathrm{sd}_t(q_t)^2}\Delta\left(\frac{g_t(q_t)q_t^2}{S_t}\right)}_{0.1\%}.$$

Figure 6: Displacement over the 20th Century in the U.S.

(a) Model-Implied Displacement $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2$



(b) Time series of $g_t(q_t)q_t^2/S_t$ and $\mathrm{sd}_t(q_t)$

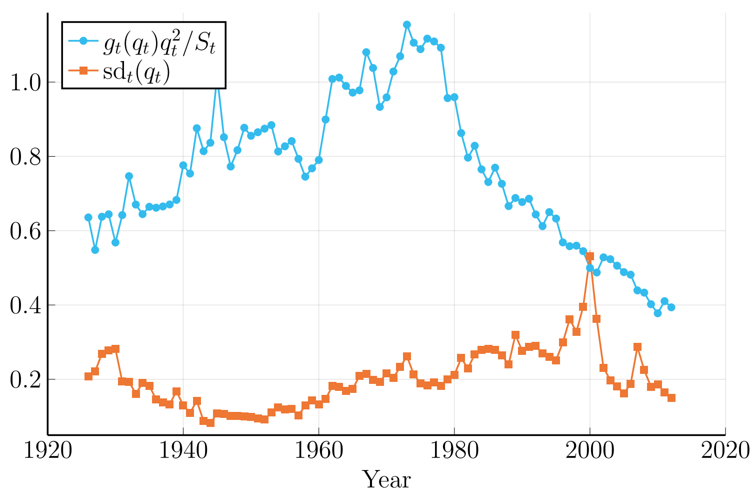


Figure 6b decomposes the model-implied displacement term into its two components, $g_t(q_t)q_t^2/S_t$ and $\mathrm{sd}_t(q_t)^2$. The product of the term equals the model-predicted term for a diffusive process. Data from Kopczuk and Saez (2004) and Saez and Zucman (2016).

6 Displacement and Mobility

The accounting decomposition can also be used to examine more normative questions, such as the relationship between wealth inequality and mobility. In this section, I show that whether a rise in wealth inequality is driven by a rise in the average wealth growth of households at the top (within term) or a rise in the dispersion of wealth shocks (displacement term) has opposite effects on mobility.

I focus on a downward concept of mobility: how long, on average, does a household remain in a top percentile? The advantage of this notion of "downward" mobility is that it only depends on the wealth dynamics of individuals in the right tail of the distribution.[25] Formally, for a household with wealth w, denote $T_q(w)$ the average time the household remains above the wealth threshold q (also called the "average first passage time"), i.e.

$$T_q(w) = \mathrm{E}[\inf\{\tau \text{ s.t. } w_{it+\tau} \leq q \text{ or } i \text{ dies }\}|w_{it} = w]. \tag{23}$$

For the remainder of this section, I assume that the law of motion of wealth is given by a diffusive process with constant drift and idiosyncratic volatility, i.e.

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu\,\mathrm{d}t + \nu\,\mathrm{d}B_{it}, \tag{24}$$

with death rate $\delta > 0$ and population growth rate $\eta > 0$. The next proposition gives a closed-form formula for the average time a household with initial wealth w remains above a wealth threshold q:

Proposition 5 (Average First Passage Time)**.** *When wealth follows the law of motion* (24)*, the average first passage time for $w \geq q$ is:*[26]

$$T_q(w) = \frac{1}{\delta}\left(1 - \left(\frac{w}{q}\right)^{-\xi}\right), \tag{25}$$

where $-\xi$ denotes the negative root of the quadratic equation $\zeta \to \zeta\mu + \frac{1}{2}\zeta(\zeta-1)\nu^2 - \delta$.

It increases with wealth w: it equals to 0 at the percentile threshold, and it converges to $1/\delta$ as wealth tends to infinity (which corresponds to the average time before death). The exponent ξ governs the speed at which the first passage time decays as wealth tends to infinity.

[25] In particular, compared to a notion of "upward" mobility, we can abstract from the role of labor income or government programs.

[26] The average first passage time of a Brownian Motion is a classic result, for instance see Karlin and Taylor (1981). This formula simply generalizes it to the case of a process with Brownian Motion with death probability.

I now examine comparative statistics of $T_q(w)$ at a given wealth level. Due to the change in ξ, the average first passage time $T_q(w)$ increases with the average wealth growth of individuals μ, while it decreases with the idiosyncratic volatility ν. Intuitively, the higher the dispersion of wealth shocks, the more likely it is to have a negative wealth shock, and therefore the more likely it is for wealth to drop below q.

While an increase in idiosyncratic volatility decreases the average first passage time at a given wealth level, it also increases the level of wealth inequality in the long-run. This may have a counterbalancing effect of mobility, by increasing the typical distance between a household in the top percentile and the percentile threshold. To determine the overall effect of idiosyncratic volatility on mobility, I consider the average first passage time for an average household in a top percentile p, $T(p)$:

$$T(p) = \mathrm{E}^g \left[T_q(w) | w \geq q(p) \right], \tag{26}$$

where $q(p)$ denotes the wealth at the lower threshold of the top percentile p

Proposition 6 (Average First Passage Time for an Average Household)**.** *Consider an economy in which individual wealth follows the process given in (8), with a reflecting barrier at some wealth level $\underline{w} < q(p)$. Then the average time someone in the top percentile p of the stationary wealth distribution remains there is:*

$$T(p) = \frac{1}{\delta} \frac{1}{1 + \frac{\zeta}{\xi}}, \tag{27}$$

where ζ denotes the Pareto exponent of the stationary wealth distribution and ξ is defined in Proposition 5.[27]

This formula characterizes in closed form the average passage time for a household in the top percentile p. One interesting property is that the average first passage time of an average household in the top percentile p, $T(p)$, does not depend on the top percentile p.

The average first passage time depends on the ratio between ζ and ξ. ξ controls the speed at which the first passage time decays as wealth tends to infinity, while ζ controls the speed at which the density of wealth decays as wealth tends to infinity. Both statistics matter to determine the first passage time for an average household in the top percentile.

As the average wealth growth of top households μ increases, T increases (i.e. mobility decreases). This is due to two reasons. First, the average first passage time at a given wealth level increases (ξ

[27] As seen in Section 3.3, it corresponds to the positive root of $\mu + \frac{1}{2}(\zeta - 1)\nu^2 - \frac{1}{\zeta}(\delta + \eta) = 0$.

increases). Second, in the long-run, the wealth distribution becomes more unequal, which increases the typical distance between a household in the top percentile and the percentile threshold (ζ decreases). These two forces combine to decrease mobility.

The effect of a rise in the dispersion of wealth shocks on mobility is more subtle. On the one hand, as ν increases, the average first passage time from a given wealth level decreases, which tends to increase mobility (ξ decreases). On the other, the wealth distribution becomes more unequal in the long-run, which increases the typical distance between a household in the top percentile and the percentile threshold (i.e. ζ decreases). However, for realistic parameters, this long-run effect on the wealth distribution is not strong enough to compensate the first force, and mobility increases.

I now use this formula to quantify the effect of the recent rise in top wealth inequality on mobility. Consider an initial (pre-1980) economy with the following parameters: the wealth growth of top households is $\mu = 1.5\%$, idiosyncratic volatility is $\nu = 10\%$, death rate is is $\delta = 2\%$, and population growth rate is $\eta = 1.5\%$.[28] Applying Proposition 6, we get that $\zeta \approx 1.8$ and $\xi \approx 3.2$, which gives that the average time a top household remains in a top percentile is $T \approx 32$ years.

Now, consider a permanent change in parameters matching the accounting decomposition for the Forbes 400, i.e. a permanent change in parameters to $\mu = 3\%$ and $\nu = 27\%$.[29] Applying Proposition 6, in the long-run, we get that $\zeta \approx 1.1$ and $\xi \approx 0.65$, which gives that the average time a top household remains in the top percentile is $T \approx 20$ years. Even though wealth inequality increases between these two states, wealth mobility *increases*. This mechanism may explain the empirical findings of Kopczuk et al. (2010), which find that, even though labor inequality increased at the end of the 20th century, labor mobility remained more or less constant.

7 Conclusion

This paper develops an accounting framework to decompose the growth in top shares into three terms: a within term, a displacement term, and a demography term. The last two terms capture the effect of changes in the composition of households in top percentiles. Empirically, displacement, i.e. the flow of households in an out top percentile, appears to be a key driver in the dynamics of top shares. I also provide an analytical framework to aid measurement of each term in absence of

[28] I pick the death rate, inheritance parameter and population growth to match the demography term of the Forbes 400 in 1983-2917 (see Table 1). I pick the idiosyncratic volatility to target the average idiosyncratic volatility for the top 0.01% in 1960-1980 from Section 5. Finally, I pick the drift μ so that the initial Pareto exponent of the stationary wealth distribution is 1.8.

