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DECOMPOSING THE GROWTH OF TOP WEALTH SHARES

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I propose an accounting framework that decomposes the growth of the average wealth in a top percentile into three terms: an *intensive* term, which is the average wealth growth of individuals initially in the top percentile; a *displacement* term, which accounts for composition changes due to changes in relative wealth rankings; and a *demography* term, which accounts for composition changes due to demographic changes. Using continuous-time methods, I obtain closed-form formulas relating each term to the statistical properties of individual wealth. Evidence from the Forbes 400 list suggests that the displacement term accounts for more than half the rise in top wealth inequality in the U.S.

KEYWORDS: Wealth inequality, accounting decomposition, transition density, Pareto distribution.

1. INTRODUCTION

What drives the recent rise in top wealth shares? One widely held view is that it reflects the high wealth growth of households in top percentiles relative to the economy (Piketty, 2014). This view implicitly assumes that the composition of top households remains constant over time. In reality, less than 10% of the households in the 1983 *Forbes* list of the 400 richest households in the United States remained on the list in 2017. These composition changes drive a wedge between the growth of top wealth shares and the average wealth growth of top households relative to the economy.

In this paper, I propose an accounting framework to decompose the growth of the wealth share of a top percentile into an *intensive* margin, which is the average wealth growth of households who initially belong to the top percentile relative to the economy, and an *extensive* margin, which accounts for composition changes in the top percentile. The extensive margin is itself the sum of *displacement* term, which accounts for changes in relative rankings among individuals (i.e., flow of individuals in and out the top percentile), and a *demography* term, which accounts for composition changes in the overall population (i.e., birth, death and population growth). Intuitively, each term corresponds to a distinct driver of top wealth inequality: top wealth shares can increase due a high growth rate of existing fortunes (*intensive*), an inflow of new fortunes in the top percentile (*displacement*), or a low rate of population renewal (*demography*).

Using continuous-time methods, I obtain closed form formulas for the displacement and demography terms in a wide range of random-growth models. These formula provide a simple statistical framework to interpret the results of the accounting decomposition. They also make it possible to quantify the displacement and demography terms in the absence of panel data, which is a common occurrence.

The displacement term depends on the dispersion of wealth growth among top households. In particular, when wealth follows a diffusion process, the displacement term equals $1/2(\zeta - 1)\nu^2$,

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where ζ denotes the local Pareto exponent of the wealth distribution and ν denotes the standard deviation of (log) wealth growth for individuals around the top percentile threshold.

The demography term depends on the rate of population renewal. In particular, in the absence of inheritance, the demography term equals $-(\delta + \eta)/\zeta$, where δ denotes the death rate of individuals in the top percentile and η denotes the population growth rate.

I then apply the accounting framework to decompose the growth of the share of wealth owned by individuals in the Forbes 400 list, for which we have panel data. The 3.8% annual growth rate of the Forbes 400 wealth share can be decomposed into an intensive margin equal to 3.0%, a displacement term equal to 2.4%, and a demography term equal to -1.5%. There are large fluctuations in the displacement term over time: displacement has slowly declined over time, from 3.2% in the 1980s to 1.5% in the 2010s.

I use the statistical framework to shed light on the displacement and demography terms. The displacement term predicted by the simple diffusion model, $1/2(\zeta - 1)\nu^2$, approximates well the actual level and dynamics of the displacement term. This provides a simple way to understand its decline over time. Part of the decline is driven by a decrease in the dispersion of wealth growth among (i.e., a decrease of ν from 0.27 to 0.25). The majority of the decline, however, is driven by a thickening of the wealth distribution (i.e., a decrease of ζ from 1.6 to 1.4). Intuitively, as wealth inequality increased, it became gradually harder for new fortunes to displace existing ones.

I use the statistical framework to quantify the displacement term for the top 1%, 0.1%, and 0.01% of the wealth distribution over the 20th century, where panel data is not available. I estimate ζ, the local Pareto exponent, using cross-sectional data on the wealth distribution. I then proxy for ν, the standard deviation of (log) wealth growth, using the standard deviation of firm-level returns. Overall, the displacement term predicted by a diffusion model matches the inverted U-shape of top wealth inequality over the 20th century. It first peaked during the Great Depression, remained low during World War II and the postwar economic boom, before peaking again during the technological revolutions of the 1980s and 1990s. This suggests that low-frequency fluctuations in displacement is a central force behind the fluctuations in wealth inequality observed during the 20th century.

Finally, I discuss the implication of my results for the relationship between wealth inequality and wealth mobility. I focus on a notion of downward mobility: the average time a rich individual remains in a top percentile. Whether a rise in the top wealth share is driven by the *intensive* or *extensive* margin has opposite effects on wealth mobility. More precisely, a rise in inequality driven by a high average wealth growth of top households (*intensive*) is associated with a lower level of mobility, whereas a rise in inequality driven by a dispersion of wealth shocks (*displacement*) is associated with a higher level of mobility, even in the long-run.

Related Literature. This paper is motivated by a large empirical literature documenting the dynamics of top wealth shares in the U.S. (Kopczuk and Saez, 2004, Saez and Zucman, 2016, Kuhn et al., 2020). This literature tends to interpret the recent rise in top wealth shares as a rise in the wealth growth of top households relative to the economy (Piketty, 2014 or Mian et al., 2020). In particular, Saez and Zucman (2016) define a "synthetic saving rate" based on the difference between the growth of the average wealth in top percentiles and the average return of top individuals. My paper clarifies that this synthetic saving rate is actually the sum of an "actual" household saving rate, and two other terms capturing composition effects: a displacement term due to the dispersion of wealth growth, and a demography term due to birth, death, and population growth.

Recent empirical papers on wealth inequality stress the importance of idiosyncratic shocks in the right tail of the wealth distribution. Benhabib et al. (2011) and Benhabib et al. (2015) examine the stationary wealth distribution in an economy with idiosyncratic returns. Fagereng et al.

(2016) and Bach et al. (2017) emphasize the heterogeneity in asset returns among households in Norway and Sweden, respectively. Benhabib et al. (2019) examine the role of idiosyncratic volatility through the lens of a quantitative model.[1] Campbell et al. (2019) decomposes the change in the *variance* of log wealth into a term due to differences in expected wealth growth and a term due to differences in unexpected wealth shocks. Relative to this literature, my contribution is to quantify, empirically and theoretically, the contribution of these idiosyncratic shocks for the growth of top wealth shares.

This work contributes to the theoretical literature that studies inequality through the lens of random growth models (Wold and Whittle, 1957, Jones, 2015, Luttmer, 2012, Gabaix et al., 2016 and Jones and Kim, 2018). A central equation in this literature is the Kolmogorov forward equation, which relates the dynamics of wealth density to the dynamics of individual wealth. A key contribution of my paper is to derive an "integrated" version of the Kolmogorov forward equation, which relates the dynamics of top wealth shares to the dynamics of individual wealth. My result is connected to Steinbrecher and Shaw (2008), which derive an equation relating the dynamics of *quantiles* to the dynamics of the underlying process.[2]

My accounting framework is related to Baily et al. (1992), Foster et al. (2008), and Melitz and Polanec (2015), who decompose the change of an average quantity (firm productivity) in the economy. Relative to this literature, I decompose the change of an average quantity *for a particular subgroup of the population*, the top percentile. This requires accounting not only for the entry and exit of individuals in and out of the economy (*demography*) but also for the flow of surviving individuals in and out of the top percentile due to changes in relative rankings (*displacement*).[3]

Roadmap. The rest of the paper is organized as follows. Section 2 presents the accounting framework. Section 3 examines the result of the decomposition in a wide range of random-growth models. In Section 4, I apply this framework to decompose the growth of the Forbes 400 wealth share, and in Section 5, I examine the role of displacement for the top 1%, 0.1%, and 0.01% in the U.S. over the 20th century. Section 6 discusses the implications of my findings for the relationship between inequality and mobility, and Section 7 concludes.

2. ACCOUNTING FRAMEWORK

I now present an accounting framework to decompose the growth of the average wealth in a top percentile. When wealth is *normalized* by the average wealth in the economy, this can be interpreted as a decomposition for the top percentile wealth *share*. To develop intuition, Section 2.1 first covers the simpler case in which the population in the economy is fixed (i.e., without demographic changes) while Section 2.2 covers the more general case with demographic changes.

2.1. *Case with Fixed Population*

To simplify the exposition, I first present the accounting framework in the case of a fixed population, that is, assuming that the set of individuals in the economy is fixed over time.

[1]Existing theories to explain the concentrated portfolios of the rich include moral hazard, expertise, taste, or asymmetric information (see, e.g., Di Tella, 2016, Haddad et al., 2014, Eisfeldt et al., 2017, or Roussanov, 2010).

[2]It is also loosely connected to Dupire's equation (Dupire et al., 1994) that relates the time derivative of the price of call option (an expectation that the price is higher than a certain threshold) to the underlying asset's volatility.

[3]The term "displacement" is borrowed from Gârleanu et al. (2012), which use it to denote the part of aggregate growth that comes from the arrival of new agents rather than from the growth of existing agents.

Let $t \in \{0,1\}$ denote time. Consider a given top percentile of the wealth distribution $p \in (0,1]$, say the top $p = 1\%$. Denote $\mathcal{I}$ the set of individuals in the economy and $\mathcal{P}_t \subset \mathcal{I}$ the subset of individuals who are the top percentile at time t. Due to changes in relative wealth rankings, the set of individuals in the top percentile typically changes over time; that is, $\mathcal{P}_0 \neq \mathcal{P}_1$. More precisely, denoting $\mathcal{X}$ the set of individuals who enter the top percentile during the time period and $\mathcal{E}$ the set of individuals who exit the top percentile during the time period, we can write $\mathcal{P}_1 = (\mathcal{P}_0 \setminus \mathcal{X}) \cup \mathcal{E}$.[4]

This equality allows me to decompose the growth of the average wealth of individuals in the top percentile as follows:

$$\frac{\sum_{i\in\mathcal{P}_1} w_{i1} - \sum_{i\in\mathcal{P}_0} w_{i0}}{\sum_{i\in\mathcal{P}_0} w_{i0}} = \underbrace{\frac{\sum_{i\in\mathcal{P}_0} w_{i1} - \sum_{i\in\mathcal{P}_0} w_{i0}}{\sum_{i\in\mathcal{P}_0} w_{i0}}}_{\text{Intensive}} + \underbrace{\frac{\sum_{i\in\mathcal{E}} w_{i1} - \sum_{i\in\mathcal{X}} w_{i1}}{\sum_{i\in\mathcal{P}_0} w_{i0}}}_{\text{Displacement}}, \tag{1}$$

where w_{it} denotes the wealth of individual i at time t.[5] The first term (*intensive*) corresponds to the wealth growth of individuals who are initially in the top percentile — whether or not they remain in the top percentile by the end of the period. The second term (*displacement*) depends on the difference between the wealth of individuals entering the top percentile and the wealth of individuals exiting it.

By definition of top percentiles, the displacement term is always positive. Put differently, the growth of the average wealth in the top percentile is systematically larger than the wealth growth of individuals initially in the top percentile (i.e., the intensive term). This can be seen as an attrition bias: only individuals with a higher-than-average wealth growth remain in the top percentile over time.

We can examine separately the contribution of entry and exit in the top percentile. Denote q_1 the wealth of the last person in the top percentile at time $t = 1$ (i.e., the top $p-$quantile), we can rewrite (1) as:[6]

$$\frac{\sum_{i\in\mathcal{P}_1} w_{i1} - \sum_{i\in\mathcal{P}_0} w_{i0}}{\sum_{i\in\mathcal{P}_0} w_{i0}} = \underbrace{\frac{\sum_{i\in\mathcal{P}_0} w_{i1} - \sum_{i\in\mathcal{P}_0} w_{i0}}{\sum_{i\in\mathcal{P}_0} w_{i0}}}_{\text{Intensive}} + \underbrace{\underbrace{\frac{\sum_{i\in\mathcal{E}}(w_{i1} - q_1)}{\sum_{i\in\mathcal{P}_0} w_{i0}}}_{\text{Inflow}} + \underbrace{\frac{\sum_{i\in\mathcal{X}}(q_1 - w_{i1})}{\sum_{i\in\mathcal{P}_0} w_{i0}}}_{\text{Outflow}}}_{\text{Displacement}}.$$

This equation further decomposes the displacement term into an *inflow* term and an *outflow* term, which respectively account for the effect of entry and exit on the growth of the average wealth in the top percentile. To understand the intuition, consider an individual below the percentile threshold who, after a higher-than-average wealth growth, enters the top percentile. Because the total number of individuals in the top percentile is constant, she displaces the last

[4] Here and elsewhere, "$\mathcal{G}_1 \setminus \mathcal{G}_2$"denotes the set of individuals in $\mathcal{G}_1$ who are not in $\mathcal{G}_2$.

[5] Note that, because population size is fixed, there is a constant number of individuals in the top percentile, which implies that the growth of the average wealth in the top percentile is the same as the growth of the total wealth.

[6] This uses the fact that there are as many individuals entering the top percentile (i.e., in $\mathcal{E}$) as there are individuals exiting it (i.e., in $\mathcal{X}$).

individual in the top percentile, with wealth q_1. The net effect of this entry is captured by the inflow term.

Conversely, consider an individuals above the percentile threshold who, after a lower-than-average wealth growth, exits the top percentile. Because the total number of individuals in the top percentile is constant, she is immediately replaced by an individual at the percentile threshold, with wealth q_1. The net effect of this exit is captured by the outflow term.

2.2. *Case with Demographic Changes*

For the sake of simplicity, the preceding analysis abstracted away from demographic changes. In reality, the set of individuals in the overall population changes over time. These demographic changes introduce additional changes in the composition of individuals in the top percentile.

I now extend the accounting framework to account for these demographic forces. Formally, denote $\mathcal{I}_t$ the overall set of individuals in the economy, which now varies over time. As above, denote $\mathcal{P}_t \subset \mathcal{I}_t$ the subset of individuals who are in the top percentile p at time $t \in \{0,1\}$.

In the presence of population growth, the number of individuals in the top percentile changes over time, and it becomes useful to introduce some notations: for any set $\mathcal{G}$, denote $\overline{w}_{\mathcal{G},t}$ the average wealth of individuals in $\mathcal{G}$ at time t, and denote $n_{\mathcal{G}}$ the number of individuals in $\mathcal{G}$ relative to the number of individuals in $\mathcal{P}_1$.

In the presence of demographic changes, the set of individuals who enter the top percentile can be split into two groups: the group of individuals who enter from another part of the distribution ($\mathcal{E}$) and the group of individuals who simultaneously enter the overall population ($\mathcal{B}$, for birth). Symmetrically, the set of individuals who exit the top percentile can be split into individuals who exit to another part of the distribution ($\mathcal{X}$) and individuals who exit the overall population ($\mathcal{D}$, for death).[7] As shown in Figure 1, these four sets fully summarize composition changes in the top percentile: $\mathcal{P}_1 = (\mathcal{P}_0 \setminus (\mathcal{D} \cup \mathcal{X})) \cup \mathcal{B} \cup \mathcal{E}$.

This equality allows one to rewrite the average wealth of individuals in the top percentile at time $t = 1$ as follows:

$$\begin{aligned}
\overline{w}_{\mathcal{P}_1,1} &= n_{\mathcal{P}_0\setminus\mathcal{D}}\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1} - n_{\mathcal{X}}\overline{w}_{\mathcal{X},1} + n_{\mathcal{B}}\overline{w}_{\mathcal{B},1} + n_{\mathcal{E}}\overline{w}_{\mathcal{E},1} \\
&= \frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\left(n_{\mathcal{P}_0}\overline{w}_{\mathcal{P}_0,0} - n_{\mathcal{D}}\overline{w}_{\mathcal{D},0}\right) - n_{\mathcal{X}}\overline{w}_{\mathcal{X},1} + n_{\mathcal{B}}\overline{w}_{\mathcal{B},1} + n_{\mathcal{E}}\overline{w}_{\mathcal{E},1}
\end{aligned}$$

As above, we compare the wealth of individuals entering or exiting the top percentile to the wealth of the last person in the top percentile, q_1. Adding and subtracting q_1 from each term gives:[8]

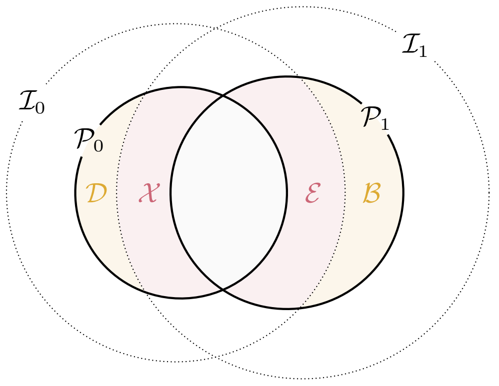
$$\begin{aligned}
\overline{w}_{\mathcal{P}_1,1} = &\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0} - (1 - n_{\mathcal{P}_0})\left(\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0} - q_1\right) - n_{\mathcal{D}}\left(\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{D},0} - q_1\right) \\
&+ n_{\mathcal{X}}(q_1 - \overline{w}_{\mathcal{X},1}) + n_{\mathcal{B}}(\overline{w}_{\mathcal{B},1} - q_1) + n_{\mathcal{E}}(\overline{w}_{\mathcal{E},1} - q_1)
\end{aligned}$$

Subtracting and dividing by the average wealth in the top percentile at time $t = 0$, and rearranging, gives:

[7] The terms birth and death should be interpreted liberally: they simply refer to composition changes in the overall population observed by the econometrician. For instance, when constructing top shares in a rotating panel (such as CEX), birth and death could refer to individuals entering and exiting the panel.