[29] See Table 1.

panel data.

What do we learn from this paper? The existing literature on wealth inequality has been focused on factors related to the average growth rate of households in the top, such as their average return or their saving rate. This paper suggests, instead, to shift the focus towards factors that can explain the increase in the dispersion of their wealth growth.

Beyond my empirical findings, there is a number of interesting economic implications as well. One such implication relates to the set of tools that can be used to curb wealth inequality. The existing literature has focused on policies that target the average wealth growth of top households, such as capital income or wealth tax. It would be interesting to compare these to policies that would target instead the dispersion of their wealth growth, such as a tax on un-diversified capital, rather than all forms of capital.

The same methodology could be used to decompose the rise in concentration in other settings. Beyond wealth and top income shares (see Appendix D), another promising application would be to examine the distribution of firms' market shares (Autor et al. (2017), Hartman-Glaser et al. (2019), Gutiérrez and Philippon (2019)). I leave this topic for future research.

A Appendix for Section 3

A.1 Proofs

Proof of Proposition 2. We first show that there is a weak solution of the SDE with a smooth density. Consider the process for log-wealth $\omega_{it} = \ln w_{it}$, which solves the SDE:

$$\mathrm{d}\omega_{it} = \left(\mu_t\left(e^{\omega_{it}}\right) - \frac{1}{2}\nu_t\left(e^{\omega_{it}}\right)^2\right)dt + \nu_t\left(e^{\omega_{it}}\right)dB_{it}.$$

Using Rogers (1985), we know that the process has a weak solution with a transition density $p_t(\omega_0, \omega_t) \in \mathcal{C}^\infty$ in t, ω_0, ω_t, and is positive everywhere. The transition density satisfies the Kolmogorov-Forward equation. Due to the fact that $\mu_t(\exp(.))$ and $\nu(\exp(.))$ are bounded, there exists constants C, d such that, for all $\omega_0, \omega, t \in [0, T], k \geq 0$, we have:

$$|\partial_t p_t(\omega_0, \omega)| \leq \frac{C}{t^{3/2}} e^{-d\frac{(\omega-\omega_0)^2}{t}} \tag{A1}$$

$$|\partial_x^k p_t(\omega_0, \omega)| \leq \frac{C}{\sqrt{t^{1+k}}} e^{-d\frac{(\omega-\omega_0)^2}{t}}. \tag{A2}$$

see, e.g., Section 9.6 of Friedman (1964). The density of log-wealth at time $t = 0$, γ_0, is given by $\gamma_0(\omega) = e^\omega g_0(e^\omega)$. The density of log-wealth at time t, γ_t, is given by the convolution of γ_0 and p_t:

$$\gamma_t(\omega) = \int_{\mathbb{R}} \gamma_0(\omega_0) p_t(\omega_0, \omega)\, \mathrm{d}\omega_0$$

Since p_t is positive everywhere, γ_t is positive everywhere. Using the dominated convergence theorem, and the upper bound Equation (A2), we obtain $\gamma_t \in \mathcal{C}^\infty$.

Let us now prove that the distribution of wealth has finite mean. We have:

$$\begin{aligned}
\int_0^x e^\omega \gamma_t(\omega)\, \mathrm{d}\omega &\leq \int_0^x e^\omega \left(\int_{\mathbb{R}} \gamma_0(\omega_0) p_t(\omega_0, \omega)\, \mathrm{d}\omega_0\right) \mathrm{d}\omega \\
&\leq \int_{\mathbb{R}} \gamma_0(\omega_0) \left(\int_0^x e^\omega \frac{C}{\sqrt{t}} e^{-d\frac{(\omega-\omega_0)^2}{t}}\, \mathrm{d}\omega\right) \mathrm{d}\omega_0 \\
&\leq \left(\int_{\mathbb{R}} \gamma_0(\omega_0) e^{\omega_0}\, \mathrm{d}\omega_0\right) \left(\int_{\mathbb{R}} e^x \frac{C}{\sqrt{t}} e^{-d\frac{x^2}{t}}\, \mathrm{d}x\right).
\end{aligned} \tag{A3}$$

Therefore, $\int_{\mathbb{R}} e^\omega \gamma_t(\omega)\, \mathrm{d}\omega < +\infty$.

Finally, the density of wealth at time t, g_t, is given by $g_t(w) = \frac{1}{w}\gamma_t(\ln w)$. Therefore, $g_t \in \mathcal{C}^\infty$, and is positive everywhere.

Let us now relate the law of motion of S_t to the law of motion of g_t. The log-quantile $\chi_t = \log(q_t)$ is defined implicitly by:

$$p = \int_{\chi_t}^{+\infty} \gamma_t(\omega)\, \mathrm{d}\omega. \tag{A4}$$

Since γ_t is positive everywhere, the function $\chi \to \int_\chi^{+\infty} \gamma_t(\omega)\, \mathrm{d}\omega$ is strictly decreasing.

Applying the implicit function theorem, we obtain the time-derivative of the log quantile χ_t

$$0 = \int_{\chi_t}^{+\infty} \mathrm{d}\gamma_t(\omega)\,\mathrm{d}\omega - \gamma_t(\chi_t)\,\mathrm{d}\chi_t. \tag{A5}$$

Note that we exchanged the time-derivative and the integral sign. This is just an application of the dominated convergence theorem (an upper bound for the time-derivative of the transition density given in (A1))

Similarly, deriving the definition of top shares (9) with respect to time gives:

$$\mathrm{d}S_t = \int_{\chi_t}^{+\infty} e^{\omega}\,\mathrm{d}\gamma_t(\omega)\,\mathrm{d}\omega - e^{\chi_t}\gamma_t(\chi_t)\,\mathrm{d}\chi_t. \tag{A6}$$

Again, this follows from the dominated convergence theorem (using (A3)).

Combining (A6) and (A6) gives

$$\begin{aligned}\mathrm{d}S_t &= \int_{\chi_t}^{+\infty} (e^{\omega} - e^{\chi_t})\,\mathrm{d}\gamma_t(\omega)\,\mathrm{d}\omega \\ &= \int_{q_t}^{+\infty} (w - q_t) g_t(w)\,\mathrm{d}w. \end{aligned}\tag{A7}$$

Now, Kolmogorov-Forward equation gives the evolution of the wealth density

$$\mathrm{d}g_t = -\partial_w(\mu_t(w) w g_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w))\,\mathrm{d}t. \tag{A8}$$

Combining (A8) with (A6) gives

$$\mathrm{d}S_t = \int_{q_t}^{+\infty} (w - q_t)\left(-\partial_w(\mu_t(w) w g_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w))\right)\mathrm{d}w.$$

Integrating by parts, we obtain:

$$\begin{aligned}\mathrm{d}S_t &= [(w - q_t)\mu_t(w) w g_t(w)]_{q_t}^{+\infty} + \int_{q_t}^{+\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w\,\mathrm{d}t \\ &\quad - \frac{1}{2}[(w - q_t)\partial_w(\nu_t^2(w) w^2 g_t(w))]_{q_t}^{+\infty} + \frac{1}{2} g_t(q_t) q_t^2 \nu_t(q_t)^2\,\mathrm{d}t. \end{aligned}\tag{A9}$$

We have $\lim_{w\to+\infty} w^2 g_t(w) = 0$ and $\lim_{w\to+\infty} w\partial_w w^2 g_t(w) = 0$.[30] Because $\mu_t(.)$ and $\nu_t(.)$ are bounded, we get $\lim_{w\to+\infty} \mu_t(w) w^2 g_t(w) = 0$ and $\lim_{w\to+\infty} w\partial_w(\nu_t^2(w) w^2 g_t(w)) = 0$. Therefore, Equation (A9) can be simplified to:

$$\mathrm{d}S_t = \int_{q_t}^{+\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w\,\mathrm{d}t + \frac{1}{2} g_t(q_t) q_t^2 \nu_t(q_t)^2\,\mathrm{d}t. \tag{A10}$$

Dividing by S_t gives Equation (10).

I now prove that the derivatives, w.r.t. τ at $\tau = 0$, of the within and displacement terms from the accounting decomposition (7) correspond to the within and displacement term defined in (10).

[30]For any t, $\omega \to e^{\omega}\gamma_t(\omega)$ tends to zero since it is integrable. Moreover, it has a bounded second derivative, therefore, its first derivative also converges to zero.