[8] Using the fact that $1 = n_{\mathcal{P}_0} - n_{\mathcal{D}} - n_{\mathcal{X}} + n_{\mathcal{B}} + n_{\mathcal{E}}$.

FIGURE 1.—Venn Diagram Representing Composition Changes in the Top Percentile



[a]The figure plots $\mathcal{I}_t$, the set of all individuals in the economy, and $\mathcal{P}_t \subset \mathcal{I}_t$, the subset of individuals in the top percentile, at time $t \in \{0, 1\}$. The intersections of these four sets delineate four subsets used in the accounting framework: $\mathcal{E} = (\mathcal{P}_1 \setminus \mathcal{P}_0) \cap \mathcal{I}_0$, the set of individuals who enter the top percentile from another part of the distribution, $\mathcal{B} = \mathcal{P}_1 \setminus \mathcal{I}_0$, the set of individuals who simultaneously enter the top percentile and the economy, $\mathcal{X} = (\mathcal{P}_0 \setminus \mathcal{P}_1) \cap \mathcal{I}_1$, the set of individuals who exit the top percentile to another part of the distribution, and $\mathcal{D} = \mathcal{P}_0 \setminus \mathcal{I}_1$, the set of individuals who simultaneously exit the top percentile and the economy. In the absence of demographic changes, $\mathcal{I}_0 = \mathcal{I}_1$, and, therefore, $\mathcal{B} = \mathcal{D} = \emptyset$.

$$\frac{\overline{w}_{\mathcal{P}_1,1}-\overline{w}_{\mathcal{P}_0,0}}{\overline{w}_{\mathcal{P}_0,0}} = \underbrace{\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}-\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}}_{\text{Intensive}} + \underbrace{\underbrace{n_{\mathcal{E}}\frac{\overline{w}_{\mathcal{E},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}}_{\text{Inflow}} + \underbrace{n_{\mathcal{X}}\frac{q_1-\overline{w}_{\mathcal{X},1}}{\overline{w}_{\mathcal{P}_0,0}}}_{\text{Outflow}}}_{\text{Displacement}}$$

$$+ \underbrace{\underbrace{n_{\mathcal{B}}\frac{\overline{w}_{\mathcal{B},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}}_{\text{Birth}} \underbrace{- n_{\mathcal{D}}\frac{\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{D},0}-q_1}{\overline{w}_{\mathcal{P}_0,0}}}_{\text{Death}} \underbrace{-(1-n_{\mathcal{P}_0})\frac{\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0}-q_1}{\overline{w}_{\mathcal{P}_0,0}}}_{\text{Pop. Growth}}}_{\text{Demography}}. \tag{2}$$

This equation generalizes the accounting decomposition (1) in the presence of demographic changes. The first term (*intensive*) corresponds to the growth of the average wealth in the top percentile, holding constant the composition of individuals in the top. The other terms account for the extensive margin: the *displacement* term accounts for composition changes in the top percentile due to changes in relative rankings, while the new *demography* term accounts for composition changes in the top percentile due to changes in the composition of individuals in the overall population.

More precisely, the intensive term is the average wealth growth of individuals in $\mathcal{P}_0 \setminus \mathcal{D}$; that is, the group of individuals who are initially in the top percentile and who do not die (since we do not observe the wealth at time $t = 1$ of individuals who do die).

The displacement term is similar to the one derived above, except that, with demographic changes, the fraction of individuals in $\mathcal{E}$ may be different from the fraction of individuals in $\mathcal{X}$; that is, $n_{\mathcal{E}} \neq n_{\mathcal{X}}$.

Finally, the new demography term accounts for composition changes in the top percentile due to individuals entering and exiting the economy. It is the sum of three terms which respectively account for birth, death, and population growth. The *birth* term measures the positive contribution of individuals who are born into the top percentile. Similarly to the inflow term, it depends on the difference between the wealth of newborn agents and the wealth at the percentile threshold. The *death* term measures the negative contribution of individuals in the top percentile who die during the time period—similar to the outflow term, it depends on the difference between the wealth at the percentile threshold and the wealth of deceased individuals (adjusted for the average wealth growth among surviving individuals). Finally, the *population growth* term measures the negative contribution of population growth: because a top percentile contains a constant fraction of the population, an increase in population size increases the number of individuals in the top percentile. This increase in population size decreases the average wealth in the top percentile, by the difference between the average wealth in the top percentile and the wealth at the percentile threshold. In the absence of demographic changes, $\mathcal{B} = \mathcal{D} = \emptyset$, while $n_{\mathcal{P}_0} = 1$, and, therefore, all of these terms are zero.

While the displacement term is always positive, the demography term can be positive or negative: it depends on whether the (positive) contribution of birth compensates the (negative) contributions of death and population growth.[9]

2.2.1. *Discussion*

Special Case $p = 100\%$. It is informative to discuss the result of my accounting framework in the special case $p = 100\%$; that is, when the top percentile group includes the entire set of individuals in the economy. In this case, the displacement term is zero: the accounting framework (2) simply decomposes the growth of the average wealth in the economy into an intensive term, which is the growth of the average wealth of individuals who do not die, and a demography term, which depends on the difference between the average wealth of newborn agents and the average wealth of deceased individuals. This decomposition boils down to the productivity decompositions developed in Baily et al. (1992), Foster et al. (2008), and Melitz and Polanec (2015). In this sense, my accounting framework extends these frameworks to decompose the growth of an average quantity not just for the whole economy, but also for *any top percentile* $p \leq 100\%$. This leads me to separate composition changes due to the flow of existing individuals in and out of the top percentile (i.e., the displacement term) from composition changes due to the entry and exit of individuals in the economy (i.e., the demography term).

Top Wealth Share. The top percentile wealth share is given by $p(\overline{w}_{\mathcal{P}_t,t}/\overline{w}_{\mathcal{I}_t,t})$. When individual wealth is normalized by the average wealth in the economy (i.e., $\overline{w}_{\mathcal{I}_t,t} = 1$), this simplifies to $p\overline{w}_{\mathcal{P}_t,t}$. As a result, when wealth is *normalized* by the average wealth in the economy, we can simply use the accounting framework (2) to decompose the growth of the top percentile wealth *share*. Note that, in this case, the intensive term should be interpreted as the average wealth growth of individuals in the top percentile *relative to the growth of the economy.*

Logarithmic Decomposition. We can also use the accounting framework to decompose the *logarithmic* growth of the average wealth in the top percentile, instead of its *arithmetic* growth. Indeed, we can modify (2) to obtain:[10]

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[9] It is negative as long as the wealth distribution among newborn individuals is less unequal than the wealth distribution among deceased individuals. See Section 3.2 for a precise statement.

[10] To see why, start by rewriting the arithmetic decomposition (2) as:

$$\frac{\overline{w}_{\mathcal{P}_1,1}}{\overline{w}_{\mathcal{P}_0,0}} = (1 + \text{Intensive})\left(1 + \frac{\text{Displacement}}{1 + \text{Intensive} + \text{Demography}}\right)\left(1 + \frac{\text{Demography}}{1 + \text{Intensive}}\right).$$

Taking log gives (3).

$$\log\left(\frac{\overline{w}_{\mathcal{P}_1,1}}{\overline{w}_{\mathcal{P}_0,0}}\right) = \underbrace{\log\left(\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\right)}_{\text{Intensive}} + \underbrace{\log\left(1+\frac{n_{\mathcal{E}}(\overline{w}_{\mathcal{E},1}-q_1)+n_{\mathcal{X}}(q_1-\overline{w}_{\mathcal{X},1})}{n_{\mathcal{P}_0\setminus\mathcal{D}}\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}+n_{\mathcal{B}}\overline{w}_{\mathcal{B},1}+(n_{\mathcal{E}}-n_{\mathcal{X}})q_1}\right)}_{\text{Displacement}}$$

$$\underbrace{+\log\left(1+\frac{n_{\mathcal{B}}(\overline{w}_{\mathcal{B},1}-q_1)-n_{\mathcal{D}}\left(\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{D},0}-q_1\right)-(1-n_{\mathcal{P}_0})\left(\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0}-q_1\right)}{\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0}}\right)}_{\text{Demography}}. \tag{3}$$

This expression decomposes the logarithmic growth of the average wealth in the top percentile into three terms, which correspond to logarithmic versions of the intensive, displacement, and demography terms defined in (2).[11]

While this expression looks slightly more complex than (2), decomposing the logarithmic growth of the average wealth in the top percentile has two advantages. First, it makes it easier to combine the accounting framework over multiple time periods. Since logarithmic growth rates are time-additive, we can directly aggregate the accounting framework over time to express the cumulative growth of the average wealth in the top percentile as a sum of a cumulative intensive, displacement, and demography term. Second, it makes it more transparent to use the accounting framework to decompose the growth of the top percentile wealth *share*. Indeed, applying (3) on wealth *normalized* by the average wealth in the economy simply comes down to decreasing the intensive term by the log growth of the average wealth in the economy, $\log(\overline{w}_{\mathcal{I}_1,1}/\overline{w}_{\mathcal{I}_0,0})$.

Over typical horizons (e.g. a year), the difference between arithmetic and logarithmic growth rates, and therefore, between the arithmetic decomposition (2) and the logarithmic decomposition (3), is minimal. In fact, as the time period converges to zero, the two decompositions converge to the same limit, which I now characterize analytically.

3. ANALYTICAL RESULTS

I now examine the result of the accounting framework in a wide range of random-growth models. I obtain closed-form formulas for the intensive, displacement, and demography terms as the time period tends to zero (i.e., in the continuous-time limit). These analytical results are useful for two reasons. First, they provide a simple statistical framework to interpret the results of the accounting decomposition. Second, they can also help researchers to quantify the displacement and demography terms in the absence of panel data, which is a common occurrence.

As in the precious section, Section 3.1 starts with the baseline case of a fixed population while Section 3.2 considers the more general case of demographic changes.

3.1. *Case with Fixed Population*

3.1.1. *Diffusion Process*

Wealth Dynamics. Time $t \in [0,\infty)$ is continuous. Consider an economy populated by a continuum of agents that is fixed over time. Denote w_{it} the wealth of individual i at time t. I

[11] In words, the intensive term is the logarithmic growth of the average wealth of individuals initially in the top percentile, the displacement term is the log ratio between the average wealth in the top at time $t = 1$ and its counterfactual value after setting the wealth of individuals in $\mathcal{E}$ and $\mathcal{X}$ to q_1, and the demography term is the log ratio between this last value and the average wealth in the top at time $t = 1$ absent any composition change.

assume that the law of motion of individual wealth follows a diffusion process:

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_t(w_{it})\,\mathrm{d}t + \nu_t(w_{it})\,\mathrm{d}B_{it}, \tag{4}$$

where B_{it} is a standard idiosyncratic Brownian motion for individual i. The function $\mu_t(\cdot)$ and $\nu_t(\cdot)$ denote respectively the geometric drift and volatility of wealth, which depend on the wealth level.

I assume the following set of regularity conditions: (i) the initial wealth density, $g_0 : \mathbb{R}^+ \to \mathbb{R}^+$, has finite mean, (ii) $\mu_t(\exp(.))$ and $\nu_t(\exp(.))$ possess bounded continuous derivatives of all orders, and (iii) the idiosyncratic volatility never vanishes; that is, there exists $\epsilon > 0$ such that $\nu_t(.) \geq \epsilon$. This set of assumptions ensures that the wealth density, denoted by g_t, is smooth, positive on $\mathbb{R}^+$, and with finite mean.[12]

Top Percentile. As in the previous section, consider a top percentile $p \in (0, 1)$ of the wealth distribution, say the top 1%. Denote q_t the wealth of the last person in the top percentile (i.e., the top p quantile) and $\overline{w}_t$ the average wealth in the top percentile p. The following proposition characterizes the dynamics of the average wealth in the top percentile, $\overline{w}_t$:

PROPOSITION 1: *Assuming that wealth follows the diffusion process* (4)*, the average wealth in the top percentile, $\overline{w}_t$, follows the law of motion:*

$$\frac{\partial_t \overline{w}_t}{\overline{w}_t} = \underbrace{\mathrm{E}^{g_t \cdot}\left[\mu_t(w) | w \geq q_t\right]}_{\textit{Intensive}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\nu_t(q_t)^2}_{\textit{Displacement}}, \tag{5}$$

where $\mathrm{E}^{g_t \cdot}[.]$ denotes the wealth-weighted expectation with respect to the wealth density g_t.[13]

This proposition relates the dynamics of the average wealth in a top percentile to the dynamics of individual wealth (i.e., its drift and volatility) in continuous time.[14] In particular, it provides analytical expressions for the intensive and displacement terms defined in the accounting framework (2). The *intensive* term is the wealth-weighted drift in the top percentile group. The *displacement* is the product between the variance of wealth growth at the percentile threshold, $\nu_t(q_t)^2$ and a term that depends on the shape of the wealth distribution around the percentile threshold, $1/2\, g_t(q_t)q_t^2/(p\overline{w}_t)$.

These analytical expressions are useful in at least two different contexts. First, for researchers with access to panel data, they can be used to *complement* the result of the accounting framework. For instance, the proposition suggests that the displacement term can be further decomposed into a term due to the dispersion of wealth shocks, $\nu_t(q_t)^2$, and a term due to the shape of the wealth distribution, $g_t(q_t)q_t^2/(p\overline{w}_t)$. I will use this idea to shed light on the fluctuations of the displacement term observed for the Forbes 400 wealth share (see Section 4).

Second, for researchers without access to panel data, the proposition can be used as a *substitute* for the accounting framework. Indeed, it provides a simple way to quantify the displacement term, given an estimate for the wealth volatility at the percentile threshold $\nu_t(q_t)$

[12] See the proof of Proposition 1 in Appendix A.

[13] That is, $\mathrm{E}^{g_t \cdot}\left[\mu_t(w) | w \geq q_t\right] = \left(\int_{q_t}^{\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w\right) / \left(p\overline{w}_t\right)$.

[14] It can be seen (and it is actually derived) as an "integrated" version of Fokker-Planck (or Kolmogorov-Forward) equation, which is a fundamental equation relating the dynamics of the wealth *density* to the dynamics of individual wealth.

— the shape of the wealth distribution, $g_t(q_t)q_t^2/(p\overline{w}_t)$, can be directly estimated using cross-sectional data. I will use this idea to quantify the displacement term for top wealth percentiles over the 20th century (see Section 5).

I now present a heuristic derivation of the proposition to understand better the analytical expressions for the intensive and displacement terms (a formal proof is given in Appendix A).

HEURISTIC PROOF: Let us consider the economy during t and $t+\Delta t$, where Δt is a short period of time. During a short period of time, the diffusion process for wealth can be approximated by the following binomial process:

$$w_{it+\Delta t} = \begin{cases} (1+\mu_t(w)\Delta t - \nu_t(w)\sqrt{\Delta t})w_{it} & \text{with probability half,} \\ (1+\mu_t(w)\Delta t + \nu_t(w)\sqrt{\Delta t})w_{it} & \text{otherwise.} \end{cases} \tag{6}$$

We apply the accounting framework (2) to decompose the growth of the average wealth in the top percentile between t and $t+\Delta t$ into the sum of a intensive and a displacement term. The intensive term, which is the wealth growth of individuals initially in the top percentile, is simply:

$$\text{Intensive} = \mathrm{E}^{g_t\cdot}\left[\mu_t(w)\Delta t | w \geq q_t\right],$$

which corresponds to the analytical expression given in (5).

The displacement term is the sum of an inflow and outflow terms. As defined in (2), the inflow term measures the contribution of individuals just below the percentile threshold who enter the top percentile group after receiving a positive wealth shock. At the first order in Δt, the inflow term can be approximated by:

$$\begin{aligned} \text{Inflow} &\approx \frac{1}{p\overline{w}_t}\int_{\frac{q_t}{1+\nu_t(q_t)\sqrt{\Delta t}}}^{q_t} \left(\left(1+\nu_t(q_t)\sqrt{\Delta t}\right)w - q_t\right)\frac{1}{2}g_t(w)\,\mathrm{d}w \\ &\approx \left(\frac{1}{2}\frac{g_t(q_t)q_t}{p}\nu_t(q_t)\sqrt{\Delta t}\right)\left(\frac{1}{2}\frac{q_t}{\overline{w}_t}\nu_t(q_t)\sqrt{\Delta t}\right), \end{aligned}$$

using the midpoint rule to approximate the integral. This expression has a simple interpretation: the first term in the product corresponds to the mass of individuals who enter the top percentile between t and Δt, relative to the mass of individuals in the top percentile, while the second term corresponds to the average effect of an entrant on the growth of the average wealth in the top percentile (denoted respectively by $n_{\mathcal{E}}$ and $(\overline{w}_{\mathcal{E},1} - q_1)/\overline{w}_{\mathcal{P}_0,0}$ in (2)).

Symmetrically, the outflow term measures the contribution of individuals just above the percentile threshold who receive a negative wealth shock and exit the top percentile. Because the mass of individuals in the top percentile remains constant, these individuals are replaced by individuals at the percentile threshold, with wealth q_t. For the process (6), the outflow ter, can be approximated by:

$$\begin{aligned} \text{Outflow} &\simeq \frac{1}{p\overline{w}_t}\int_{q_t}^{\frac{q_t}{1-\nu_t(q_t)\sqrt{\Delta t}}} \left(q_t - \left(1-\nu_t(q_t)\sqrt{\Delta t}\right)w\right)\frac{1}{2}g_t(w)\,\mathrm{d}w \\ &\simeq \left(\frac{1}{2}\frac{g_t(q_t)q_t}{p}\nu_t(q_t)\sqrt{\Delta t}\right)\left(\frac{1}{2}\frac{q_t}{\overline{w}_t}\nu_t(q_t)\sqrt{\Delta t}\right). \end{aligned}$$

This expression also has a simple interpretation: the first term in the product corresponds to the mass of individuals who exit the top percentile between t and Δt, relative to the mass of individuals in the top, while the second term corresponds to the average effect of an exit on the growth of the average average wealth in the top percentile (denoted respectively by $n_{\mathcal{X}}$ and $(q_1 - \overline{w}_{\mathcal{X},1})/\overline{w}_{\mathcal{P}_0,0}$ in (2)). The inflow term equals the outflow term; adding them together gives the expression for the displacement term in (5). *Q.E.D.*

Stationary Case. One of the most ubiquitous regularities in economics and finance is that the right tail of many distributions, including the wealth distribution, is well approximated by a power law, i.e.,

$$g(w) \sim Cw^{-\zeta-1} \text{ as } w \to \infty, \tag{7}$$

where $C > 0$ is a constant.[15] We say that the distribution has a Pareto tail, and the exponent $\zeta > 0$ is called the Pareto exponent of the wealth distribution: the lower the ζ, the higher the level of wealth inequality.

The following set of assumptions ensures that the stationary distribution corresponding to the wealth process (4) has a Pareto tail: (i) the drift and volatility of wealth are constant over time (i.e., $\mu_t(w) = \mu(w)$ and $\nu_t(w) = \nu$) (ii) they converge to constant at high wealth level (i.e., $\mu(w) \to \mu < 0$ and $\nu(w) \to \nu$ as $w \to \infty$), and (iii) the wealth drift is large enough at low wealth level (i.e., $\mu(w) - \nu(w)^2/2 \geq K|\ln w|^\beta$ near 0, for some $\beta > 1$).[16] In particular, the assumption that $\mu < 0$ ensures that the wealth distribution has a finite mean; that is, its Pareto exponent is such that $\zeta > 1$.

A key property of a distribution with a Pareto tail is that it is "scale-free". In particular, it implies that the ratio between the mass of individuals around a percentile threshold and the mass of individual in the top percentile converges to a constant in the right tail; that is, $g(q)q/p \to \zeta$ as $p \to 0$. It also implies that the ratio between wealth at a top percentile threshold and the average wealth above the threshold is constant in the right tail; that is, $q/\overline{w} \to 1 - 1/\zeta$ as $p \to 0$.[17]

Given these results, taking the limit $p \to 0$ in Proposition 2 gives the following balance equation for top wealth shares:ves the following balance equation

$$0 = \underbrace{\mu}_{\text{Intensive}} + \underbrace{\frac{\zeta - 1}{2}\nu^2}_{\text{Displacement}}. \tag{8}$$

In the right tail of the distribution, the displacement term only depends on ζ, the Pareto exponent of the wealth distribution, as well as ν, the idiosyncratic volatility of wealth growth in the right tail.

The displacement term increases with the Pareto exponent ζ (that is, it decreases with the level of wealth inequality). This comes from two reasons. First, when inequality increases, the wealth distribution becomes more spread out around the percentile threshold, which decreases the fraction of individuals who enter or exit the top percentile. Second, and more importantly, when inequality increases, wealth at the percentile threshold becomes smaller relative to the average wealth in the top percentile, which decreases the contribution of each entry or exit

[15] Here and elsewhere, "$f(x) \sim g(x)$" for two functions f and g means $\lim_{x\to\infty} f(x)/g(x) = 1$.

[16] See Karlin and Taylor (1981). Alternatively, we could also have a reflecting boundary at some wealth level, as in Gabaix et al. (2016).

[17] Here, and elsewhere, I remove the t subscript to refer to quantities associated with the stationary wealth distribution.

to the top percentile.[18] In the limit where the Pareto exponent converges to $\zeta = 1$ (Zipf's law), wealth at the percentile threshold becomes infinitesimally small compared to the average wealth in the top percentile, and, therefore, the displacement term converges to zero.

This expression gives a simple rule of thumb to quantify the displacement term, even outside the steady-state. In the U.S., the Pareto exponent of the wealth distribution is approximately $\zeta = 1.5$.[19] This implies that, for an idiosyncratic volatility of wealth growth $\nu = 0.15$, we can expect the displacement term to be around 0.5% per year. Doubling the idiosyncratic volatility to $\nu = 0.30$ quadruples the displacement term, to 2% per year. This simple calculation suggests that the displacement term can be large, and that changes in idiosyncratic volatility can generate sizable changes in the displacement term.

Finally, note that we can use Equation (8) to pin down the Pareto exponent ζ of the stationary distribution. The corresponding expression, $\zeta = 1 - 2\mu/\nu^2$, is well-known in the inequality literature.[20] Equation (8)allows us to interpret it as a balance equation for the average wealth in a top percentile: the steady state Pareto exponent ζ must adjust so that the (positive) displacement term $^1/_2(\zeta - 1)\nu^2$ exactly compensates the (negative) intensive term μ.

3.1.2. *More General Wealth Processes*

We have derived an analytical expression for the displacement term when wealth follows a simple diffusion process. In Appendix B, I obtain similar expressions when wealth follows more complex processes. I now discuss briefly these results.

Type Heterogeneity. The preceding analysis assumed that the drift and volatility of wealth only depended on the wealth level. In reality, they also depend on other characteristics, such as age or preferences. To handle this case, I consider an economy composed of several groups of individuals, where groups potentially differ in their drifts and idiosyncratic volatilities. I find that, in this economy, the intensive and displacement terms remain essentially the same: as in the baseline model, they depend respectively on the wealth-weighted average drift of individuals in the top percentile and on the average idiosyncratic variance of individuals at the percentile threshold — heterogeneous drifts only have a second order effects on the displacement term.

Aggregate Shocks. The preceding analysis assumed that individuals were only exposed to idiosyncratic shocks. In reality, individuals are also differently exposed to aggregate shocks, such as industry specific shocks. To handle this case, I consider an economy in which the process for individual wealth is the sum of an idiosyncratic and an aggregate Brownian Motions. I find that, in this economy, the intensive term is stochastic: its exposure to aggregate shocks is given by the wealth-weighted average exposure of individuals in the top percentile. The displacement term now depends not only on the average idiosyncratic variance of individuals at the percentile threshold, but also on the cross-sectional variance of their exposures to aggregate shocks.

Jump-Diffusion Model The preceding analysis assumed that individual wealth followed a diffusion process (e.g., normal innovations). This implied that, during a short period of time, only individuals close to the percentile threshold could enter or exit the top percentile. In reality,

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[18] As shown in the heuristic proof of Proposition 1, during a short period of time Δt, the first term corresponds to $(g(q)q/p)\,\nu\sqrt{\Delta t} \to \zeta\nu\sqrt{\Delta t}$, while the second term corresponds to $^1/_2(q/\overline{w})\nu\sqrt{\Delta t} \to {}^1/_2(1-1/\zeta)\nu\sqrt{\Delta t}$ as $p \to 0$. The product between the two is equal to the displacement term, $^1/_2\left(g(q)q^2/(p\overline{w})\right)\nu^2\Delta t \to {}^1/_2(\zeta-1)\nu^2\Delta t$.

[19] See, for instance, Klass et al. (2006).

[20] See, for instance, Gabaix (2009).

the rapid rise of a few individuals at the top of the distribution suggests that jumps may play an important role in composition changes in the top percentile.

To understand how jumps (e.g., non-normal innovations) affect the formula for the displacement term, I consider an economy in which individual wealth follows a jump-diffusion process. In this case, the displacement term does not only depend on variance of wealth growth and the wealth density at the percentile threshold: it also depends on all of the higher-order cumulants of wealth growth as well as well as on the higher-order derivatives of the wealth density around the percentile threshold.

Sill, the expression drastically simplifies when higher-order cumulants do not depend on the wealth level and when the stationary wealth distribution has a Pareto tail (with Pareto exponent $\zeta \geq 1$). In this case, in the right tail of the distribution, the displacement term converges to:[21]

$$\begin{aligned}\text{Displacement} = &\frac{\zeta-1}{2}\text{sd}^2 + \frac{\zeta^2-1}{3!}\cdot\text{skewness}\cdot\text{sd}^3 + \frac{\zeta^3-1}{4!}\cdot\text{excess kurtosis}\cdot\text{sd}^4 \\ &+ \text{higher-order terms.}\end{aligned} \tag{9}$$

This equation expresses the displacement term as a infinite series. While the first term is the same as the one obtained in the diffusion model, the other terms in the sum reflect the contribution of higher-order cumulants of log wealth growth, such as its skewness and excess kurtosis.

Interestingly, the relative importance of higher-order cumulants increases with ζ; that is, it decreases with the level of wealth inequality. Intuitively, the higher the level of wealth inequality, the more spread out the wealth distribution is, and, therefore, the less likely it is to enter the top percentile far away from the percentile threshold.[22] Empirically, the Pareto exponent of the wealth distribution in the U.S. is close to one ($\zeta = 1.5$), which suggests that the effect of higher-order cumulants on the displacement term may be limited — I will return to this topic when decomposing the growth of the Forbes 400 wealth share in Section 4.

3.2. *Case with Demographic Changes*

For the sake of simplicity, the preceding analysis abstracted away from demographic changes. In reality, birth, death, and population growth changes the composition of individuals in the economy. As in Section 2, these demographic changes generate additional composition changes in the top percentile, which affect the law of motion of the average wealth in a top percentile.

I assume that individuals die with a hazard rate $\delta_t > 0$ (independent on wealth) and that population grows at rate $\eta_t > 0$, where both rates are continuous with respect to time. Moreover, I assume that individual are born with wealth density g_{Bt}, which is continuous with respect to time and such that that the average wealth of newborn agents is finite; that is, $\int_{\mathbb{R}+} w g_{Bt}(w)\,\mathrm{d}w < \infty$. The following proposition characterizes the dynamics of the average wealth in the top percentile, $\overline{w}_t$, in terms of the dynamics of individual wealth:

[21] In a completely different context, Martin (2013) and Schmidt (2016) derive similar expressions for the risk-free rate and the equity premium in representative agent models in terms of the higher-order cumulants of (log) consumption growth. In these papers, the relative effect of higher-order cumulants depend on the curvature of the utility function (as summarized by the Relative Risk Aversion), rather than the curvature of the wealth distribution, as summarized by ζ.

[22] To take a quantitative example, going from a distribution with $\zeta = 2.5$ (the approximate Pareto exponent of the labor income distribution) to $\zeta = 1.5$ (the approximate Pareto exponent of the wealth distribution), the term due to the variance of wealth shocks is divided by 3, the term due to skewness is divided by 4, while the term due to excess kurtosis is divided by 6.

PROPOSITION 2: *Assuming that wealth follows the diffusion process (4) with death rate δ_t, population growth η_t, and wealth density of newborn agents g_{Bt}, the average wealth in the top percentile $\overline{w}_t$ follows the law of motion:*

$$\begin{aligned}\frac{\partial_t \overline{w}_t}{\overline{w}_t} &= \underbrace{\mathrm{E}^{g_t \cdot}\left[\mu_t(w) | w \geq q_t\right]}_{\textit{Intensive}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\nu_t(q_t)^2}_{\textit{Displacement}} \\ &\quad + \underbrace{\underbrace{\frac{1}{p\overline{w}_t}\left(\int_{q_t}^{\infty}(w-q_t)g_{Bt}(w)\,\mathrm{d}w\right)(\delta_t+\eta_t)}_{\textit{Birth}} \underbrace{-\frac{\overline{w}_t - q_t}{\overline{w}_t}\delta_t}_{\textit{Death}} \underbrace{-\frac{\overline{w}_t - q_t}{\overline{w}_t}\eta_t}_{\textit{Pop. Growth}}}_{\textit{Demography}}.\end{aligned} \tag{10}$$

The new demography term accounts for the effect of birth, death, and population growth. As in the heuristic derivation for Proposition 1, we can obtain the expressions for the birth, death, and population growth terms by applying the accounting framework (2) between t and Δt, and approximating at the first order in Δt. For instance, the death term depends on the relative fraction of individuals in the top percentile that die, $\delta_t \Delta t$, times the difference between the average wealth of individuals who die and the wealth of individuals who replace them, relative to the average wealth in the top percentile, $-(\overline{w}_t - q_t)/\overline{w}_t$. A similar derivation holds for the terms due to population growth or birth.

Because wealth at the threshold is always lower or equal to the average wealth above the threshold, the terms due to death and population growth are both negative. In contrast, the term due to birth is always positive. I now examine a particular model of inheritance to characterize more precisely the sign of the demography term.

A Simple Model of Inheritance. I assume that each individual has k children, where k is a bounded random variable, taking values in $\mathbb{N}^*$. For simplicity, I assume that children are born exactly when their parents die. Note that, to be consistent with a population growth rate of η_t, the average number of children per parent must satisfy $\mathrm{E}_k[k]\delta_t = \eta_t + \delta_t$, where $\mathrm{E}_k[\cdot]$ denotes the average with respect to k. When individuals die, they bequest a proportion $\chi \in (0, 1]$ of their wealth to their children, which is shared equally.[23]

This simple model of inheritance allows me to relate the wealth distribution of newborns, g_{Bt}, to the wealth distribution of existing agents, g_t, and, therefore, to re-express the birth term as:[24,25]

$$\text{Birth} = \frac{1}{p\overline{w}_t}\mathrm{E}_k\left[\int_{\frac{k}{\chi}q_t}^{\infty}\frac{k}{\mathrm{E}_k[k]}\left(\frac{\chi}{k}w - q_t\right)g_t(w)\,\mathrm{d}w\right](\delta_t+\eta_t). \tag{11}$$

[23] Menchik (1980) provides evidence of equal sharing among children. Cowell (1998) studies a related model of inheritance, and its effect on the Pareto exponent of the wealth distribution.

[24] Formally, we have

$$g_{Bt}(w) = \mathrm{E}_k\left[\frac{k}{\mathrm{E}_k[k]}g_t\left(\frac{k}{\chi}w\right)\frac{k}{\chi}\right].$$

Plugging into the expression for the birth term in (5) gives the result.

[25] A similar expression obtains if the inheritance rate χ itself is stochastic — we simply need to replace $\mathrm{E}_k[\cdot]$ by $\mathrm{E}_{k,\chi}[\cdot]$, the joint expectation with respect to k and χ.