The accounting decomposition (7) defines the within term between t and $t+\tau$ to be the average growth of households in the top percentile between t and $t+\tau$. When wealth follows (8), the within term is:

$$\frac{1}{S_t}\int_{\chi_t}^{+\infty}\left(\int_{\mathbb{R}} e^{\omega'} p_{t\to t+\tau}(\omega,\omega')\,\mathrm{d}\omega'\right)\gamma_t(\omega)\,\mathrm{d}\omega - 1. \tag{A11}$$

The derivative of this expression with respect to τ, taken at $\tau=0$, is $\frac{1}{S_t}\int_{q_t}^{+\infty}\mu_t(w)g_t(w)\,\mathrm{d}w$.

The derivative of the displacement term at $\tau=0$ equals the instantaneous growth in top wealth share minus the derivative of the within term at $\tau=0$. Using Equations (10) and (A11), we obtain that the derivative of the displacement term at $\tau=0$ is $\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}$. $\square$

Proof of Proposition 3. Denote f_U the density of jumps U. Kolmogorov Forward equation is

$$\begin{aligned}\mathrm{d}g_t &= -\partial_w(\mu_t(w)wg_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2\left(\nu_t^2(w)w^2g_t(w)\right)\mathrm{d}t\\ &+\left(\int_{\mathbb{R}+} f_U\left(\frac{\log(w/x)}{\phi_t(x)}\right)g_t(x)\,\mathrm{d}x - g_t(w) + \partial_w\left(E_U\left[e^{\phi_t(w)U}-1\right]wg_t(w)\right)\right)\lambda\,\mathrm{d}t.\end{aligned}$$

Plugging this into (A6) and integrating by parts the first two terms:

$$\begin{aligned}\mathrm{d}S_t &= \int_{q_t}^{+\infty}\mu(w)wg_t(w)\,\mathrm{d}w\,\mathrm{d}t + \frac{1}{2}g_t(q_t)q_t^2\nu_t(q_t)^2\,\mathrm{d}t\\ &+\int_{q_t}^{+\infty}(w-q_t)\left(\int_{\mathbb{R}+} f_U\left(\frac{\log(w/x)}{\phi_t(x)}\right)g_t(x)\,\mathrm{d}x - g_t(w) + \partial_w\left(\lambda E_U\left[e^{\phi_t(w)U}-1\right]wg_t(w)\right)\right)\mathrm{d}w\lambda\,\mathrm{d}t.\end{aligned}$$

Let us focus on the last term

$$\begin{aligned}&\int_{q_t}^{+\infty}(w-q_t)\left(\int_{\mathbb{R}+} f_U\left(\frac{\log(w/x)}{\phi_t(x)}\right)g_t(x)\,\mathrm{d}x - g_t(w) + \partial_w\left(\lambda E_U\left[e^{\phi_t(w)U}-1\right]wg_t(w)\right)\right)\mathrm{d}w\lambda\,\mathrm{d}t\\ &=\left(\int_{\mathbb{R}}\int_0^{\infty}(e^{\phi_t(w)U}w-q_t)^+ f_U(U)\,\mathrm{d}Ug(w)\,\mathrm{d}w\,\mathrm{d}U - \int_{q_t}^{+\infty}E_U\left[e^{\phi_t(w)U}-1\right]wg_t(w)\,\mathrm{d}w\right)\lambda\,\mathrm{d}t\\ &=E_U\left[\int_0^{\infty}(e^{\phi_t(w)U}w-q_t)^+g(w)\,\mathrm{d}w - \int_{q_t}^{+\infty}(e^{\phi(w)U}w-q_t)g_t(w)\,\mathrm{d}w\right]\lambda\,\mathrm{d}t\\ &=E\left[\int_0^{q_t}(e^{\phi_t(w)U}w-q_t)^+g(w)\,\mathrm{d}w + \int_{q_t}^{+\infty}(q_t-e^{\phi_t(w)U}w)^+g(w)\,\mathrm{d}w\right]\lambda\,\mathrm{d}t.\end{aligned}$$

This concludes the demonstration. $\square$

Proof of Corollary 1. Let us rewrite the displacement term in Proposition 3:

$$
\begin{aligned}
&\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + E\left[\int_0^{q_t}(e^{\phi_t(w)U}w-q_t)^+g(w)\,\mathrm{d}w + \int_{q_t}^{+\infty}(q_t-e^{\phi_t(w)U}w)^+g(w)\,\mathrm{d}w\right]\lambda\,\mathrm{d}t\\
&= \frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \left(\int_0^{q_t}\int_U(e^{\phi_t(w)U}w-q_t)^+g(w)f_U(U)\,\mathrm{d}U\,\mathrm{d}w\right.\\
&\qquad\qquad\left.+\int_{q_t}^{+\infty}\int_U(q_t-e^{\phi_t(w)U}w)^+g(w)f_U(U)\,\mathrm{d}U\,\mathrm{d}w\right)\lambda\,\mathrm{d}t\\
&= \frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \left(\int_0^{q_t}\int_J(e^Jw-q_t)^+\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)g(w)\,\mathrm{d}J\,\mathrm{d}w\right.\\
&\qquad\qquad\left.+\int_{q_t}^{+\infty}\int_J(q_t-e^Jw)^+\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)g(w)\,\mathrm{d}J\,\mathrm{d}w\right)\lambda\,\mathrm{d}t\\
&= \frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \left(\int_J\int_{q_te^{-J}}^{q_t}(e^Jw-q_t)g_t(w)\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)\mathrm{d}w\,\mathrm{d}J\right)\lambda\,\mathrm{d}t\\
&= \frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \left(\int_J\sum_{j\geq 2}\frac{1}{j!}\partial_v^j\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}J^j\,\mathrm{d}J\right)\lambda\,\mathrm{d}t\\
&= \frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \sum_{j\geq 2}\frac{1}{j!}\partial_v^j\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\left(\lambda\int_J J^j\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)\mathrm{d}J\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}\mathrm{d}t\\
&= \frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \sum_{j\geq 2}\frac{1}{j!}\partial_v^j\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\left(\lambda\int_U\phi_t(w)^jU^jf_U(U)\,\mathrm{d}U\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}\mathrm{d}t\\
&= \frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\nu_t(q_t)^2\,\mathrm{d}t + \sum_{j\geq 2}\frac{1}{j!}\partial_v^j\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\left(\lambda\phi_t(w)^jE[U^j]\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}\mathrm{d}t\\
&= \sum_{j\geq 2}\frac{1}{j!}\frac{1}{S_t}\partial_v^j\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\kappa_{jt}(w)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}\mathrm{d}t\\
&= \sum_{j\geq 2}\frac{1}{j!}\frac{1}{S_t}\partial_v^{k-1}\left(\int_{q_te^{-v}}^{q_t}e^vw\kappa_{jt}(w)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}\mathrm{d}t\\
&= \sum_{j\geq 2}\frac{1}{j!}\frac{q_t}{S_t}\sum_{0\leq l\leq j-2}\partial_v^l\left(q_te^{-v}g_t(q_te^{-v})\kappa_{jt}(q_te^{-v})\right)\Big|_{v=0}\mathrm{d}t\\
&= \sum_{j\geq 2}\frac{1}{j!}\frac{q_t}{S_t}\sum_{0\leq l\leq j-2}(-1)^l(w\partial_w)^l\left(\kappa_{jt}(w)wg_t(w)\right)\mathrm{d}t.
\end{aligned}
$$

□

Proof of Proposition 4. As in the proof of Proposition 2, denote ω_{it} the process defined by

$$
\mathrm{d}\omega_{it} = \left(\mu_t\left(e^{\omega_{it}}\right)-\frac{1}{2}\nu_t\left(e^{\omega_{it}}\right)^2\right)dt+\nu_t\left(e^{\omega_{it}}\right)dB_{it}.
$$

Denote $p_{t\to s}(\omega_t,\omega_s)$ the transition density between t and s. Denote $\gamma_0(\omega)=e^{\omega}g_0(e^{\omega})$ the density of log-wealth at $t=0$, and $\gamma_{Bt}(\omega)=e^{\omega}g_{Bt}(e^{\omega})$ the density of arriving agents at time t.

$$
\gamma_t(\omega)=\int_0^t(\delta_s+\eta_s)e^{-\int_s^t(\delta_u+\eta_u)\,\mathrm{d}u}\left(\int_{\mathbb{R}}\gamma_{Bs}(\omega')p_{s\to t}(\omega',\omega)\,\mathrm{d}\omega'\right)\mathrm{d}s+e^{-\int_0^t(\delta_u+\eta_u)\,\mathrm{d}u}\int_{\mathbb{R}}\gamma_0(\omega_0)p_{0\to t}(\omega_0,\omega)\,\mathrm{d}\omega_0.
$$

Similar arguments as in Proposition 2 ensure that the density γ_t is positive everywhere, infinitely differentiable, and such that $\int_{\mathbb{R}}e^{\omega}\gamma_t(\omega)\,\mathrm{d}\omega<+\infty$.

The derivative of the density with respect to time is

$$\partial_t \gamma_t(\omega) = \int_0^t (\delta_s + \eta_s) e^{-\int_s^t (\delta_u+\eta_u)\,\mathrm{d}u} \left(\int_{\mathbb{R}} \gamma_{Bt}(\omega') \partial_t p_{s\to t}(\omega', \omega)\,\mathrm{d}\omega' \right) \mathrm{d}s + (\delta_t + \eta_t)(\gamma_{Bt} - \gamma_t).$$

Using similar arguments as Proposition 2, the derivative is bounded by a function f such that $e^{\omega} f(\omega)$ is integrable, which ensures that we can invert the integral and derivative sign to obtain (A6):

$$\mathrm{d}S_t = \int_{q_t}^{+\infty} (w - q_t)\,\mathrm{d}g_t(w)\,\mathrm{d}w.$$

The Kolmogorov Forward Equation for g_t is

$$\mathrm{d}g_t = -\partial_w(\mu_t(w) w g_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w))\,\mathrm{d}t + (\delta_t + \eta_t)\,(g_{Bt} - g_t)\,\mathrm{d}t. \tag{A12}$$

Plugging this equation into (A6) and integrating by parts, we obtain (18).

I now prove that the derivatives, w.r.t. τ at $\tau = 0$, of the death, population growth, and birth terms defined in (7) correspond to the death, population growth, and birth terms defined in (10).

The death term between t and $t+\tau$ is

$$\left(1 - e^{-\int_t^{t+\tau} \delta_s\,\mathrm{d}s}\right) e^{-\int_t^{t+\tau} \eta_s\,\mathrm{d}s} \left(\frac{q_{t+\tau}}{S_t/p} - \frac{1}{S_t} \int_{\chi_t}^{+\infty} \left(\int_{\mathbb{R}} e^{\omega'} p_{t\to t+\tau}(\omega, \omega')\,\mathrm{d}\omega' \right) \gamma_t(\omega)\,\mathrm{d}\omega \right). \tag{A13}$$

In particular, the derivative of the death term at $\tau = 0$ is $\delta_t \left(\frac{q_t p}{S_t} - 1 \right)$.

The population growth term between t and $t+\tau$ is:

$$\left(e^{\int_t^{t+\tau} \eta_s\,\mathrm{d}s} - 1\right) e^{-\int_t^{t+\tau} \eta_s\,\mathrm{d}s} \left(\frac{q_{t_\tau}}{S_t/p} - \frac{1}{S_t} \int_{\chi_t}^{+\infty} \left(\int_{\mathbb{R}} e^{\omega'} p_{t\to t+\tau}(\omega, \omega')\,\mathrm{d}\omega' \right) \gamma_t(\omega)\,\mathrm{d}\omega \right). \tag{A14}$$

In particular, the derivative of the population growth term at $\tau = 0$ is $\eta_t \left(\frac{q_t p}{S_t} - 1 \right)$.

The birth term between t and $t+\tau$ is:

$$\int_s^{t+\tau} e^{-\int_s^{t+\tau} (\delta_u+\eta_u)\,\mathrm{d}u} (\delta_s + \eta_s) \left(\frac{1}{S_t} \int_{\mathbb{R}} \left(\int_{\mathbb{R}} (e^{\omega'} - q_{t+\tau})^+ p_{s\to t+\tau}(\omega, \omega')\,\mathrm{d}\omega' \right) \gamma_{Bs}(\omega)\,\mathrm{d}\omega \right) \mathrm{d}s. \tag{A15}$$

In particular, the derivative of the birth term at $\tau = 0$ is $(\delta_t + \eta_t) \int_{q_t}^{+\infty} (w - q_t) g_{Bt}(w)\,\mathrm{d}w$.

Finally, the derivative of the within and displacement term can be obtained similarly to Proposition 2. □

A.2 Type Heterogeneity

Heterogeneous Types Until now, we have assumed that the law of motion of wealth only depended on the wealth level. In reality, individuals with the same wealth may have different portfolios, or saving rates, and therefore different wealth dynamics.

To model this heterogeneity in a parsimonious way, I assume that the dynamics of wealth depends on type $1 \le n \le N$.

Assume that the law of motion of the normalized wealth of individuals in group n is

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_{nt}(w_{it})\,\mathrm{d}t + \nu_{nt}(w_{it})\,\mathrm{d}B_{it}. \tag{A16}$$

$\mu_{nt}(.)$ and $\nu_{nt}(.)$ satisfy the same regularity conditions as above. Individuals can transition between different states with a Markov transition matrix $\mathcal{T}$.

Proposition 7. *Consider the wealth process* (A16)*. The top wealth share S_t follows the law of motion:*

$$\frac{\mathrm{d}S_t}{S_t} = \underbrace{\mathrm{E}_w\left[\mu_{nt}(w) \mid w_{it} \geq q_t\right]\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\mathrm{E}_w[\nu_{nt}(q_t)^2 \mid w_{it} = q_t]\,\mathrm{d}t}_{\textit{Displacement}}. \tag{A17}$$

In case of individual heterogeneity, the within term depends on the wealth-weighted drift of individuals in the top percentile, while the displacement term depends on the average idiosyncratic volatility of individuals at the top percentile threshold. In other words, heterogeneity does not fundamentally change the expression for the within and displacement term relative to the baseline case.

In particular, heterogeneity in growth rate does not affect the displacement term. Heuristically, this comes from the fact that, during a short period of time, the cross-sectional variance of wealth growth is driven by unexpected wealth shocks, rather than heterogeneous expected wealth growth (the first term is in $\mathrm{d}t$, whereas the second is in $\mathrm{d}t^2$).

Aggregate Shocks Different individuals may also have different exposures to aggregate shocks (see, e.g., Gomez (2016), Kuhn et al. (2017)). To model this heterogeneity in a parsimonious way, I assume that individuals of different types are differently exposed to an aggregate Brownian Motion:

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_{nt}(w_{it})\,\mathrm{d}t + \sigma_{nt}(w_{it})\,\mathrm{d}Z_t + \nu_{nt}(w_{it})\,\mathrm{d}B_{it}, \tag{A18}$$

where $Z_t = \{Z_t \in \mathbb{R}^d, \mathcal{F}_t, t \geq 0\}$ is a d-dimensional aggregate Brownian motion, and, for any $1 \leq n \leq N$, $\mu_{nt}(.), \sigma_{nt}(.), \nu_{nt}(.)$ satisfy the same conditions in (8). Because the aggregate Brownian motion is multidimensional, this setup includes situations in which individuals are differently exposed to the same aggregate risk (e.g. heterogeneous leverages), or in which individuals are exposed different aggregate risks (e.g. exposure on heterogeneous factors).

In this case, the distribution of wealth is stochastic. The mathematics involved in keeping track of a stochastic distribution are more involved (see, e.g., Carmona and Delarue (2018)). Heuristically, we can still write the growth rate of the top share in term of two terms:

Proposition 8. *Consider the wealth process* (A18)*. The top wealth share S_t follows the law of motion:*

$$\begin{aligned}\frac{\mathrm{d}S_t}{S_t} =& \underbrace{\mathrm{E}_w\left[\mu_{nt}(w) \mid w_{it} \geq q_t\right]\mathrm{d}t + \mathrm{E}_w\left[\sigma_{nt}(w) \mid w_{it} \geq q_t\right]\mathrm{d}Z_t}_{\textit{Within}} \\ &+ \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{S_t}\left(\mathrm{E}_w[\nu_{nt}(q_t)^2 \mid w_{it} = q_t] + \mathrm{Var}_w[\sigma_{nt}(q_t) \mid w_{it} = q_t]\right)\mathrm{d}t}_{\textit{Displacement}}.\end{aligned} \tag{A19}$$

Heterogeneous exposure to aggregate shock affects both the within term and the displacement term. The within term now depends on the aggregate Brownian motion $\mathrm{d}Z_t$: the exposure of the top share is given by the wealth-weighted exposure of individuals in the top percentile. Moreover, the displacement term now depends not only on the average idiosyncratic volatility at the percentile threshold, but also on the variance of their exposures to aggregate risks. Intuitively, both contribute to the cross-sectional variance of wealth growth at the percentile threshold.