This expression reflects the fact that, when an individual with wealth w and a number of children k dies, her children enter the top percentile as long as their wealth at birth, $(\chi/k)w$, is higher than the wealth at the top percentile threshold, q_t.

The birth term increases with the inheritance rate χ. It also increases after a mean-preserving spread in the distribution of children k: this is because only children who inherit enough enter the top percentile.[26]

Finally, with this particular model of inheritance, the birth term never fully compensates the terms due to death and population growth; that is, the demography term is always negative. Even when individuals can fully pass their wealth to their children (i.e., $\chi = 1$), the wealth of parents must still be shared across multiple children.

Stationary Case. As above, the demography term takes a particularly simple form when the wealth distribution has a Pareto tail. As above, assume that $\mu_t(w) = \mu(w)$, $\nu_t(w) = \nu(w)$, with $\mu(w) \to \mu < \eta + \delta$ and $\nu(w) \to \nu$ as $w \to \infty$), as well as $\delta_= \delta$ and $\eta_t = \eta$. Under these assumptions, the stationary distribution, if it exists, has a Pareto tail with Pareto exponent $\zeta > 1$.

As seen above, for a distribution with a Pareto tail, the ratio between wealth at the threshold and wealth above the threshold converge to a constant, $q/\overline{w} \to 1 - 1/\zeta$ as $p \to 0$. This implies that the expression for the death and population growth terms (given in Proposition 2) converge respectively to $-\delta/\zeta$ and $-\eta/\zeta$. Similarly, we can also simplify the birth term obtained in the inheritance model (11) since:

$$\frac{1}{p\overline{w}} \int_{\frac{k}{\chi}q}^{\infty} \left(\frac{\chi}{k} w - q\right) g(w) \,\mathrm{d}w \to \frac{1}{\zeta} \left(\frac{\chi}{k}\right)^{\zeta} \text{ as } p \to 0.$$

As a result, taking the limit $p \to 0$ in Proposition 2 and (11) gives the following balance equation for top wealth shares:

$$0 = \underbrace{\mu}_{\text{Intensive}} + \underbrace{\frac{\zeta - 1}{2}\nu^2}_{\text{Displacement}} + \underbrace{\underbrace{\frac{1}{\zeta}\mathrm{E}_k\left[\frac{k}{\mathrm{E}_k[k]}\left(\frac{\chi}{k}\right)^{\zeta}\right](\delta + \eta)}_{\text{Birth}} \underbrace{-\frac{1}{\zeta}\delta}_{\text{Death}} \underbrace{-\frac{1}{\zeta}\eta}_{\text{Pop. Growth}}}_{\text{Demography}} . \tag{12}$$

The shape of the wealth distribution is simply summarized by its Pareto exponent, ζ. Moreover, the demography term increases with the Pareto exponent ζ; that is, it decreases with the level of wealth inequality. This comes from the fact that, when wealth inequality increases, wealth at the percentile threshold becomes smaller relative to the average wealth in the top percentile, which magnifies the negative effect of death and population growth.

This expression gives a simple rule of thumb to quantify the demography term, even outside the steady-state. To take an example, consider an economy in which the wealth distribution has a Pareto tail with exponent $\zeta = 1.5$, the death rate is 2%, the population growth rate is 1%, and the inheritance rate $\chi = 50\%$ (which is roughly consistent with the U.S. economy with a marginal estate tax rate is close to 50%). In this case, the formula yields a demography term

[26] This comes from the fact that the following function is convex:

$$k \to \int_{\frac{k}{\chi}q_t}^{\infty} k\left(\frac{\chi}{k} w - q_t\right) g_t(w) \,\mathrm{d}w.$$

equals to -1.5%.[27] As shown below, this is close to the actual magnitude of the demography term for the Forbes 400 wealth share.

4. EMPIRICAL RESULTS

In this section, I decompose the growth of the wealth share of the Forbes 400 using the accounting framework developed in Section 2. I interpret the results through the lens of the statistical framework developed in Section 3. In particular, I show that the simple diffusion model presented in the previous section captures well the level and cyclicality of the displacement term.

Section 4.1 presents the data. Section 4.2 discusses the result of the accounting decomposition. Section 4.3 discusses the robustness of my results.

4.1. *Data*

I focus on the list of the wealthiest 400 American households constructed by *Forbes Magazine* annually since 1983.[28] The list is created by a dedicated staff of the magazine, based on a mix of public and private information.[29] *Forbes* nominatively identifies the 400 wealthiest individuals in the U.S., allowing me to track the wealth of individuals from one year to the next, which is key for the accounting decomposition.[30] By contrast, other data sources used to track the level of wealth inequality in the U.S. rely on repeated cross-sections.[31] The Forbes 400 list includes 1,518 distinct households between 1983 and 2017. I remove from the entire sample all households that were later removed by *Forbes* due to posterior corrections or revisions in methodology (73 households).

I obtain the total wealth of U.S. households from the Financial Accounts of the United States and the number of U.S. households from the U.S. Census Bureau. I convert all wealth to 2017 dollars using the PCE price index.

I focus on the percentile group that includes the entirety of households in the Forbes 400 list in 2017. Because a percentile includes a constant fraction of total population, this top percentile only includes 264 households in 1983. With a slight abuse of language, I use the term "Forbes 400 percentile" to refer to this top percentile and "Forbes 400 wealth share" to refer to its share of aggregate wealth.

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[27] For this computation, I assume that each households gives birth to either one or two households with equal probability, i.e. $\mathrm{P}(k=1)=\mathrm{P}(k=2)=0.5$.

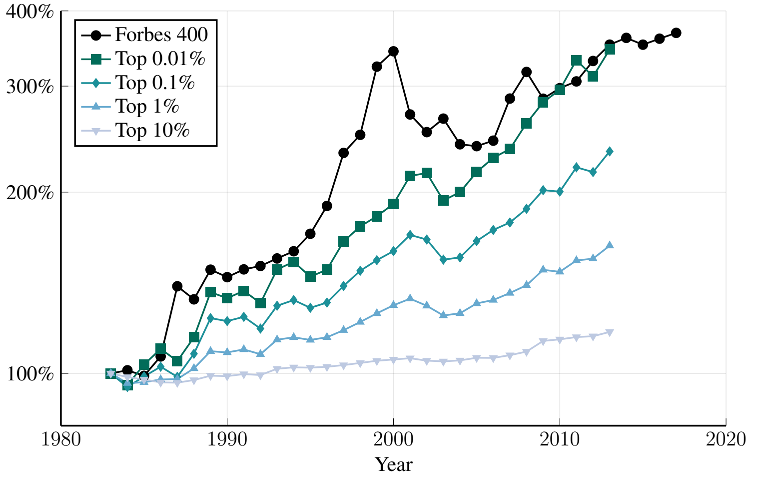
[28] This dataset has been examined by economists in the past. For instance, Kaplan and Rauh (2013a), Kaplan and Rauh (2013b), and Capehart (2014) examine characteristics of households in the Forbes 400. Saez and Zucman (2016) compare their estimates for the top 0.01% and the ones implied by *Forbes*. Gârleanu and Panageas (2017) stress the growth of self-made billionaires over the long run compared to pre-existing billionaires.

[29] *Forbes Magazine* reports that "we pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don't pretend to know what is listed on each billionaire's private balance sheet, although some candidates do provide paperwork to that effect."

[30] I extend the construction from Capehart (2014) to the 2012–2017 period. In certain cases, *Forbes* does not report the wealth of individuals who exit the Forbes 400. For these cases, I use a Kaplan and Meier (1958) estimator to obtain an estimate for the average wealth conditioning on exiting the Forbes 400. Appendix D discusses the accuracy of this imputation method.

[31] The three main datasets on the wealth distribution in the U.S. are the Survey of Consumer Finances, Estate Tax Returns (see Kopczuk and Saez, 2004), and Income Tax Returns (see Saez and Zucman, 2016), which all correspond to repeated cross-sections.

FIGURE 2.—Cumulative Growth of Top Wealth Percentiles in the U.S. [a]



[a]The figure plots the cumulative growth of the Forbes 400 wealth share and of the wealth share of the top 10%, 1%, 0.1%, 0.01% from Saez and Zucman (2016).

While this top percentile accounts for a small percentage of the total U.S. population (3% of the top 0.01%), it accounts for a substantial share of total U.S. wealth (approximately 3% in 2017). Figure 2 plots the cumulative growth of the Forbes 400 wealth share since 1983 as well as the cumulative growth of the wealth share of the top 0.01%, 0.01%, 1%, and 10% from Saez and Zucman (2016). As discussed in their paper, most of the increase of wealth inequality during the period is concentrated in the top 0.01%. Moreover, the growth of the top 0.01% tracks closely the growth of the Forbes 400 wealth share, which suggests that understanding the growth of the Forbes 400 wealth share can shed light on the overall rise in top wealth inequality in the U.S. during this period.

4.2. *Results*

I apply the accounting framework (3) for the Forbes 400 percentile. Because I am interested in decomposition the Forbes 400 wealth *share*, I apply the accounting framework on wealth *normalized* by the average wealth in the economy. As discussed in Section 2, this means that the intensive term corresponds to the difference between the growth of the (raw) average welath of individuals in the top percentile and the growth of the average wealth in the economy.

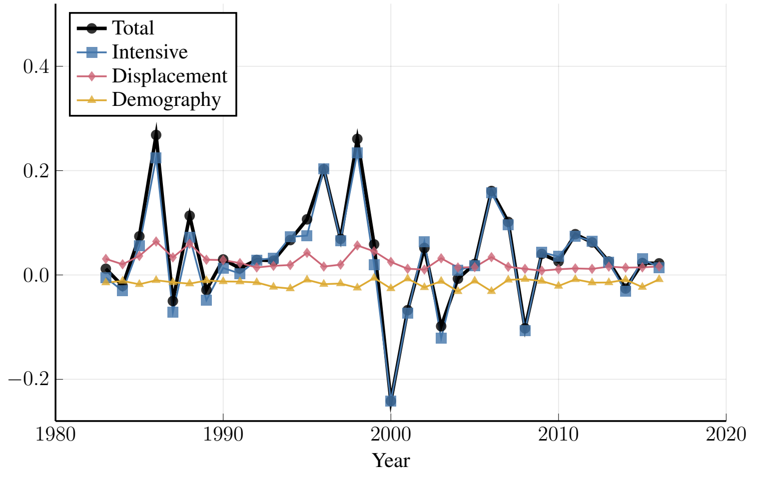
I classify as birth (i.e., $\mathcal{B}$) any entry in the top percentile following the death of a family number. Conversely, I also classify as death (i.e., $\mathcal{D}$), any exit from the top percentile which is linked to an in-vivo transfer. This allows me to obtain measures of the intensive and displacement terms that are not impacted by within-family transfers.

The first row of Table Ia shows each term averaged over the entire time period. I find that the 3.8% yearly growth of the Forbes 400 wealth share during the time period is the sum of a intensive term equal to 3.0%, a displacement term equal to 2.4%, and a demography term equal to -1.5%. This means that displacement (i.e., change in relative rankings among households) had a first-order effect on the growth of the top wealth share: without it, the growth of the top wealth share would have been 1.4% instead of 3.8%.

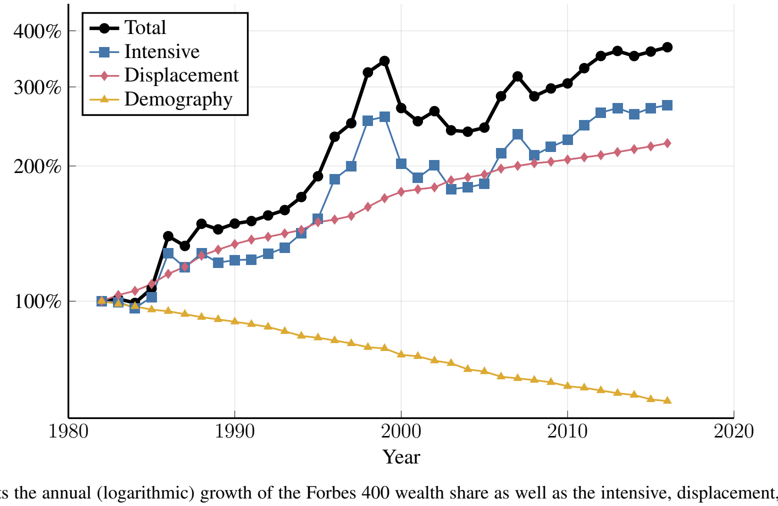
Figure 3 plots the result of the accounting decomposition every year as well as the cumulative sum of each term over time. Yearly fluctuations in the Forbes 400 wealth share are almost entirely driven by the intensive margin. These large fluctuations of the intensive term reflect the high fluctuations of stock market returns — indeed, top households tend to be overexposed to stock market returns relative to the average household in the economy.[32] By comparison, fluctuations in the displacement or demography terms appear to be more muted.

FIGURE 3.—Decomposing the Growth of the Forbes 400 Wealth Share[a]

(a) Annual Growth



(b) Cumulative Growth



[a]Figure 3a plots the annual (logarithmic) growth of the Forbes 400 wealth share as well as the intensive, displacement, and demography terms defined in the logarithmic accounting framework (3). Figure 3b plots the same series cumulated over time. Data are from *Forbes* and the Financial Accounts of the United States.

[32]See, for instance, Wolff (2002) or Gomez (2016).

TABLE I

DECOMPOSING THE GROWTH OF THE FORBES 400 WEALTH SHARE[a]

(a) Summary

	Total (%)	Intensive (%)			Displacement (%)	Demography (%)
		Total	Top	−U.S.		
All Years	3.8	3.0	5.5	–2.5	2.4	–1.5
1983-1994	4.2	2.5	5.1	–2.6	3.2	–1.4
1994-2005	3.7	2.8	6.3	–3.5	2.6	–1.8
2005-2017	3.6	3.5	5.1	–1.6	1.5	–1.4

(b) Displacement (Details)

	Inflow (%)			Outflow (%)		
	Total	$n_{\mathcal{E}}$	$\frac{\overline{w}_{\mathcal{E},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{X}}$	$\frac{q_1-\overline{w}_{\mathcal{X},1}}{\overline{w}_{\mathcal{P}_0,0}}$
All Years	1.9	12.3	15	0.6	9.7	6
1983-1994	2.4	15.4	15	0.9	12.5	7
1994-2005	2.2	12.8	16	0.5	10.2	5
2005-2017	1.2	9.0	13	0.4	6.8	5

(c) Demography (Details)

	Birth (%)			Death (%)			Pop. Growth (%)		
	Total	$n_{\mathcal{B}}$	$\frac{\overline{w}_{\mathcal{B},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{D}}$	$\frac{q_1-\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{D},0}}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$1-n_{\mathcal{P}_0}$	$\frac{q_1-\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0}}{\overline{w}_{\mathcal{P}_0,0}}$
All Years	0.4	0.8	66	–1.1	2.2	–51	–0.8	1.2	–68
1983-1994	0.4	1.2	76	–1.0	2.7	–38	–0.7	1.3	–59
1994-2005	0.4	0.9	42	–1.2	2.1	–57	–1.0	1.4	–72
2005-2017	0.4	0.5	77	–1.1	1.8	–59	–0.7	0.9	–73

[a] Table Ia reports the average of the annual (logarithmic) growth of the Forbes 400 wealth share as well as the intensive, displacement, and demography terms defined in the (logarithmic) accounting framework (3). Table Ib and Table Ic report the average of the inflow, outflow, birth, death, and population growth terms defined (2). Data are from *Forbes* and the Financial Accounts of the United States.

To focus on their low-frequency fluctuations, Table Ia shows each term averaged across three time periods of equal duration since 1983, which roughly correspond to distinct business cycles. The first period covers 1983–1994, which includes the 1990–1991 recession. The second period covers 1994–2005, which includes the 2001 recession, and the third period covers 2005–2017, which includes the 2007–2009 recession. The key finding is that the displacement term exhibits important fluctuations over time. In particular, the displacement term has been steeply decreasing over time: it goes from 3.2% in the first part of the sample (1983–1994) to 1.5% in the third part of the sample (2005–2017). In the rest of the section, I examine separately the displacement and demography terms through the lens of the statistical model from Section 3.

4.2.1. *Displacement*

In this section, I examine the displacement term through the lens of the statistical model developed in Section 3. I show that a simple diffusion model captures well the magnitude and the dynamics of the displacement term. I then use the model to analyze the dynamics of the displacement term over time.

Variance. The diffusion model predicts that, at short horizons, the displacement term should be close to $1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$, where $g_t(q_t)$ denotes the wealth density at the percentile threshold, q_t denotes the wealth at the percentile threshold, $\overline{w}_t$ denotes the average wealth in the top, and $\nu_t(q_t)$ denotes the yearly standard deviation of wealth growth at the percentile threshold.