Proof of Proposition 7. Denote $\boldsymbol{\mu}_t(w) = (\mu_{nt}(w))_{1\leq n\leq N}$ the vector that $N\times 1$ vector giving the drift of each type. Similarly, denote $\boldsymbol{\nu}_t(w) = (\nu_{nt}(w))_{1\leq n\leq N}$ the $N\times 1$ vector giving the idiosyncratic volatility of each type. Finally, denote $\mathbf{g}_t(w)$ the vector giving the density of wealth for each type.

Denote $\mathcal{T}$ the Markov generator encoding the transition probability between different types. The Kolmogorov Forward equation is:

$$\mathrm{d}\mathbf{g_t} = -\partial_w(w\boldsymbol{\mu}_t(w)\circ\mathbf{g}_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2\left(w^2\boldsymbol{\nu}_t(w)^2\circ\mathbf{g}_t(w)\right)\mathrm{d}t + \mathcal{T}'\mathbf{g}_t\,\mathrm{d}t. \tag{A20}$$

where $\circ$ denotes the elementwise product.

The overall wealth density is $g_t = \mathbf{g}_t'\mathbf{e}$ where $\mathbf{e}$ denotes a vector of one. We can write the instantaneous change in g_t as:

$$\mathrm{d}g_t = -\partial_w\left(w\boldsymbol{\mu}_t(w)'\boldsymbol{g}_t(w)\right)\mathrm{d}t + \frac{1}{2}\partial_w^2\left(w^2\boldsymbol{\nu}_t^2(w)'\boldsymbol{g}_t(w)\right)\mathrm{d}t. \tag{A21}$$

As in the proof Proposition 2, we plug this equation into the $\mathrm{d}S_t = \int_{q_t}^{+\infty}(w-q_t)\,\mathrm{d}g_t$ and integrate by parts to obtain

$$\mathrm{d}S_t = \int_{q_t}^{+\infty} w\boldsymbol{\mu}_t(w)'\mathbf{g}_t(w)\,\mathrm{d}w\,\mathrm{d}t + \frac{1}{2}q_t^2\boldsymbol{\nu}_t^2(q_t)'\mathbf{g}_t(q_t)\,\mathrm{d}t.$$

This concludes the demonstration. $\square$

Proof of Proposition 8. We follow the steps of the proof of Proposition 2, except that, now, the density is stochastic. As pointed out in the main text, the derivation is only heuristic. Applying Ito's lemma on the implicit definition of quantile $p = \int_{q_t}^{+\infty} g_t(w)\,\mathrm{d}w$ gives the law of motion of the quantile q_t

$$0 = -g_t(q_t)\,\mathrm{d}q_t + \int_{q_t}^{+\infty}\mathrm{d}g_t(w)\,\mathrm{d}w\,\mathrm{d}t - \sigma_t[\mathrm{d}g_t(q_t)]'\sigma_t[\mathrm{d}q_t]\,\mathrm{d}t \tag{A22}$$

where $\sigma_t[\mathrm{d}g_t(q_t)]$ and $\sigma_t[\mathrm{d}q_t]$ denote the exposure of $g_t(q_t)$ and q_t to aggregate shocks.

Applying Ito's lemma on $S_t = \int_{q_t}^{+\infty} g_t(w)\,\mathrm{d}w$ gives the law of motion of S_t:

$$\mathrm{d}S_t = -q_tg_t(q_t)\,\mathrm{d}q_t + \int_{q_t}^{+\infty} w\,\mathrm{d}g_t(w)\,\mathrm{d}w - q_t\sigma_t[\mathrm{d}g_t(q_t)]'\sigma_t[\mathrm{d}q_t]\,\mathrm{d}t - \frac{1}{2}g_t(q_t)\sigma_t[\mathrm{d}q_t]'\sigma_t[\mathrm{d}q_t]\,\mathrm{d}t \tag{A23}$$

$$\mathrm{d}S_t = \int_{q_t}^{+\infty}(w-q_t)\,\mathrm{d}g_t(w)\,\mathrm{d}w - \frac{1}{2}\frac{1}{g_t(q_t)}\left(\int_{q_t}^{+\infty}\sigma_t[\mathrm{d}g_t(w)]\,\mathrm{d}w\right)'\left(\int_{q_t}^{+\infty}\sigma_t[\mathrm{d}g_t(w)]\,\mathrm{d}w\right)\mathrm{d}t$$

Injecting (A22) into (A23), we obtain:

$$\begin{aligned}\mathrm{d}S_t &= \int_{q_t}^{+\infty}(w-q_t)\,\mathrm{d}g_t(w)\,\mathrm{d}w - \frac{1}{2}g_t(q_t)\sigma_t[\mathrm{d}q_t]'\sigma_t[\mathrm{d}q_t]\,\mathrm{d}t\\ &= \int_{q_t}^{+\infty}(w-q_t)\,\mathrm{d}g_t(w)\,\mathrm{d}w - \frac{1}{2}\frac{1}{g_t(q_t)}\left(\int_{q_t}^{+\infty}\sigma_t[\mathrm{d}g_t(w)]\,\mathrm{d}w\right)'\left(\int_{q_t}^{+\infty}\sigma_t[\mathrm{d}g_t(w)]\,\mathrm{d}w\right)\mathrm{d}t\end{aligned} \tag{A24}$$

The density of wealth for households of type n evolves following the equation (Kurtz and Xiong (1999), Carmona and Delarue (2018)):

$$\begin{aligned}
\mathrm{d}\mathbf{g_t} = & -\partial_w(w\boldsymbol{\mu}_t(w)\circ\mathbf{g}_t(w))\,\mathrm{d}t - \partial_w(w\boldsymbol{\sigma}_t(w)\circ\mathbf{g}_t(w))\,\mathrm{d}Z_t \\
& + \frac{1}{2}\partial_w^2\left(w^2(\boldsymbol{\nu}_t(w)^2+\boldsymbol{\sigma}_t(w)^2)\circ\mathbf{g}_t(w)\right)\mathrm{d}t + \mathcal{T}'\mathbf{g}_t\,\mathrm{d}t
\end{aligned} \tag{A25}$$

The overall wealth density is $g_t = \mathbf{g}_t'\mathbf{e}$ where $\mathbf{e}$ denotes a vector of one. We can write the instantaneous change in g_t as:

$$\begin{aligned}
\mathrm{d}g_t = & -\partial_w\left(w\boldsymbol{\mu}_t(w)'\boldsymbol{g}_t(w)\right)\mathrm{d}t - \partial_w\left(w\boldsymbol{\sigma}_t(w)'\boldsymbol{g}_t(w)\right)\mathrm{d}Z_t \\
& + \frac{1}{2}\partial_w^2\Big(w^2\left(\boldsymbol{\nu}_t(w)^2+\boldsymbol{\sigma}_t(w)^2\right)'\boldsymbol{g}_t(w)\Big)\,\mathrm{d}t
\end{aligned} \tag{A26}$$

Plugging this equation into (A24) gives

$$\begin{aligned}
\mathrm{d}S_t = & -\left(\int_{q_t}^{+\infty}(w-q_t)\partial_w(w\boldsymbol{\mu}_t(w)'\mathbf{g}_t(w))\,\mathrm{d}w\right)\mathrm{d}t - \left(\int_{q_t}^{+\infty}(w-q_t)\partial_w(w\boldsymbol{\sigma}_t(w)'\mathbf{g}_t(w))\,\mathrm{d}w\right)\mathrm{d}Z_t \\
& + \int_{q_t}^{+\infty}(w-q_t)\frac{1}{2}\partial_w^2\left(w^2(\boldsymbol{\nu}_t(w)^2+\boldsymbol{\sigma}_t(w)^2)'\mathbf{g}_t(w)\right)\mathrm{d}w\,\mathrm{d}t - \frac{1}{2}\frac{1}{g_t(q_t)}\left(\int_{q_t}^{+\infty}\partial_w\left(w\boldsymbol{\sigma}_t(w)'\mathbf{g}_t(w)\right)\mathrm{d}w\right)^2\mathrm{d}t
\end{aligned}$$

Integrating by parts gives the law of motion of the top wealth share S_t:

$$\begin{aligned}
\mathrm{d}S_t = & \left(\int_{q_t}^{+\infty}w\boldsymbol{\mu}_t(w)'\mathbf{g}_t(w)\,\mathrm{d}w\right)\mathrm{d}t + \left(\int_{q_t}^{+\infty}w\boldsymbol{\sigma}_t(w)'\mathbf{g}_t(w)\,\mathrm{d}w\right)\mathrm{d}Z_t \\
& + \frac{1}{2}q_t^2\boldsymbol{\nu}_t^2(q_t)'\mathbf{g}_t(q_t)\,\mathrm{d}t + \frac{1}{2}q_t^2\left(\boldsymbol{\sigma}_t^2(q_t)'\mathbf{g}_t(q_t) - (\boldsymbol{\sigma}_t(q_t)'\mathbf{g}_t(q_t))^2\right)\mathrm{d}t
\end{aligned}$$

This concludes the demonstration. □

B Appendix for Section 4

B.1 Left Censoring

The decomposition in Section 2 requires us to know the wealth of households that drop off the top percentile. However, before 2012, Forbes only rarely reported the wealth of individuals who dropped out from the Forbes 400.