Since all of these quantities can be estimated in the data, we can construct a model-implied displacement term and compare it to the actual displacement term. Doing this will provide evidence on whether the diffusion model provides a good fit to explain the data. Indeed, the diffusion model made a certain number of assumptions that are not necessarily satisfied in the data, including that wealth follows a diffusion, the time period tends to zero, and the number of agents tends to infinity.

I first estimate $\nu_t(q_t)^2$ every year by regressing (log) wealth growth and its square on log wealth around the percentile threshold.[33] I obtain an estimate for the yearly standard deviation by combining the local estimates for the first and second moments. As shown in Table IIb, I obtain an average standard deviation equal to 0.27. While this term tends to decline over time, Figure 4a shows there are important fluctuations at the business cycle frequency, with a particularly large spike during the dot-com bubble.

I then estimate $g_t(q_t)q_t^2/(p\overline{w}_t)$ every year. Remember that this term can be written as the product of $q_t/\overline{w}_t$, the ratio between wealth at the percentile threshold and average wealth above the threshold, and $g_t(q_t)q_t/p$, the elasticity of the mass of households above a wealth threshold with respect to the wealth threshold. The first term can be directly observed in the data, while the second term can be estimated by regressing the log number of households above a given wealth level on log wealth around the percentile threshold.[34] Table IIa reports the results of this estimation. The ratio between wealth at the percentile threshold and wealth above the threshold, $q_t/\overline{w}_t$, averages to 0.34. The average elasticity of the counter-cumulative distribution function to wealth, $g_t(q_t)q_t/\overline{w}_t$, averages to 1.29. Finally, the product, of these two terms, $g_t(q_t)q_t^2/(p\overline{w}_t)$, averages to 0.45—if the distribution had a Pareto tail, this would correspond to a Pareto exponent ζ of 1.45, which is consistent with existing studies.[35] As shown in Figure 4a, the term decreases slowly over time, indicating a rise in wealth concentration.

I then combine these two estimates to construct a model-implied displacement term $1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$. Table IIc shows that the model-implied displacement term averages to 1.7%, which tends to be lower than the actual displacement term, 2.4%. Figure 4b plots the model-implied displacement term and the actual displacement term every year. The model-implied term tracks well the dynamics of the displacement term and, in particular, its decline over time.

As discussed in Section 3, we can decompose the decline of the model-implied displacement term between 1983–1994 and 2005–2017 into a term due to the decline in the variance of wealth growth and a term due to a rise in wealth concentration:

$$\underbrace{\Delta\left(\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\nu_t(q_t)^2\right)}_{-1.3\%} = \underbrace{\left\langle\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\right\rangle\Delta\left(\nu_t(q_t)^2\right)}_{-0.3\%} + \underbrace{\langle\nu_t(q_t)^2\rangle\Delta\left(\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\right)}_{-1.0\%},$$

[33] I use a local polynomial of degree one, with a triangular kernel with a bandwidth of one.

[34] Indeed, we have $\partial_{\ln q}\int_q^\infty g_t(w)\,\mathrm{d}w = g_t(q_t)q_t/p$. I use a kernel with a bandwidth of one around the percentile threshold.

[35] See, for instance, Klass et al. (2006).

where Δx (resp. $\langle x \rangle$) denotes the difference (resp. average) of a variable x between the two time periods. The result of this decomposition suggests that most of the decline in the displacement term is driven by a rise in wealth concentration, which made it harder for new fortunes to displace existing fortunes, rather than a decline in idiosyncratic shocks.

What explains the dispersion of wealth growth for households around the percentile threshold? One simple hypothesis is that it is driven by the cross-sectional dispersion of stock market returns: most households in the Forbes 400 tend to only invest in a few firms. To test this idea, I regress the variance of household-level wealth growth on the equal-weighted variance of firm-level returns, computed using stock-level returns from the Center for Research in Security Prices (CRSP). Table III shows that the resulting $R^2 \approx 0.73$ is high, suggesting the yearly fluctuations in the variance of firm returns explain very well the yearly fluctuations in the variance of wealth growth. The coefficient of 0.34 can be interpreted as saying that the average household in the Forbes 400 holds $\sqrt{0.34} \approx 60\%$ of its wealth in a firm (and the rest in a diversified portfolio of firms). Finally, the estimate for the intercept is close to zero, meaning that the level of portfolio concentration, identified purely from time-series variation, also accounts for the level of the variance of wealth growth. This suggests that the variance of wealth growth is almost entirely driven by the variance of firm-level returns.

Higher-Order Cumulants. As discussed in Section 3, when the process for wealth follows a jump-diffusion process, the displacement term depends not only on the variance of wealth growth but also on all higher-order cumulants of wealth growth. To simplify the analysis, I only estimate the contributions of the third and fourth cumulants. Moreover, I assume higher-order cumulants are independent of wealth and that the wealth distribution has a Pareto tail with Pareto exponent ζ. In this case, Equation (9) gives that the term due to skewness is $^1/_{3!}(\zeta^2 - 1) \cdot \text{skewness} \cdot \text{sd}^3$, while the term due kurtosis is $^1/_{4!}(\zeta^3 - 1) \cdot \text{excess kurtosis} \cdot \text{sd}^4$.

As above, I use local polynomial regressions to estimate the third and fourth moments of wealth growth at the percentile threshold. I estimate the Pareto exponent ζ as one plus the estimated $g_t(q_t)q_t^2/(p\overline{w}_t)$ to be consistent with the diffusive term estimated above. As shown in Table IIb, the average estimated skewness is negative at -0.34 (i.e., more downward realizations compared to the log-normal distribution), while the average excess kurtosis is positive at 6.6 (i.e., more extreme realizations compared to the log-normal distribution). After plugging these estimates into Equation (9), the skewness decreases the displacement term by 0.2%, while the excess kurtosis increases the displacement term by 0.4% annually (see Table IIc). This suggests that the effect of higher-order cumulants on the displacement term is relatively small. As seen in Section 3.1, the intuition is that wealth inequality is so high that, at short horizons, most of the entry in the top percentile is driven by households already close to the top percentile rather than by entrepreneurs from the bottom of the wealth distribution with extremely high wealth realizations.

Figure 4b plots the displacement term implied by the diffusion model (i.e., due to the variance of wealth growth) and the one implied by the jump-diffusion model (i.e., adding skewness and kurtosis). Both series remain close over time, suggesting higher-order cumulants do not matter much for the dynamics of the displacement term. One exception is 2000, when the term due to variance drastically overestimates the actual displacement term, while the term including skewness and kurtosis is much closer. This happens because during the burst of the dot-com bubble, the distribution of wealth shocks had a very high variance together with a very negative skewness. Overall, the effect of wealth growth dispersion on the displacement term was limited.

4.2.2. *Demography*

As shown in Table Ia, the demography term averages to -1.5%. It can be decomposed into a birth term, which averages to 0.4%; a death term, which averages to -1.1%; and a population

TABLE II

DISPLACEMENT IN THE FORBES 400 THROUGH THE LENS OF THE STATISTICAL MODEL [a]

(a) Shape of the Wealth Distribution

	$q_t/\overline{w}_t$	$g_t(q_t)q_t/p$	$g_t(q_t)q_t^2/(p\overline{w}_t)$
All Years	0.34	1.29	0.45
1983-1994	0.42	1.49	0.63
1994-2005	0.30	1.23	0.38
2005-2017	0.30	1.15	0.35

(b) Dispersion of Wealth Growth

	Std. Dev.	Skewness	Excess Kurt.
All Years	0.27	–0.34	6.58
1983-1994	0.27	–0.53	6.42
1994-2005	0.31	–0.26	7.70
2005-2017	0.25	–0.23	5.72

(c) Displacement implied by Higher-Order Cumulants

	Displacement Term				
	Actual (%)	Model Implied (%)			
		Total	Due to Std Dev.	Due to Skewness	Due to Excess Kurt.
All Years	2.4	2.0	1.7	–0.2	0.4
1983-1994	3.2	2.8	2.4	–0.2	0.5
1994-2005	2.6	1.9	1.8	–0.5	0.6
2005-2017	1.5	1.2	1.1	–0.1	0.2

[a]Table IIa shows summary statistics on the shape of the wealth distribution at the percentile threshold using local regression techniques. Table IIb shows the standard deviation, skewness and excess kurtosis of log wealth growth at the percentile threshold using local regression techniques. Table IIc uses these statistics as input in (9) to report the displacement term implied by the second, third, and fourth cumulants of log wealth growth. Data are from *Forbes*.

TABLE III

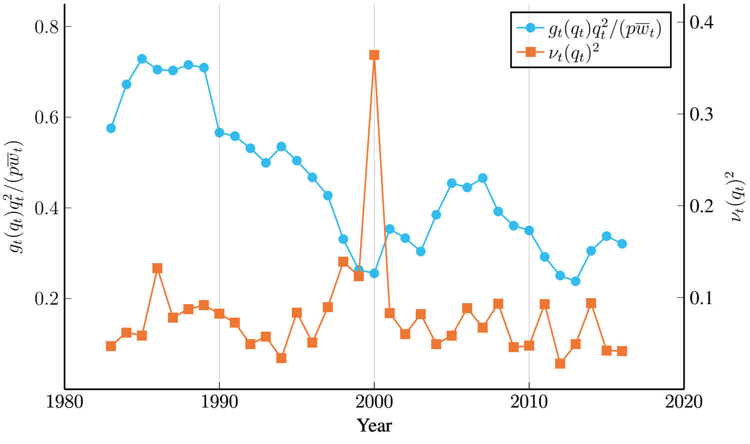
DISPERSION OF WEALTH GROWTH AND DISPERSION OF FIRM-LEVEL RETURNS[a]

	Variance of Wealth Growth
	(1)
Variance of Firm Returns	0.34
	(0.05)
Constant	–0.01
	(0.01)
R^2	0.73
Period	1983-2016
N	34

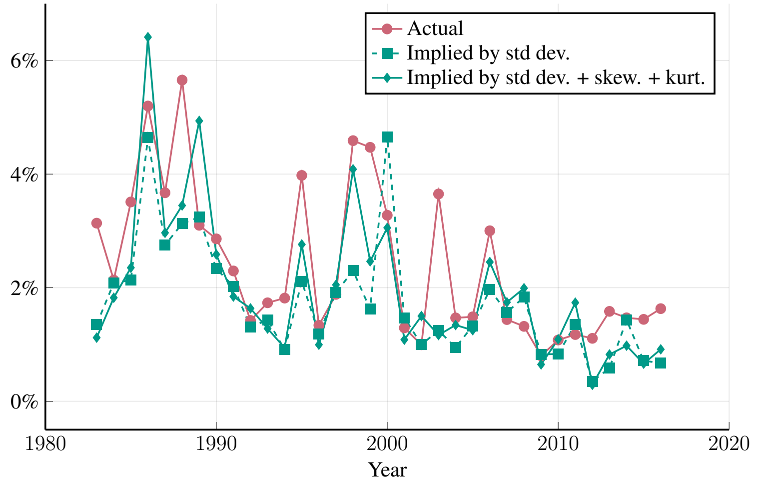
[a] The table shows the result of regressing the cross-sectional variance of (log) wealth growth $\nu_t(q_t)^2$ for the Forbes 400 on the cross-sectional variance of firm-level (log) returns. Estimation is done via OLS. Standard errors are in parentheses and estimated using Newey-West with three lags. Data are from *Forbes* and CRSP.

FIGURE 4.—Displacement in the Forbes 400 Through the Lens of the Statistical Model[a]

(a) Estimates of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$



(b) Displacement Term Implied by the Model



[a]Figure 4a plots the time series of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and the cross-sectional variance of log wealth growth at the percentile threshold $\nu_t(q_t)^2$. Figure 4b plots the displacement term, the one implied by the variance of log wealth growth (diffusion model), and the one implied by variance, skewness and excess kurtosis of log wealth growth (jump-diffusion model). Data are from *Forbes*.

growth term, which averages to -0.8% (Table Ic). I now discuss each term through the lens of the statistical model discussed in Section 3.

I start with the terms due to death and population growth. Proposition 2 predicts that, at short horizons, the terms due to death and population growth should be close to $-(1-q_t/\overline{w}_t)\delta_t$ and $-(1-q_t/\overline{w}_t)\eta_t$, respectively. As reported in Ic, the death rate of households in the Forbes 400 averages to 2.2%, while the population growth rate averages to 1.2% (as estimated by the average $n_{\mathcal{D}}$ and $1-n_{\mathcal{P}_0}$). Given that the ratio between wealth at the threshold and wealth above the threshold $q_t/\overline{w}_t$ averages to 0.34 (Table IIa), these estimates imply a model-implied death term equal to -1.5% and a model-implied population growth term equal to -0.8%. In the data, these terms are -1.1% and -0.8%, respectively. In other words, while the model perfectly matches the term due to population growth, it underestimates the term due to death. This reflects the fact that, during the time period, households that died tended to be less wealthy than the average household in the Forbes 400.

As shown in Table Ic, the terms due to death and population growth have remained pretty much constant over time, even though the rates of death in the Forbes 400 and the population growth rate have declined over time. The reason is that this decline in rates has been counterbalanced by a rise in wealth inequality (i.e., a decrease in $q_t/\overline{w}_t$), which magnifies the negative effect of death and population growth on the average wealth in the top percentile.

I now turn to the term due to birth. When the distribution has a Pareto tail, Equation (2) shows that the birth term should be close to $\mathrm{E}_k[(k/\mathrm{E}_k[k])(\chi/k)^\zeta](\delta+\eta)/\zeta$. I use the estimate for ζ obtained for the displacement term; that is, $1+g_t(q_t)q_t^2/(p\overline{w})$, which gives an average ζ of 1.45. I use an inheritance rate $\chi=0.5$, which is roughly consistent with the marginal estate tax during the period. Finally, I assume that the distribution for the number of children k is binomial, with $\mathrm{P}(k=1)=1-\mathrm{P}(k=2)=0.45$ to match the rate of population growth.[36] After combining these estimates, the model implies an average birth term equal to 0.5%, which is very close to the average birth term measured by the accounting decomposition, 0.4%. In summary, the simple inheritance model discussed in Section 3 explains well the magnitude of the demography term, as well as its decomposition between a birth term, a death term, and a population growth term.

4.3. *Robustness*

The wealth of the richest individuals in the economy is inevitably measured with errors. I now discuss the effect of these measurement errors for the results of the accounting decomposition.

The first concern is that *Forbes* may systematically overestimate the wealth of the top 400 households. Along these lines, Atkinson (2008) argues the magazine may give inflated values of the wealth of top households, because debts are harder to track than assets. Empirically, Raub et al. (2010) show that the wealth of deceased households reported on estate tax returns is approximately half of the wealth estimated by *Forbes*.[37] Fortunately, this type of measurement error in levels does not impact the growth of top wealth shares or its accounting decomposition.

A more serious concern is that *Forbes* measures the wealth of top households with noise. If the measurement error is completely persistent, as noted in Luttmer (2002), this would lead *Forbes* to overestimate the level of top wealth shares. As discussed above, this would not affect the growth of top wealth shares or its accounting decomposition. If, however, the measurement error has a transitory component, it may generate artificial entry and exit in the top percentile. This would lead me to underestimate the intensive term and overestimate the displacement term.

I deal with this potential bias in two ways. First, as explained above, I removed from my sample every household that was later removed by *Forbes* due to methodological error. Second, I check that wealth growth is close to a random walk in the remaining sample.[38] Table D.II in Appendix D shows that the autocorrelation of wealth growth at the individual level is close to zero, suggesting there is little mean reversion in wealth growth.[39]

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[36] Indeed, remember that we must have $\mathrm{E}_k[k]\delta=\eta+\delta$.

[37] Alternatively, this may reflect the fact that these households under-report their wealth on these tax returns.

[38] Indeed, economic theory suggests that wealth should be close to a random walk at large levels of wealth (see, e.g., Achdou et al. (forthcoming)).

[39] In Appendix D, I show that the relative bias in the displacement term is well approximated by -2ρ, where ρ is the AR(1) coefficient of wealth growth. With an estimated ρ approximately equal to -0.01, this suggests a measurement error of only four basis points.

A final concern is that *Forbes*'s coverage may become more and more extensive over time, and therefore the magazine gradually discovers rich households that were not reported earlier. This would lead me to overestimate the displacement term as well as the growth of top wealth shares. To mitigate this effect, I start my decomposition in 1983, even though the Forbes 400 list started one year earlier. Moreover, I find that the actual displacement term is well approximated by the model-implied displacement term, which only uses the distribution of wealth growth among existing households. This suggests that the discovery of new fortunes is not a key driver of the displacement term.[40]

5. QUANTIFYING DISPLACEMENT IN THE ABSENCE OF PANEL DATA

When individual wealth follows a diffusion process, Proposition 1 says that the displacement term should be close to $1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$. As shown in the previous section, this approximation works well for the case of Forbes 400. This suggests that one can use this formula to quantify the displacement term in settings where panel data are not available. I now use this idea to quantify the displacement term for the top 1%, 0.1%, and 0.01% of the wealth distribution over the 20th century, for which panel data are not available.

Methodology I first describe how I estimate the displacement term implied by a diffusion model, $1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$, at the top 1%, 0.1%, and 0.01% percentiles from 1916 to 2012.

I first estimate the yearly standard deviation of wealth growth at each percentile threshold $\nu_t(q_t)$ by taking the product of the share of wealth invested in equity (using data from Kopczuk and Saez, 2004 for 1916–1962 and Saez and Zucman, 2016 for 1962–2012) and the cross-sectional standard deviation of firm-level returns. This is motivated by the fact that in the Forbes 400, the variance of wealth growth correlates well with the variance of returns (Table III). I scale this product so that the standard deviation of the top 0.01% matches the standard deviation of the Forbes 400 in 1983–2012. Table IV shows the estimated standard deviation for top percentiles. Over the time period, the standard deviation equals 0.14 for the top 1% and 0.21 for the top 0.01%. Note that the average standard deviation of wealth increases in the right tail of the wealth distribution, which is driven by the fact that top percentiles tend to invest more in equity.

I then estimate the shape of the wealth distribution $g_t(q_t)q_t^2/(p\overline{w}_t)$ at the top 1%, 0.1%, and 0.01% percentiles using cross-sectional data on the wealth distribution available from Kopczuk and Saez (2004) for 1916–1962 and Saez and Zucman (2016) for 1962–2012. More precisely, the ratio $q_t/\overline{w}_t$ can be estimated from the data, while $g_t(q_t)q_t/p$ can be estimated by comparing the ratio between two log top percentiles and the respective log quantiles.[41] Table IV shows the estimated $g_t(q_t)q_t^2/(p\overline{w}_t)$ for top percentiles. The estimate does not vary much across top percentiles, reflecting that the wealth distribution is close to a Pareto distribution above the top 1%.

[40]Formally, in a world in which wealth follows a diffusion model and in which *Forbes* discovers new fortunes with hazard rate ι, the displacement term would become

$$\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\nu_t(q_t)^2 + \left(1 - \frac{q_t}{\overline{w}_t}\right)\iota.$$

[41]This is a discretized version of the regression of the log top percentile on log quantiles performed in the previous section.

Results. Figure 5b plots the displacement term implied by a diffusion model, $1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$, for the top 1%, top 0.1%, and top 0.01% percentiles from 1916 to 2012. The displacement term roughly follows a U-shape over time for all top percentiles. The displacement term for the top 0.01% peaked at 2% during the Great Depression and then steadily decreased, reaching its minimum in 1945. The displacement term again increased starting in 1960 and reached its maximum at the height of the dot-com bubble. Overall, the displacement term was roughly twice as high in 1983–2012 as it had been for the rest of the century.

To better understand what drives the displacement term over time, Figure 5a plots separately the term due to the idiosyncratic variance of wealth $\nu_t(q_t)^2$ and the term due to the shape of the wealth distribution $g_t(q_t)q_t^2/(p\overline{w}_t)$ for the top 0.01%. Relative fluctuations in $\nu_t(q_t)^2$ are much larger than the relative fluctuations in $g_t(q_t)q_t^2/(p\overline{w}_t)$.[42] In other words, most of the time-series fluctuations in displacement are driven by fluctuations in the idiosyncratic volatility of wealth growth rather than by changes in the shape of the wealth distribution.

I now examine the variation of the displacement term across top percentiles. According to Saez and Zucman (2016), the yearly growth rate of the top 0.01%'s wealth share in 1982–2012 averaged to 4.3%, while the yearly growth rate of the top 1% averaged to 1.9%, a difference of 2.4% per year. The results of Table IV suggest that the differences in the displacement term between the two percentiles can explain a 1.3% differential. The difference in the average displacement term between the top 0.01% and the top 1% is almost fully driven by differences in the standard deviation of wealth growth across the two percentiles rather than by differences in the shape of the wealth distribution. This reflects the fact that the wealth distribution is close to a Pareto distribution for top percentiles.

TABLE IV

DISPLACEMENT TERM FOR THE TOP 0.01% (DIFFUSION MODEL)[a]

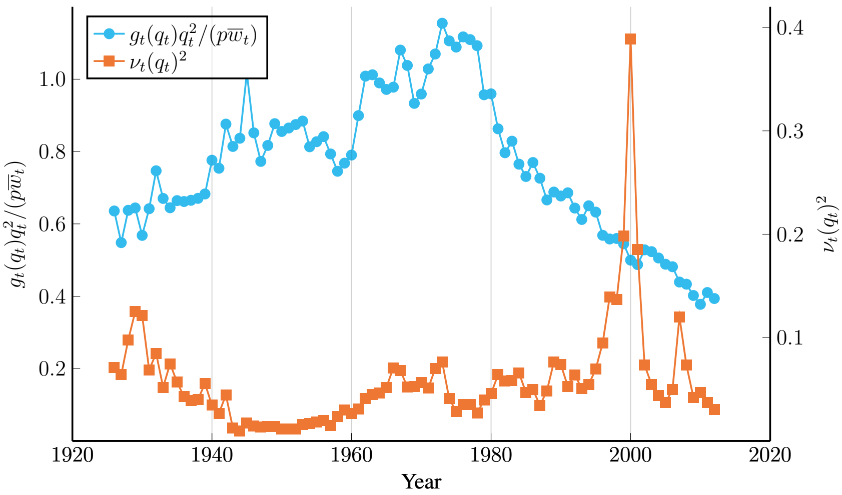
	Top 1%	Top 0.1%	Top 0.01%	Top 400
Panel A: 1926–1982				
$g_t(q_t)q_t^2/(p\overline{w}_t)$	0.64	0.70	0.86	
$\nu_t(q_t)^2$	0.02	0.03	0.04	
$1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$	0.6%	1.0%	1.8%	
Panel B: 1983–2012				
$g_t(q_t)q_t^2/(p\overline{w}_t)$	0.55	0.56	0.58	0.47
$\nu_t(q_t)^2$	0.04	0.06	0.08	0.08
$1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$	1.0%	1.6%	2.3%	1.9%

[a] Table IV uses the diffusion model to quantify the displacement term over the 20th century for the top 1%, 0.1%, and 0.01%. The shape of the wealth distribution $g_t(q_t)q_t^2/(p\overline{w}_t)$ is estimated using cross-sectional data on the wealth distribution. The variance of log wealth growth $\nu_t(q_t)^2$ is proxied by multiplying the share of wealth invested in equity with the cross-sectional standard deviation of stock market returns. Data are from Kopczuk and Saez (2004), Saez and Zucman (2016), and CRSP.

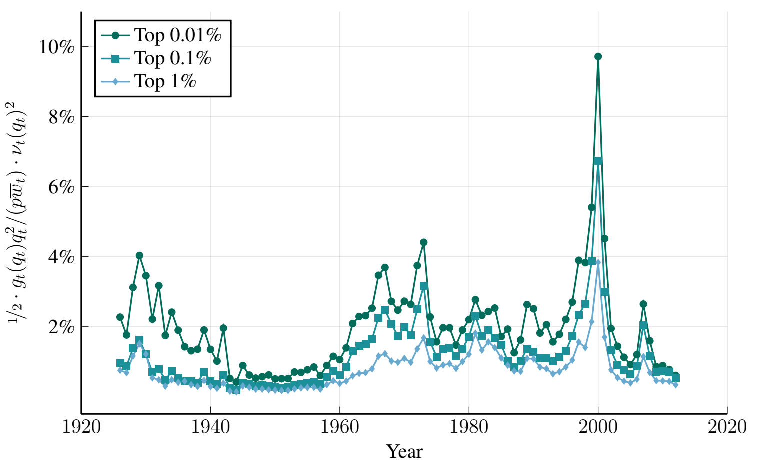
[42] More precisely, the ratio between the 90th and 10th quantile of the realizations of $\nu_t(q_t)^2$ is approximately equal to eight, while it is only two for $g_t(q_t)q_t^2/(p\overline{w}_t)$. These large fluctuations in the idiosyncratic volatility of stock market returns have been examined by Campbell et al. (2001) and Herskovic et al. (2016). Theories to explain these fluctuations have been discussed in Brandt et al. (2009), Fink et al. (2010), and Herskovic et al. (2020).

FIGURE 5.—Displacement Term for Top Wealth Percentiles in the U.S. over the 20th Century[a]

(a) Estimates of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$ for the Top 0.01%



(b) Displacement Term for Top Percentiles Implied by a Diffusion Model



[a]Figure 5a plots the estimated $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$ over time for the top 0.01%. The shape of the wealth distribution $g_t(q_t)q_t^2/(p\overline{w}_t)$ is estimated using cross-sectional data on the wealth distribution. The variance of log wealth growth $\nu_t(q_t)^2$ is proxied by multiplying the share of wealth invested in equity with the cross-sectional standard deviation of stock market returns. Figure 5b plots the diffusion term implied by the diffusion model, $1/2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$, for the top 1%, top 0.1%, and top 0.01%. Data are from Kopczuk and Saez (2004), Saez and Zucman (2016), and CRSP.

6. IMPLICATIONS FOR LONG RUN WEALTH MOBILITY

The accounting decomposition has direct implications on the relationship between wealth inequality and mobility. In this section, I show that a rise in the average wealth growth of households at the top (i.e., in the intensive term) increases wealth inequality and decreases wealth mobility. In contrast, a rise in the dispersion of wealth shocks (i.e., in the displacement term) increases both wealth inequality and wealth mobility.

I focus on a downward concept of mobility: how long, on average, does a household remain in a top percentile? The advantage of this notion of "downward" mobility is that it only depends on the wealth dynamics of individuals in the right tail of the wealth distribution.[43]

For the remainder of this section, I assume that the law of motion of wealth is given by a diffusive process with constant drift and idiosyncratic volatility; that is,

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu \,\mathrm{d}t + \nu \,\mathrm{d}B_{it}, \tag{13}$$

with a reflecting boundary at $\underline{w}$. Moreover, I assume that individuals die with rate $\delta > 0$, population grows at rate $\eta > 0$, and newborn agents are born with initial wealth $\underline{w}$. Under these assumptions, the stationary wealth distribution is Pareto; more precisely, we have $g(w) = \zeta(w/\underline{w})^{-\zeta-1}$ for $w \geq \underline{w}$, where the Pareto exponent ζ is the positive solution of the quadratic equation $\zeta\mu + {}^1\!/_2\zeta(\zeta-1)\nu^2 - \delta - \eta = 0$.[44]

Consider a top percentile $p \in (0, 1)$ of the stationary wealth distribution, and denote q the associated top quantile (i.e., the wealth at the top percentile threshold). Denote $T(w)$ the average time an individual with wealth $w \geq q$ remains in the top percentile (also called the "average first passage time"); that is,

$$T(w) = \mathrm{E}_t[\inf\{\tau \text{ s.t. } w_{it+\tau} \leq q \text{ or } i \text{ dies}\}|w_{it} = w].$$

PROPOSITION 3—Average First Passage Time: *When wealth follows the law of motion* (13)*, the average first passage time for $w \geq q$ is*[45]

$$T(w) = \frac{1}{\delta}\left(1 - \left(\frac{w}{q}\right)^{-\xi}\right), \tag{14}$$

where $-\xi$ denotes the negative solution of the quadratic equation $\zeta\mu + {}^1\!/_2\zeta(\zeta-1)\nu^2 - \delta = 0$.

This average first passage time naturally increases with wealth w: it equals 0 at the percentile threshold and converges to $1/\delta$ as wealth tends to infinity (which corresponds to the average time before death). The exponent ξ governs the speed at which the average first passage time converges to $1/\delta$ as wealth tends to infinity.

Moreover, the average first passage time $T(w)$ increases with the average wealth growth of individuals μ, and it decreases with the idiosyncratic volatility ν (through change in ξ). Intuitively, the higher the dispersion of wealth shocks, the more likely it is for wealth to drop below q. Yet, this does not mean that an increase in idiosyncratic volatility necessarily leads to an increase in wealth mobility. Indeed, an increase in idiosyncratic volatility decreases the average first passage time at a given wealth level, and it also tends to increase wealth inequality in the long run, which increases the typical distance between a household in the top percentile and the percentile threshold. Overall, this long-run effect may actually decrease mobility.

To formalize this effect, denote T the average first passage time for an *average* household in the top percentile p:

$$T = \frac{1}{p}\int_q^{\infty} T(w)g(w)\,\mathrm{d}w. \tag{15}$$

[43] In particular, compared to a notion of "upward" mobility, we can abstract from the role of labor income or government programs.

[44] See, for instance, Gabaix et al. (2016).

[45] The average first passage time of a Brownian motion is a classic result; for instance, see Karlin and Taylor (1981). This formula simply generalizes it to the case of a process with death.

PROPOSITION 4—Average First Passage Time for an Average Household: *Consider an economy in which individual wealth follows the process given in* (4). *The average time an average household in the top percentile* p *remains in the top is*

$$T = \frac{1}{\delta} \frac{1}{1 + \frac{\zeta}{\xi}}. \tag{16}$$

This formula characterizes in closed form the average passage time for an average household in the top percentile p. One interesting property is that T does not depend on the top percentile p, which reflects the "scale-free" property of Pareto distributions.

The average first passage time depends on the ratio between ζ and ξ. As seen in Proposition 3, ξ controls the speed at which the first passage time decays as wealth tends to infinity, while the Pareto exponent ζ controls the speed at which the density of wealth decays as wealth tends to infinity. The average time an average household remains in the top depends on the ratio between these two exponents, ζ/ξ.

As the average wealth growth of top households μ increases, T increases (i.e., mobility decreases). This is due to two reasons. First, the average first passage time at a given wealth level increases (i.e., ξ increases). Second, in the long run, the wealth distribution becomes more unequal, which increases the typical distance between a household in the top percentile and the percentile threshold (i.e., ζ decreases). These two forces combine to decrease mobility.

The effect of a rise in the dispersion of wealth shocks on mobility is more subtle. On the one hand, as ν increases, the average first passage time from a given wealth level decreases, which tends to increase mobility (i.e., ξ decreases). On the other, the wealth distribution becomes more unequal in the long run, which increases the typical distance between a household in the top percentile and the percentile threshold (i.e., ζ decreases). For realistic parameters, this adjustment is not important enough to compensate the first force, and mobility increases.

A Stylized Calibration. I now use the proposition to quantify the effect of the observed rise in top wealth shares on long-run wealth mobility. I calibrate the death rate δ and population growth rate η to match to the respective rates measured in Table Ic, that is, $\delta = 2.2\%$ and $\eta = 1.2\%$. For the initial (pre-1980) steady state, I pick an idiosyncratic volatility $\nu = 0.10$, to match the average idiosyncratic volatility for the top 0.01% in 1960–1980 and a wealth drift $\mu = 1.5\%$, to match an initial Pareto exponent of the stationary wealth distribution $\zeta = 1.8$ (from Section 5). These parameters give $\xi = 3.3$, which implies that an average time a top household remains in the top percentile, $T = 30$ years (using Proposition 4).

I consider a permanent increase in the drift and volatility of wealth consistent with the results of the accounting framework for the Forbes 400 in Table Ia; that is, a permanent change to $\mu = 3\%$ and $\nu = 0.27$. In this new steady state, the Pareto exponent ζ is approximately equal to 1.1, while ξ is approximately equal to 0.7. Using Proposition 4, this implies that the average time a top household remains in the top percentile, T, is now 18 years. While this calibration is very stylized, it suggests that the recent rise in top wealth shares is associated with an *increase* in long-run wealth mobility, even as wealth inequality continues to increase.[46]

7. CONCLUSION

This paper develops a framework to better understand the drivers of top wealth shares over time. The growth of a top percentile wealth share can be decomposed into three distinct terms:

[46]This relates to the empirical findings of Kopczuk et al. (2010), who find that even though labor inequality increased at the end of the 20th century, labor mobility remained more or less constant.

an intensive term (the growth of the top wealth share absent any composition change), a displacement term (which accounts for the flow of households in and out of the top percentile), and a demography term (which accounts for the entry and exit of households in the economy). Each term corresponds to a distinct driver of top wealth inequality: top wealth shares can increase due to a high average growth rate of top individuals (intensive), a high dispersion in their wealth growth (displacement), or a lack of population renewal (demography).