First, 70% households that drop off the top percentile actually stay in the Forbes 400. Indeed, the top percentile used in this paper is composed of only 264 households in 1983 (it was chosen so that, with population growth, it includes 400 households in 2017). Because wealth is so concentrated in the top, there is usually a great difference between the last individual in this top percentile and the wealth of the last individual in the top 400. Therefore, most households that drop off this top percentile stay in the top 400.

I now focus on the remaining 30% of households that drop off the Forbes 400. Formally, the problem boils down to estimating the average of a variable (the wealth growth of top households) that is left censored. In this particular setting, the Kaplan and Meier (1958) estimator gives tight bounds to estimate this quantity.

The idea is to estimate the average growth rate of drop-offs using the observed negative growth rates of households in the top percentile. The identifying assumption is that the distribution of growth rates is homogeneous for households within the top percentile.

More precisely, the Kaplan and Meier (1958) method is to first estimate the survival function, i.e. in my setting the probability that wealth growth is lower than a certain threshold $\mathbb{P}(\frac{w_{t+1}-w_t}{w_t} \leq x)$. This survival function can then be used to estimate the conditional expectation of wealth growth, given that it is lower than a certain threshold, i.e. $\mathrm{E}[\frac{w_{t+1}-w_t}{w_t} | \frac{w_{t+1}-w_t}{w_t} \leq x]$.

I check the validity of this imputation method by focusing on years where Forbes reports the wealth of drop-offs. Starting in 2012, Forbes systematically reported the wealth of drop-offs. In these years, I compare the result obtained from the estimated method and the result obtained using the real wealth of drop-offs. The results are reported in Table A1. Columns (2) and (3) report the average return of these drop-offs using the imputed method and the actual data reported by wealth. The estimates differ by only 2 percentage points on average (-25.76% vs -27.93%). The fact that the Kaplan-Meier estimator gives such a good result is intuitive: because wealth is so concentrated, households at the very top of the distribution hold ten times more wealth than the households at the margin, and therefore I actually observe a large part of the distribution of negative growth rates, even for households that remain in the top.

The last four columns report the estimates for the within and displacement term using imputed and real data. The estimates differ by less than 0.1 percentage points. The bias is small because, as discussed above, the Kaplan-Meier method gives accurate estimates of the wealth growth of imputed households. Moreover, the wealth share represented by the imputed households is small to begin with.

Table A1: Comparison Method using Imputed Wealth of Drop-offs vs Reported Wealth

Year	$E\left[\frac{w_{t+1}-w_t}{w_t} \vert \text{Drop-off}\right]$ (%)		Within (%)		Displacement (%)	
	Imputed	Actual	Imputed	Actual	Imputed	Actual
2011-2012	−36.59	−33.67	7.25	7.29	1.25	1.21
2012-2013	−29.70	−27.21	6.68	6.72	1.17	1.13
2013-2014	−19.44	−23.90	2.30	2.23	1.60	1.67
2014-2015	−30.33	−26.49	−3.20	−3.11	1.41	1.33
2015-2016	−32.51	−26.44	3.13	3.27	1.45	1.32
2016-2017	−18.99	−16.82	1.61	1.66	1.64	1.60
2011-2017	−25.76	−27.93	3.01	2.96	1.38	1.42

Notes. The table compares the within and the displacement term using imputed data and actual data about the wealth of drop-offs.

B.2 Measurement Error

I study the relation between the persistence of wealth growth and measurement error. Suppose the process for wealth is given by $w_{it+1} = w_{it}e^{r_{it+1}}$ where r_{it+1} is an i.i.d. process independent of wealth. Moreover, suppose the observed wealth $\tilde{w}_{it} = w_{it}e^{\epsilon_{it}}$ where ϵ_{it} is an i.i.d process independent of wealth capturing measurement error and $\mathrm{E}[e^{\epsilon_{it}}] = 1$. Denote ψ the ratio between the variance of measurement error and the variance of wealth growth, i.e.

$$\psi = \frac{\mathrm{Var}(\epsilon_{it})}{\mathrm{Var}(r_{it})} \tag{A27}$$

The log change in observed wealth can be written as

$$\log\left(\frac{\tilde{w}_{it+1}}{\tilde{w}_{it}}\right) = r_{it+1} + \epsilon_{it+1} - \epsilon_{it} \tag{A28}$$

A regression of observed wealth growth on the lagged observed wealth growth estimates the slope coefficient ρ:

$$\begin{aligned}
\rho &= \frac{\mathrm{cov}(\log(\tilde{w}_{it+1}/\tilde{w}_{it}), \log(\tilde{w}_{it+1}/\tilde{w}_{it}))}{\mathrm{Var}(\log(\tilde{w}_{it+1}/\tilde{w}_{it}))} \\
&= \frac{\mathrm{cov}(r_{it+1} + \epsilon_{it+1} - \epsilon_{it}, r_{it} + \epsilon_{it} - \epsilon_{it-1})}{\mathrm{Var}(r_{it} + \epsilon_{it} - \epsilon_{it-1})} \\
&= -\frac{\mathrm{Var}(\epsilon_{it})}{\mathrm{Var}(r_{it}) + 2\mathrm{Var}(\epsilon_{it})} \\
&= -\frac{\psi}{1 + 2\psi}
\end{aligned} \tag{A29}$$

Testing whether ρ is equal to zero is a test on whether ψ is different from zero, i.e. that there is measurement error.

The slope coefficient ρ is negative and decreasing in ψ. Moreover, for ψ close to zero, ρ can be well approximated by the opposite of ψ, i.e. $\rho \approx -\psi$.

I now examine how the displacement term depends on ρ. Even though I cannot reject that ρ is statistically different from zero, this allows me to check that small values of ρ do not have a disproportionate effect on the displacement term. To make further progress, I assume that ϵ_{it} and r_{it} are normal variables.

The variance of $\tilde{w}_{t+1}/\tilde{w}_t$ is higher than the variance of w_{t+1}/w_t

$$\mathrm{Var}\log\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right) = \mathrm{Var}\log\left(\frac{w_{t+1}}{w_t}\right) + 2\mathrm{Var}\left(\epsilon_{it}\right) \tag{A30}$$

This suggests that the bias between the observed displacement term and the actual displacement term is:

$$\frac{\widetilde{\text{displacement}} - \text{displacement}}{\text{displacement}} = -\frac{2\rho}{1 + 2\rho}$$

In particular, when ρ is close to zero, the relative bias is close to -2ρ. Since ρ is very small, we can conclude that the relative bias is also very small.

Table A2: Trends in the Growth of the Forbes 400 Wealth Share

	Total (%)	Within (%)	Displacement (%)	Demography (%)
	(1)	(2)	(3)	(4)
Year	-0.13	-0.05	-0.08^{***}	0.00
	(0.11)	(0.10)	(0.02)	(0.01)
R^2	0.02	0.00	0.30	0.00
Period	1983-2016	1983-2016	1983-2016	1983-2016
N	34	34	34	34

† *Notes.* This table reports the result of a regression of the terms in the accounting decomposition in Proposition 1 on a linear trend. Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,**,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively. Data from Forbes.

B.3 Within Term

I use a factor model to decompose the growth rate of households in the top percentile. It can be decomposed into the risk free rate, a term due to the exposure to priced factors, a positive term due to labor income, a negative term due to tax paid as a proportion of wealth, and a residual that corresponds to the consumption rate, i.e.:

$$R_f + \sum_{1 \le k \le K} \beta_k \times (R_k - R_f) + \text{wage income rate} - \text{tax rate} - \text{consumption rate}, \tag{A31}$$

where R_f denotes the risk free rate (in real term), R_k denotes the return of asset k, wage income rate denotes the ratio between the total wage income of households in the top percentile and their total wealth, while the tax rate denotes the total tax paid by households in the top percentiles, divided by their total wealth.