I provide a statistical framework to interpret the results of this accounting framework. The continuous-time of this decomposition corresponds to an "integrated" version of Kolmogorov-Forward equation. This allows me to obtain simple formulas relating the displacement and demography terms to the statistical properties of wealth.

After applying this framework on growth of the Forbes 400 wealth share, I find that displacement has played a key role in the recent rise in top wealth inequality: more than half of the recent rise in top wealth inequality is due to the arrival of new fortunes in top percentiles. I find similar results after applying the accounting framework to examine the rise in billionaires in China and Russia (Appendix E) or to examine the rise in top income shares (Appendix F). These findings have direct implications for our understanding of top wealth inequality. In particular, while the existing literature has focused on factors driving the average growth of existing fortunes (i.e., the average return on capital or their average saving rate), this paper stresses the role of the dispersion in their wealth growth.

Finally, the tools developed in this paper could be used to decompose the rise in concentration in other settings. Beyond wealth and top income shares (see Appendix F), another promising application would be to examine the distribution of firms' market shares (Autor et al. (2017), Hartman-Glaser et al. (2019), Gutiérrez and Philippon (2019)). I leave this topic for future research.

APPENDIX A: PROOFS

PROOF OF PROPOSITION 1: *Step 1— Existence, smoothness, and upper bounds for the wealth density.* I first show that there is a weak solution of the stochastic differential equation (SDE) with a smooth density. Let $\omega_{it} = \ln w_{it}$ denote log wealth. Using Ito's lemma, the process for log wealth ω_{it} solves the following SDE:

$$\mathrm{d}\omega_{it} = \left(\mu_t\left(e^{\omega_{it}}\right) - \frac{1}{2}\nu_t\left(e^{\omega_{it}}\right)^2\right)\mathrm{d}t + \nu_t\left(e^{\omega_{it}}\right)\mathrm{d}B_{it}.$$

As shown in Rogers (1985), the solution of this SDE process has a weak solution and its transition density between s and t, denoted by $\pi_{s\to t}(\omega_s, \omega_t)$, is $\mathcal{C}^\infty$ in t, ω_s, ω_t, positive everywhere, and satisfies the Kolmogorov forward equation.

As discussed in Friedman (1964) (Section 9.6), the fact that $\mu_t(\exp(.))$ and $\nu_t(\exp(.))$ are bounded gives upper bounds on the derivatives of the transition density: there exist constants A, d such that for all $\omega_0, \omega, t \in [0, T]$, we have

$$|\partial_t \pi_{0\to t}(\omega_0, \omega)| \leq \frac{A}{t^{3/2}} e^{-d\frac{(\omega-\omega_0)^2}{t}}, \tag{17}$$

$$|\partial_\omega^n \pi_{0\to t}(\omega_0, \omega)| \leq \frac{A}{\sqrt{t^{1+n}}} e^{-d\frac{(\omega-\omega_0)^2}{t}}, \tag{18}$$

for any integer $n \geq 0$.

Denote γ_t the density of log wealth, which is given by $\gamma_0(\omega) = e^{\omega} g_0(e^{\omega})$ and

$$\gamma_t(\omega) = \int_{\mathbb{R}} \gamma_0(\omega_0)\pi_{0\to t}(\omega_0, \omega)\,\mathrm{d}\omega_0,$$

for any time $t > 0$. Since $\pi_{0\to t}$ is positive everywhere, γ_t is positive everywhere. Moreover, applying the dominated convergence theorem gives that γ_t inherits the smoothness of $\pi_{0\to t}$; that is, $\gamma_t \in \mathcal{C}^{\infty}$.[47]

Next, we prove that the wealth distribution has finite mean at any time $t \geq 0$. We have, for $t \geq 0$ and $x \geq 0$,

$$\begin{aligned}
\int_0^x e^{\omega}\gamma_t(\omega)\,\mathrm{d}\omega &= \int_0^x e^{\omega}\left(\int_{\mathbb{R}} \gamma_0(\omega_0)\pi_{0\to t}(\omega_0, \omega)\,\mathrm{d}\omega_0\right)\mathrm{d}\omega \\
&\leq \int_{\mathbb{R}} \gamma_0(\omega_0)\left(\int_0^x e^{\omega}\frac{A}{\sqrt{t}}e^{-d\frac{(\omega-\omega_0)^2}{t}}\,\mathrm{d}\omega\right)\mathrm{d}\omega_0 \\
&\leq \left(\int_{\mathbb{R}} \gamma_0(\omega_0)e^{\omega_0}\,\mathrm{d}\omega_0\right)\left(\int_{\mathbb{R}} e^{u}\frac{A}{\sqrt{t}}e^{-d\frac{u^2}{t}}\,\mathrm{d}u\right),
\end{aligned} \tag{19}$$

where the second line uses the upper bound (18) with $n = 0$. Taking the limit $x \to \infty$ shows that the $\int_0^{\infty} e^{\omega}\gamma_t(\omega)\,\mathrm{d}\omega < \infty$. Therefore, we can conclude that the density of wealth, $g_t(w) = \gamma_t(\ln w)/w$, exists, is in $\mathcal{C}^{\infty}$, is positive everywhere, and has finite mean.

Step 2—Law of motion of the average wealth in the top percentile. I now relate the time derivative of the average wealth in the top percentile, $\partial_t \overline{w}_t$, to the time derivative of the wealth density, $\partial_t g_t$. Since g_t is positive everywhere, the function $q \to \int_q^{\infty} g_t(w)\,\mathrm{d}w$ is strictly decreasing, so there is a unique quantile q_t associated with the top percentile p, which is defined implicitly by

$$p = \int_{q_t}^{\infty} g_t(w)\,\mathrm{d}w. \tag{20}$$

The average wealth in the top percentile, $\overline{w}_t$, is defined by

$$p\overline{w}_t = \int_{q_t}^{\infty} w g_t(w)\,\mathrm{d}w. \tag{21}$$

Differentiating both of these equations with respect to time gives:[48]

$$\begin{aligned}
0 &= \int_{q_t}^{\infty} \partial_t g_t(w)\,\mathrm{d}w - g_t(q_t)\partial_t q_t, \\
p\partial_t \overline{w}_t &= \int_{q_t}^{\infty} w\partial_t g_t(w)\,\mathrm{d}w - q_t g_t(q_t)\partial_t q_t.
\end{aligned}$$

[47] Equation (18) gives an upper bound for the space derivative of the transition density.

[48] The dominated convergence theorem ensures that we can differentiate with respect to time under the integral — (17) implies that, over a finite time period, $\partial_t g_t(w)$ and $w\partial_t g_t(w)$ are uniformly dominated by an integrable function.

Combining both equations gives the time derivative of the average wealth in the top percentile in terms of the time derivative of the wealth density:

$$p\partial_t \overline{w}_t = \int_{q_t}^{\infty} (w - q_t)\partial_t g_t(w)\,\mathrm{d}w. \tag{22}$$

This equation relates to instantaneous change in the average wealth in the top percentile to the instantaneous change in the wealth density. It corresponds directly to the spirit of the accounting framework (2): individual entering the top percentile increases wealth in the top percentile by the difference between their wealth and the wealth at the percentile threshold, q_t.

Step 3—Integrating Kolmogorov-Forward Equation. The Kolmogorov Forward equation relates the law of motion of the wealth density to the law of motion of individual wealth:

$$\partial_t g_t(w) = -\partial_w(\mu_t(w) w g_t(w)) + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w)).$$

Plugging it into (22) gives

$$p\partial_t \overline{w}_t = \int_{q_t}^{\infty} (w - q_t)\left(-\partial_w(\mu_t(w) w g_t(w)) + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w))\right)\mathrm{d}w.$$

Integrating by parts gives:

$$\begin{aligned} p\partial_t \overline{w}_t = & \Big[(w - q_t)\,\mu_t(w) w g_t(w)\Big]_{q_t}^{\infty} - \frac{1}{2}\Big[(w - q_t)\partial_w(\nu_t^2(w) w^2 g_t(w))\Big]_{q_t}^{\infty} \\ & + \int_{q_t}^{\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w + \frac{1}{2}\nu_t(q_t)^2 q_t^2 g_t(q_t). \end{aligned} \tag{23}$$

The first two terms on the right-hand size equal to zero because, since the wealth distribution's mean is finite, we have $w^2 g_t(w) \to 0$ and $w\partial_w w^2 g_t(w) \to 0$ as $w \to \infty$ (and $\mu_t(.)$ and $\nu_t(.)$ are bounded). Therefore, Equation (23) simplifies to

$$p\partial_t \overline{w}_t = \int_{q_t}^{\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w + \frac{1}{2}\nu_t(q_t)^2 q_t^2 g_t(q_t). \tag{24}$$

Dividing by $p\overline{w}_t$ gives (5).

Step 4— Limit of the accounting framework as the time period tends to zero. I now show that, as the time period tends to zero, the intensive term and displacement term defined in the accounting framework (1) converge to the respective terms in (5). Between two time periods t and $t + \Delta t$, the intensive term in (1) corresponds to

$$\text{Intensive}_{t,t+\Delta t} = \frac{1}{p\overline{w}_t}\int_{\log q_t}^{\infty}\left(\int_{\mathbb{R}} e^{\omega'}\pi_{t\to t+\Delta t}(\omega,\omega')\,\mathrm{d}\omega'\right)\gamma_t(\omega)\,\mathrm{d}\omega - 1. \tag{25}$$

Differentiating this expression with respect to Δt, taken at $\Delta t = 0$, gives the intensive term in (5). The expression for the derivative of the displacement term is obtained as a residual. *Q.E.D.*

PROOF OF PROPOSITION 2: We follow the steps of the proof of Proposition 1, with the key difference that the population of individuals changes over time. *Step 1— Existence, smoothness,* *and upper bounds for the wealth density.* As in the proof of Proposition 1, let $\omega_{it} = \ln(w_{it})$ be log wealth. Applying Ito's lemma gives that the law of motion of ω_{it} is given by

$$\mathrm{d}\omega_{it} = \left(\mu_t\left(e^{\omega_{it}}\right) - \frac{1}{2}\nu_t\left(e^{\omega_{it}}\right)^2\right)\mathrm{d}t + \nu_t\left(e^{\omega_{it}}\right)\mathrm{d}B_{it}.$$

Denote $\pi_{s\to t}(\omega_s, \omega_t)$ the transition density of log wealth between t and s. Denote $\gamma_{Bt}(\omega) = e^{\omega} g_{Bt}(e^{\omega})$ the density of arriving agents at time t. We have $\gamma_0(\omega) = e^{\omega} g_0(e^{\omega})$ and

$$\begin{aligned}\gamma_t(\omega) = &\int_0^t (\delta_s + \eta_s) e^{-\int_s^t (\delta_u + \eta_u)\,\mathrm{d}u} \left(\int_{\mathbb{R}} \gamma_{Bs}(\omega')\pi_{s\to t}(\omega', \omega)\,\mathrm{d}\omega'\right)\mathrm{d}s \\ &+ e^{-\int_0^t (\delta_u + \eta_u)\,\mathrm{d}u} \int_{\mathbb{R}} \gamma_0(\omega_0)\pi_{0\to t}(\omega_0, \omega)\,\mathrm{d}\omega_0\end{aligned}$$

for $t > 0$. Using similar arguments as in the proof of Proposition 1, we obtain that the density γ_t is positive everywhere, infinitely differentiable, with $\int_{\mathbb{R}} e^{\omega}\gamma_t(\omega)\,\mathrm{d}\omega < \infty$.

Step 2—Law of motion of average wealth in the top percentile. Using similar arguments as in the proof of Proposition 1, we obtain the same equation for the time-derivative of the average wealth in the top percentile in terms of the time-derivative of the wealth density, (22).

Step 3-Integrating Kolmogorov-Forward Equation. In presence of demographic changes, the Kolmogorov forward equation becomes

$$\partial_t g_t(w) = -\partial_w(\mu_t(w) w g_t(w)) + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w)) + (\delta_t + \eta_t)\left(g_{Bt}(w) - g_t(w)\right).$$

Plugging this equation into (22) and integrating by parts gives

$$\begin{aligned}p\partial_t \overline{w}_t = &\int_{q_t}^{\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w + \frac{1}{2}\nu_t(q_t)^2 q_t^2 g_t(q_t) \\ &+ \left(\int_{q_t}^{\infty} (w - q_t)(g_{Bt}(w) - g_t(w))\,\mathrm{d}w\right)(\delta_t + \eta_t) \\ = &\int_{q_t}^{\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w + \frac{1}{2}\nu_t(q_t)^2 q_t^2 g_t(q_t) \\ &+ \left(\int_{q_t}^{\infty} (w - q_t) g_{Bt}(w)\,\mathrm{d}w - p\left(\overline{w}_t - q_t\right)\right)(\delta_t + \eta_t).\end{aligned}$$

Dividing by $p\overline{w}_t$ gives (10).

Step 4— Limit of the accounting framework as the time period tends to zero. I now show that, as the time period tends to zero, the intensive, displacement, and demography terms defined in the accounting framework (2) converge to the respective terms in (10). Between two time periods t and $t + \Delta t$, the terms defined in the accounting framework correspond to:[49]

$$\mathrm{Intensive}_{t,t+\Delta t} = \frac{1}{p\overline{w}_t}\int_{\log q_t}^{\infty}\left(\int_{\mathbb{R}} e^{\omega'}\pi_{t\to t+\Delta t}(\omega, \omega')\,\mathrm{d}\omega'\right)\gamma_t(\omega)\,\mathrm{d}\omega - 1,$$

$$\mathrm{Birth}_{t,t+\Delta t} = \int_t^{t+\Delta t} e^{-\int_s^{t+\Delta t}(\delta_u + \eta_u)\,\mathrm{d}u}(\delta_s + \eta_s)$$

[49]Here, and elsewere, "x^+" denotes the positive part of x; that is, $x^+ = \max(x, 0)$.

$$\times \left(\frac{1}{p\overline{w}_t} \int_{\mathbb{R}} \left(\int_{\mathbb{R}} (e^{\omega'} - q_{t+\Delta t})^+ \pi_{s\to t+\Delta t}(\omega, \omega')\, \mathrm{d}\omega' \right) \gamma_{Bs}(\omega)\, \mathrm{d}\omega \right) \mathrm{d}s,$$

$$\text{Death}_{t,t+\Delta t} = -\left(1 - e^{-\int_t^{t+\Delta t} \delta_s\, \mathrm{d}s}\right) e^{-\int_t^{t+\Delta t} \eta_s\, \mathrm{d}s} \frac{(1+\text{Intensive}_{t,t+\Delta t})\overline{w}_t - q_{t+\Delta t}}{\overline{w}_t},$$

$$\text{Pop. Growth}_{t,t+\Delta t} = -\left(1 - e^{-\int_t^{t+\Delta t} \eta_s\, \mathrm{d}s}\right) \frac{(1+\text{Intensive}_{t,t+\Delta t})\overline{w}_t - q_{t+\Delta t}}{\overline{w}_t}.$$

Differentiating with respect to Δt, at $\Delta t = 0$, gives the intensive, birth, death, and population growth terms defined in (10). The displacement term is obtained as a residual. *Q.E.D.*

PROOF OF PROPOSITION 3: We can express the average time $T(w_{it})$ by backward induction between t and $t + \Delta t$, where Δt is a short time period:

$$T(w_{it}) = \delta \Delta t \times 0 + (1 - \delta \Delta t) \times (\Delta t + \mathrm{E}_t[T(w_{it+\Delta t})]).$$

Rearranging terms,

$$0 = (1 - \delta \Delta t)(\Delta t + \mathrm{E}_t[T(w_{it+\Delta t}) - T(w_{it})]) - T(w_{it})\delta \Delta t.$$

Keeping only terms at the first order in Δt gives the following expression for $T(w_{it})$:

$$0 = \mathrm{d}t + \mathrm{E}_t[\mathrm{d}T(w_{it})] - T(w_{it})\delta\, \mathrm{d}t.$$

Applying Ito's lemma gives an ordinary differential equation (ODE) satisfied by T:

$$1 + T'(w) w \mu + \frac{1}{2} T''(w) w^2 \nu^2 - T(w)\delta = 0.$$

The solution of this ODE has the general form:

$$T(w) = c_1 w^{\lambda_1} + c_2 w^{\lambda_2} + \frac{1}{\delta}, \tag{26}$$

where λ_1 and λ_2 are, respectively, the positive and negative roots of $\zeta \to \mu\zeta + {}^1\!/_2\zeta(\zeta - 1)\nu^2 - \delta$.[50] We have the following limit conditions,

$$T(q) = 0,$$

$$\lim_{w\to+\infty} T(w) = \frac{1}{\delta}.$$

This imposes $c_1 = 0$ and $c_2 = -1/(\delta q^{\lambda_2})$. Therefore, the solution of the ODE is:

$$T(w) = \frac{1}{\delta}\left(1 - \left(\frac{w}{q}\right)^{\lambda_2}\right).$$

which gives (14) after defining $\xi = -\lambda_2$. *Q.E.D.*

[50]Note that this function is convex, converges to infinity as ζ converges to infinity, and equals $-\delta$ in zero; therefore there are exactly two zeros for this function, one negative and one positive.