I determine β_k, the exposure of the wealth of top households to priced factors, as regressing wealth growth on on a variety of factors in Table A4. Because the wealth of top households may contain illiquid assets that are difficult to valuate, one concern is that the true volatility of wealth is higher than the volatility reported by Forbes.[31] To avoid this issue, I estimate the exposure of top households by regressing three-year horizon wealth growth on one year factors returns. After obtaining a beta, I compute a constant term as the average of the difference between the wealth growth of top households and the return predicted by factor exposures. I compute the standard errors of factor exposures and of the constant term by bootstrapping.[32].

Column (1) reports the results where the only factor is the stock market. The slope coefficient, which reflects the exposure of top households to the stock market, is close to one. Column (2) reports the results for the Fama-French three factor models that add the value factor and the size factor. The exposure to

[31] This problem is known as the "stale pricing" problem in the private equity literature, see for instance Emery (2003).

[32] More precisely, I use block-bootstrap to correct for the serial correlations of the returns across time.

the size factor is weakly negative, significant at 10%, which reflects that households at the top tend to own bigger firms. The exposure to the value factor is not significant. Similarly, Column (3) reports the results for the Fama-French five factor models that add profitability and investment factors. Similarly, only the exposure market is significant. Finally, in column (4), I add the excess returns of long-term bonds, as well as the excess returns of corporate bonds. Similarly, only the exposure to the market is significant. Overall, the stock market appears to be the main factor for the average wealth growth of top households. Moreover, the exposure to the market is relatively constant around 1.0 across specifications.

I also compare the within term to a representative portfolio of industries at the top, rather than simply the market return in Column (5) of Table A4. I classify households in the Forbes 400 based on the 49 industries of Fama-French. The industries of households in top percentiles are not representative of the market (in particular, Real Estate, Printing and Publishing, Computer Software, and Petroleum play a more important role at the top compared to the market). I construct a benchmark portfolio that weights each industry similar to the industry represented in the top. I find a similar exposure of 1 to this industry weighted portfolio. This suggests that the exact industry composition of individuals at the top does not matter much for the growth of top wealth shares.

I obtain the total tax paid and total wage income received by the top 400 individuals by income from the IRS.[33] The dataset is only available after 1992, so I use the average of this term in 1992-1995 to impute it starting from 1983.

Panel A of Table A3 reports decomposition (A31) using the market return as the only factor. Because labor income and taxes are very small as a proportion of total wealth, they play a very small role in the within term. I estimate the consumption rate as the residual in Equation (A31). Note that it looks like it decreases over time, although it is very imprecisely estimated.

Table A3: Decomposing the Within Term

Year	Within						
	Total (%)	Top (%)					- Per Capita (%)
		Total	R_f	$\beta_M(R_M - R_f)$	Wage-Tax Rate	-Consumption Rate	
All Years	3.0	5.6	1.6	6.5	−0.8	−1.7	−2.5
1983-1994	2.5	4.7	3.6	5.5	−0.8	−3.5	−2.0
1994-2005	2.9	7.2	2.0	6.8	−0.7	−0.9	−4.0
2005-2017	3.6	5.1	−0.6	7.3	−0.8	−0.6	−1.4

Notes. The table reports the decomposition of the within term according to Equation (A31). All returns in real terms. Data for the risk-free rate and market returns come from Fama-French Data Library. Data from Forbes.

[33] https://www.irs.gov/pub/irs-soi/13intop400.pdf.

Table A4: Factor Model

	Wealth Growth Top Households				
	Market	FF 3-factors	FF 5-factors	Bond Factors	Industry
	(1)	(2)	(3)	(4)	(5)
market	0.97***	1.08***	0.98***	0.98***	
	(0.26)	(0.28)	(0.29)	(0.26)	
smb		−0.86*	−0.82		
		(0.51)	(0.50)		
hml		0.31	0.62		
		(0.36)	(0.55)		
cma			−0.78		
			(0.69)		
rmw			−0.06		
			(0.46)		
ltg				0.42	
				(0.55)	
crd				−0.02	
				(0.61)	
industry					1.00***
					(0.26)
Constant	−0.03	−0.05	−0.02	−0.06	−0.03
	(0.06)	(0.06)	(0.07)	(0.06)	(0.06)
R^2	0.31	0.39	0.41	0.33	0.32
Period	1983-2016	1983-2016	1983-2016	1983-2016	1983-2016
N	32	32	32	32	31

Notes. The table reports the results of regressing the wealth growth of top households on the excess stock market return, and a set of other factors. The left-hand side is the three-year excess wealth growth of top households. Portfolio returns of Fama-French factor models, as well as industry portfolios, are from the Fama-French Data Library. Corporate bond returns are obtained from Ibbotson's Stocks, Bonds, Bills and Inflation Yearbook. Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,**,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

C Appendix for Section 6

Proof of Proposition 5. We can express the average time $T_q(w_{it})$ by backward induction.

$$T_q(w_{it}) = \delta\Delta t \times 0 + (1 - \delta\Delta t) \times (\Delta t + \mathrm{E}[T_q(w_{it+\Delta t})]). \tag{A32}$$

Therefore

$$0 = (1 - \delta\Delta t)(\Delta t + \mathrm{E}[T_q(w_{it+\Delta t}) - T_q(w_{it})]) - \delta\Delta t T_q(w_{it}). \tag{A33}$$

Taking $\Delta t \to 0$, we obtain a forward-looking expression for $T_q(w_{it})$:

$$0 = \mathrm{d}t + \mathrm{E}[dT_q(w_{it})] - T_q(w_{it})\delta\,\mathrm{d}t. \tag{A34}$$

Applying Ito's lemma, we obtain an ODE satisfied by T_q:

$$1 + T_q'(w)\mu w + T_q''(w)\frac{\nu^2 w^2}{2} - \delta T_q(w) = 0. \tag{A35}$$

The solution has the form:

$$T_q(w) = c_1 w^{\lambda_1} + c_2 w^{\lambda_2} + \frac{1}{\delta}, \tag{A36}$$

where λ_1 and λ_2 are the positive and negative roots of $\zeta \to \mu\zeta + \frac{\zeta(\zeta-1)}{2}\nu^2 - \delta$. Note that this function is convex, converges to infinity as ζ converges to infinity, and equals $-\delta$ in zero, therefore there are exactly two zeros for this function, one negative, and one positive.

Using the limit condition:

$$T_q(q) = 0 \tag{A37}$$

$$\lim_{w \to +\infty} T_q(w) = \frac{1}{\delta}. \tag{A38}$$

This gives $c_1 = 0$ and $c_2 = -1/(\delta q^{\lambda_2})$.Denoting $\xi = -\lambda_2$, we obtain:

$$T_q(w) = \frac{1}{\delta}\left(1 - \left(\frac{w}{q}\right)^{-\xi}\right). \tag{A39}$$

This concludes the proof. □

Proof of Proposition 6. The stationary wealth distribution is Pareto, i.e. $g(w) = C\left(\frac{w}{\underline{w}}\right)^{-\zeta}$, where ζ is given by the positive root of $\mu\zeta + \frac{1}{2}\zeta(\zeta-1)\nu^2 - (\delta + \eta)$ (see, e.g., Gabaix et al. (2016)).

The average time is

$$
\begin{aligned}
T(p) &= \mathrm{E}^g[T_q(w_{it})|w_{it} \geq q] \\
&= \frac{\int_q^{+\infty} T_q(w)g(w)\,\mathrm{d}w}{\int_q^{+\infty} g(w)\,\mathrm{d}w} \\
&= \frac{1}{\delta}\left(1 - \frac{\int_q^{+\infty}(w/q)^{-\xi}w^{-\zeta-1}\,\mathrm{d}w}{\int_q^{+\infty} w^{-\zeta-1}\,\mathrm{d}w}\right) \\
&= \frac{1}{\delta}\left(1 - \frac{1}{q^{\xi}}\frac{\frac{q^{-\xi-\zeta}}{\zeta+\xi}}{\frac{q^{-\zeta}}{\zeta}}\right) \\
&= \frac{1}{\delta}\frac{1}{1+\frac{\zeta}{\xi}}.
\end{aligned} \tag{A40}
$$

The derivative of $T(p)$ with respect to idiosyncratic variance ν^2 is

$$
\begin{aligned}
\partial_{\nu^2} T &= -\frac{1}{\delta}\left(1+\frac{\zeta}{\xi}\right)^{-2}\partial_{\nu^2}\left(\frac{\zeta}{\xi}\right) \\
&= \frac{1}{\delta}\left(1+\frac{\zeta}{\xi}\right)^{-2}\frac{\zeta}{\xi}\left(\frac{\partial_{\nu^2}\xi}{\xi} - \frac{\partial_{\nu^2}\zeta}{\zeta}\right).
\end{aligned} \tag{A41}
$$

As ν^2 increases, $T(p)$ decreases only if the percentage decrease of ξ is higher than the percentage decrease of ζ. This concludes the proof. □

D Decomposing the Growth of Top Income Shares

While the paper has focused on decomposing the rise in top wealth shares, this framework can also be used to decompose the rise in top income shares. One key difference, however, is that income is a flow, not a stock. In particular, in comparison with wealth, transitory shocks play a much larger role in driving the dynamics of income in the short run, although they have little impact on long-run inequality.[34] Models such as the ones introduced in Section 3 describe better the permanent component of income (most recently by Jones and Kim (2016) and Gabaix et al. (2016)).

Data. I use the IRS public use panel files created by the Statistics of Income Division from 1979 to 1990. This dataset includes a random subsample of taxpayers who can be followed over time between 1979 and 1990. Following Piketty and Saez (2003), I construct a comprehensive measure of earnings, which includes wages, salary, and entrepreneurial income.[35] I focus on tax filers that file jointly. In this context, birth and death represent the fact that a taxpayer starts or stops filing jointly. The sample contains 5,523 distinct tax payers.

[34] See Heathcote et al. (2010) and Blundell et al. (2008)

[35] More precisely, I construct earnings as AGI minus capital gain, dividend income, interest income, rental income, and royalties.

Methodology. As mentioned earlier, I want to focus on the permanent component of income.[36] To do so, I average income over three year periods before applying the accounting on the sample. I then annualize each term to obtain yearly terms. While three years sounds arbitrary, using a six year period gives similar results. This suggests that three years is enough to focus on the permanent component of the rise in top shares (see Figure A2a.)

Results. Table A5 reports the result of the accounting decomposition for the top 100%, the top 10%, and the top 2% (I stop at the top 2% since it is only constituted of 100 households). I find that the annual growth rate of top income share during the time period for the top 2% is 4.0%. This number can be decomposed as follows: a within term of 0.3%, a displacement term of 4.2%, and a demography term of -0.5%. In other words, the displacement term accounts for the whole increase in the top income share.

Table A5 plots the result of the decomposition over all percentiles. A striking finding is that the within term and the demography terms are relatively stable across the income distribution, while the displacement term gradually increases with top percentiles. Moreover, the slope of the displacement term for top percentiles exactly lines up with the slope of the growth in the top share, which suggests that displacement has been a key driver of the rise in top income shares during the time period.

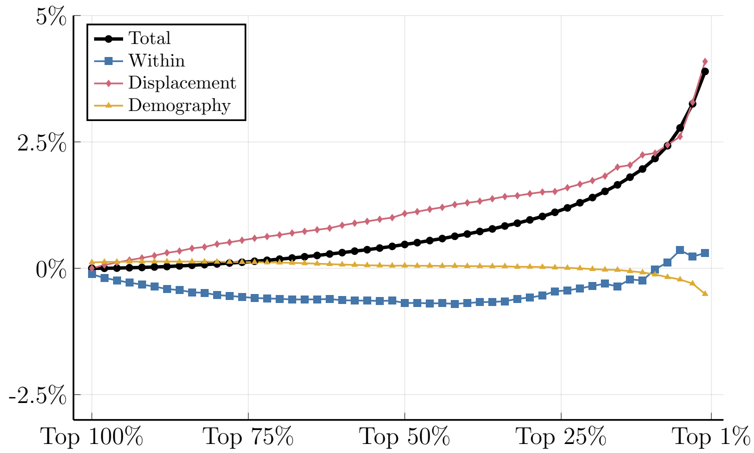
To understand why the displacement term increases with top percentiles, Figure A1a plots the contributions of $g_t(q_t)q_t^2/S_t$ and $\mathrm{sd}_t(q_t)$ over all percentiles. The standard deviation of log income follows a U-shape, as in Guvenen et al. (2015). Moreover, $g_t(q_t)q_t^2/S_t$ increases almost monotonically with the top percentile, except at the very top.

Overall, these results suggest that the framework can shed light on the drivers of top income inequality. Of course, this is simply a sketch of what can be done, and, hopefully, more work will be done to understand better the evolution of the top income over time.

[36]The transitory component is exactly similar to the problem of observing wealth with error, which was discussed in Appendix B.2

Figure A1: Decomposing the Growth of Income Shares (1979-1990)

(a) Within, Displacement, and Demography



(b) Time series of $g_t(q_t)q_t^2/S_t$ and $\mathrm{sd}_t(q_t)$

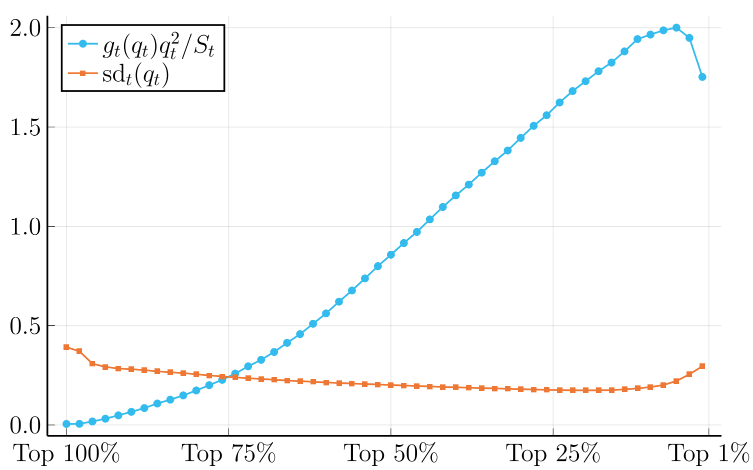


Table A5: Decomposition Income Distribution

(a) Summary

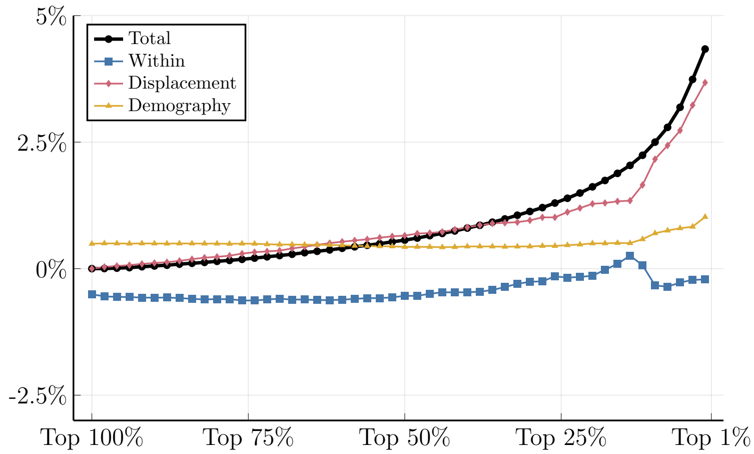
	Total (%)	Within (%)			Displacement (%)			Demography (%)			
		Total	Top	-Per Capita	**Total**	Entry	Exit	**Total**	Death	Pop. Growth	Birth
Top 100%	0.0	−0.1	2.6	−2.6	0.0	0.0	0.0	0.1	−3.0	−0.9	3.8
Top 10%	2.2	0.0	2.7	−2.6	2.3	1.2	1.2	−0.1	−1.0	−0.3	1.1
Top 2%	4.0	0.3	3.0	−2.6	4.2	2.5	1.8	−0.5	−1.2	−0.3	0.9

(b) Details

	Displacement				Demography					
	Entry		Exit		Death		Pop. Growth		Birth	
	s_E (%)	$\frac{\overline{w}_{E,2}-q_2}{\overline{w}_{T,1}}$	s_X (%)	$\frac{q_2-\overline{w}_{X,2}}{\overline{w}_{T,1}}$	s_D (%)	$\frac{q_2-\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{D,1}}{\overline{w}_{T,1}}$	$1-s_T$ (%)	$\frac{q_2-\frac{\overline{w}_{T\setminus D,2}}{\overline{w}_{T\setminus D,1}}\overline{w}_{T,1}}{\overline{w}_{T,1}}$	s_B (%)	$\frac{\overline{w}_{B,2}-q_2}{\overline{w}_{T,1}}$
Top 100%	0.0		0.0		4.2	−0.67	1.0	−0.91	5.1	0.74
Top 10%	7.3	0.15	7.4	0.15	2.4	−0.36	1.0	−0.35	3.5	0.32
Top 2%	9.2	0.24	8.8	0.19	2.6	−0.35	1.0	−0.30	3.0	0.32

Figure A2: Decomposing the Growth of Income Shares (6-year income) (1979-1990)

(a) Within, Displacement, and Demography



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