PROOF OF PROPOSITION 4: Since the wealth distribution is Pareto, (15) becomes:

$$\begin{aligned}T &= \frac{1}{\delta}\left(1-\frac{\int_q^\infty (w/q)^{-\xi} w^{-\zeta-1}\,\mathrm{d}w}{\int_q^\infty w^{-\zeta-1}\,\mathrm{d}w}\right)\\ &= \frac{1}{\delta}\left(1-\frac{\zeta}{\zeta+\xi}\right),\end{aligned}$$

which gives (16).

Note that the derivative of T with respect to the idiosyncratic variance of wealth, ν^2, is

$$\begin{aligned}\partial_{\nu^2} T &= -\frac{1}{\delta}\left(1+\frac{\zeta}{\xi}\right)^{-2}\partial_{\nu^2}\left(\frac{\zeta}{\xi}\right)\\ &= \frac{1}{\delta}\left(1+\frac{\zeta}{\xi}\right)^{-2}\frac{\zeta}{\xi}\left(\frac{\partial_{\nu^2}\xi}{\xi}-\frac{\partial_{\nu^2}\zeta}{\zeta}\right).\end{aligned}$$

As ν^2 increases, T decreases only if the percentage decrease of ξ is higher than the percentage decrease of ζ.

Q.E.D.

APPENDIX B: EXTENSIONS

I now derive the law of motion of the average wealth in a top percentile in three models that extend the diffusion model presented in Section 3.

B.1. *Type Heterogeneity*

The main text considers the case in which the drift and the volatility of wealth only depends on the wealth level. In reality, individuals at the same wealth level may have different portfolios or consumption rates, and, therefore, different wealth dynamics.

To model this heterogeneity in a parsimonious way, I assume that individuals are split in $J \geq 1$ groups, with group-specific drift and volatility.[51] More precisely, I assume that the law of motion of the wealth for individuals in group $1 \leq j \leq J$ is

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_{jt}(w_{it})\,\mathrm{d}t + \nu_{jt}(w_{it})\,\mathrm{d}B_{it}, \tag{27}$$

where $\mu_{jt}(.)$ and $\nu_{jt}(.)$ satisfy the same regularity conditions as in Section 3. Individuals transition between the different groups $1 \leq j \leq J$ with an arbitrary Markov transition matrix $\mathbb{T}$. Denote $\mathbf{g}_t = (g_{1t}, \ldots, g_{Jt})$ the joint density of wealth w and type j at time t and $g_t = \sum_{j=1}^J g_{jt}$ the overall wealth density at time t.

[51] See Luttmer (2011) or Gabaix et al. (2016) for similar models.

PROPOSITION 5: *Assume that individual wealth follows the diffusion process* (27). *The average wealth in the top percentile $\overline{w}_t$ follows the law of motion:*

$$\frac{\partial_t \overline{w}_t}{\overline{w}_t} = \underbrace{\mathrm{E}^{\mathbf{g}_t \cdot}\left[\mu_{jt}(w) \mid w \geq q_t\right]}_{\textit{Intensive}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\mathrm{E}^{\mathbf{g}_t \cdot}\left[\nu_{jt}(w)^2 \mid w = q_t\right]}_{\textit{Displacement}},$$

where $\mathrm{E}^{\mathbf{g}_t \cdot}[.]$ *denotes the wealth-weighted expectation with respect to the density* $\mathbf{g}_t$.

The expressions for the intensive and displacement terms are essentially the same as in the baseline model. The intensive term simply depends on the wealth-weighted *average* drift of individuals in the top percentile, while the displacement term depends on the *average* idiosyncratic volatility at the top percentile threshold. Note that drift heterogeneity does not appear in the displacement term: this comes from the fact that, during a short period of time Δt, the cross-sectional variance of wealth growth due to heterogeneous drifts is in Δt^2, rather than Δt.

B.2. *Aggregate Shocks.*

The main text considers the case in which wealth shocks are purely idiosyncratic. In reality, individuals are also differently exposed to aggregate shocks.[52]. To model this heterogeneity in a parsimonious way, I assume that individual wealth follows the law of motion:

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_{jt}(w_{it})\,\mathrm{d}t + \sigma_{jt}(w_{it})\,\mathrm{d}Z_t + \nu_{jt}(w_{it})\,\mathrm{d}B_{it}, \tag{28}$$

where Z_t is an aggregate Brownian motion and, for any $1 \leq j \leq J$, $\mu_{jt}(.), \sigma_{jt}(.), \nu_{jt}(.)$ satisfy the same regularity conditions as in Section 3.[53]

In this case, the distribution of wealth is stochastic. While the mathematics involved in keeping track of a stochastic distribution are more involved (see, e.g., Carmona and Delarue (2018)), we can still derive informally the law of motion of the average wealth in the top percentile:

PROPOSITION 6: *Assuming that individual wealth follows the diffusion process* (28)*, the average wealth in the top percentile $\overline{w}_t$ follows the law of motion:*

$$\begin{aligned}\frac{\mathrm{d}\overline{w}_t}{\overline{w}_t} = &\underbrace{\mathrm{E}^{\mathbf{g}_t \cdot}\left[\mu_{jt}(w) \mid w \geq q_t\right]\mathrm{d}t + \mathrm{E}^{\mathbf{g}_t \cdot}\left[\sigma_{jt}(w) \mid w \geq q_t\right]\mathrm{d}Z_t}_{\textit{Intensive}} \\ &+ \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\left(\mathrm{E}^{\mathbf{g}_t \cdot}[\nu_{jt}(w_{it})^2 \mid w_{it} = q_t] + \mathrm{Var}^{\mathbf{g}_t \cdot}[\sigma_{jt}(w_{it}) \mid w_{it} = q_t]\right)\mathrm{d}t}_{\textit{Displacement}},\end{aligned}$$

where $\mathrm{E}^{\mathbf{g}_t \cdot}$ *(resp.* $\mathrm{Var}^{\mathbf{g}_t \cdot}$*) denotes the wealth-weighted expectation (resp. variance) with respect to the density* $\mathbf{g}_t$.

Heterogeneous exposure to aggregate shocks affects both the intensive term and the displacement term. The intensive term now loads on the aggregate Brownian motion: the exposure of

[52] See, for instance, Di Tella (2016) or Wolff (2002)

[53] At the cost of more cumbersome notations, this can be easily extended to the case when Z_t is a multi-dimensional Brownian to model (i.e., heterogeneous exposure to multiple aggregate shocks, such as industry-specific shocks).

the average wealth is given by the wealth-weighted exposure of individuals in the top percentile. More interestingly, the displacement term now depends not only on the average idiosyncratic volatility of individuals at the percentile threshold but also on the variance of their exposures to aggregate risks. Intuitively, both terms contribute to the cross-sectional variance of wealth growth at the percentile threshold.

B.3. *Jump-Diffusion*

The main text considers the case in which wealth is a continuous process. In reality, the wealth process may be discontinuous.[54] I now consider the more general case in which the wealth process follows a jump-diffusion model. Formally, I assume that wealth follows the law of motion:

$$\begin{aligned}\frac{\mathrm{d}w_{it}}{w_{it\text{-}}} &= \mu_t(w_{it\text{-}})\,\mathrm{d}t + \nu_t(w_{it\text{-}})\,\mathrm{d}B_{it} \\ &\quad + \left(e^{\phi_t(w_{it\text{-}})U_i} - 1\right)\mathrm{d}N_{it} - \mathrm{E}_U\left[e^{\phi_t(w_{it\text{-}})U_i} - 1\right]\lambda\,\mathrm{d}t,\end{aligned} \tag{29}$$

where N_{it} is a compound Poisson process with intensity λ and U_i is a bounded random variable with density f_U. The function ϕ_t allows the effective log sizes of jumps, $\phi_t(w)U_i$, to depend on wealth w and time t. Without loss of generality, I compensate the compound Poisson process by $\mathrm{E}_U\left[e^{\phi_t(w_{it\text{-}})U_i} - 1\right]\lambda\,\mathrm{d}t$, where E_U denotes the expectation with respect to the random variable U, so that $\mu_t(w)$ still corresponds to the instantaneous expected growth rate of wealth.

I assume the same regularity conditions for μ_t and ν_t as in Section 3, with the additional assumptions that (i) ϕ_t is positive, (ii) $\phi_t(\exp(.))$ possesses bounded derivatives of all orders and (iii) the resulting wealth density is smooth enough, that is, $g_t \in \mathcal{C}^\infty$, and, over a finite time period, $w\partial_t g_t(w)$ is uniformly dominated by an integrable function.[55,56]

PROPOSITION 7: *Assuming that wealth follows the jump-diffusion process* (28)*, the average wealth in the top percentile $\overline{w}_t$ follows the law of motion:*[57]

$$\begin{aligned}\frac{\partial_t \overline{w}_t}{\overline{w}_t} &= \underbrace{\mathrm{E}^{g_t\cdot}\left[\mu_t(w)|w \geq q_t\right]}_{\textit{Intensive}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\nu_t(q_t)^2}_{\textit{Displacement (Diffusion Part)}} \\ &\quad + \underbrace{\frac{\lambda}{p\overline{w}_t}\mathrm{E}_U\left[\int_0^{q_t}\left(e^{\phi_t(w)U}w - q_t\right)^+ g_t(w)\,\mathrm{d}w + \int_{q_t}^{\infty}\left(q_t - e^{\phi_t(w)U}w\right)^+ g_t(w)\,\mathrm{d}w\right]}_{\textit{Displacement (Jump Part)}}.\end{aligned} \tag{30}$$

In the presence of jumps, the displacement term is the sum of two terms: the first term is due to the diffusive part of the process (i.e., the Brownian motion B_{it}) while the second term is due

[54] A large literature in finance argues for the presence of jumps in asset prices, which would imply jumps in individual wealth (e.g., Ait-Sahalia (2004), Barro and Ursúa (2012)).

[55] That is, for any finite time $T > 0$, there exists f_T such that $\int_{\mathbb{R}+} f_T(w)\,\mathrm{d}w < \infty$, and for any $t < T$, $w\partial_t g_t(w) \leq f_T(w)$. As in the proof of Proposition 1, this will be needed to use the dominated convergence theorem to relate the time derivative of $\overline{w}_t$ to the time derivative of the wealth density g_t.

[56] While I believe it to be a consequence of the other assumptions, I was not able to find a proof in the existing literature. The closest paper is Cass (2006), which examines the case of a time-homogeneous process.

[57] Here, and elsewere, "x^+" denotes the positive part of x; that is, $x^+ = \max(x, 0)$.

to the jump part of the process (i.e., the compound Poisson process N_{it}). The jump part is the sum of an inflow term and an outflow term, which correspond respectively to the contribution of positive and negative jumps on the law of motion of the average wealth in the top percentile. In contrast with the term due to the diffusion part of the process, the jump term depends on the density of wealth and on the distribution of jump sizes *across the whole distribution*, not just at the percentile threshold. This reflects the fact that, during an infinitesimal period of time $\mathrm{d}t$, individuals can now enter the top percentile from any part of the wealth distribution.

Higher-Order Cumulants. When the distribution of jumps is regular enough, we can rewrite the displacement term as a Taylor expansion of the wealth density and jump sizes around the percentile threshold.

Formally, for $n \geq 2$, denote $\kappa_{nt}(w)$ the "instantaneous" cumulant of (log) wealth growth, defined as the n-th derivative of the instantaneous cumulant generating function:

$$\kappa_{nt}(w) = \partial_{\Delta t=0}\partial^n_{\theta=0} \log \mathrm{E}_t \left[e^{\theta \log\left(\frac{w_{it+\Delta t}}{w_{it\text{-}}}\right)} | w_{it\text{-}} = w \right].$$

An application of Ito's lemma, combined with (29), gives the following expression for the instantaneous cumulants:

$$\kappa_{nt}(w) = \begin{cases} \nu_t(w)^2 + \lambda \phi_t(w)^2 \mathrm{E}_U \left[U^2\right] & \text{for } n = 2, \\ \lambda \phi_t(w)^n \mathrm{E}_U \left[U^n\right] & \text{for } n > 2. \end{cases} \tag{31}$$

The second cumulant (i.e., the instantaneous variance) is the sum of a term due to the diffusive part of the process, $\nu_t(w)^2$, and a term that depends on the second moment of the random variable U. For $n > 2$, the instantaneous cumulant of order n depends on the n-th moment of the random variable U. When the process follows a diffusion (i.e., $\lambda = 0$), all cumulants of order higher than two are zero.

Assuming that the function $v \to \int_{q_t e^{-v}}^{q_t} (e^v w - q_t) \frac{1}{\phi_t(w)} f_U(\frac{J}{\phi_t(w)}) g_t(w) \,\mathrm{d}w$ is analytic, the displacement term can be rewritten as a sum of all higher-order cumulants:

COROLLARY 8: *Assuming that wealth follows the jump-diffusion process* (29)*, the average wealth in the top percentile* $\overline{w}_t$ *follows the law of motion:*

$$\frac{\partial_t \overline{w}_t}{\overline{w}_t} = \underbrace{\mathrm{E}^{g_t \cdot} \left[\mu_t(w) | w \geq q_t\right]}_{\textit{Intensive}} + \underbrace{\sum_{n=2}^{\infty} \frac{1}{n!} \frac{q_t}{\overline{w}_t} \sum_{m=0}^{n-2} (-w\partial_w)^m \left(\kappa_{nt}(w) w g_t(w)/p\right)|_{w=q_t}}_{\textit{Displacement}}. \tag{32}$$

The displacement term can be rewritten as an infinite series of higher-order terms. The term of order $n = 2$ corresponds to the contribution of the variance of (log) wealth growth. It is equal to $1/2(g_t(q_t) q_t^2 / p\overline{w}_t)\kappa_{2t}(q_t)$, which is similar to the term obtained for a diffusion process — except that $\kappa_{2t}(q_t)$ now reflects the total variance of (log) wealth growth at the percentile threshold, as shown in Equation (31).

The term of order $n > 2$ depends on the n-th cumulant of (log) wealth growth κ_{nt}. When the wealth process follows a diffusion, these cumulants are zero, and therefore these terms are equal to zero.

Note that the term of order n depends on the derivatives up to order $n - 2$ of the higher-order cumulant κ_{nt} and of the wealth density g_t at the percentile threshold. This comes from the fact that it reflects the contribution of large wealth realizations, which depends on the wealth density as well as on the distribution of jump sizes far from the percentile threshold, as captured by their higher-order derivatives at the percentile threshold.

Stationary Case. As in the diffusion model, the displacement term takes a particularly simple form when the wealth distribution has a Pareto tail. Assume that $\mu_t(w) = \mu(w)$, $\nu_t(w) = \nu(w)$, and $\phi_t(w) = \phi(w)$, with $\mu(w) \to \mu < 0$, $\nu(w) \to \nu$, and $\phi(w) \to \phi$ as $w \to \infty$. Under these assumptions, the stationary distribution, if it exists, has a Pareto tail with Pareto exponent $\zeta > 1$.

One can check that, for a distribution with a Pareto tail, we have, as $p \to 0$:

$$\frac{1}{p\overline{w}}\mathrm{E}_U\left[\int_0^q \left(e^{\phi(w)U}w - q\right)^+ g(w)\,\mathrm{d}w\right] \to \mathrm{E}_U\left[\left(\frac{e^{\zeta\phi U}-1}{\zeta} - \left(e^{\phi U}-1\right)\right)1_{U\geq 0}\right],$$
$$\frac{1}{p\overline{w}}\mathrm{E}_U\left[\int_q^\infty \left(q - e^{\phi(w)U}\right)^+ g(w)\,\mathrm{d}w\right] \to \mathrm{E}_U\left[\left(\frac{e^{\zeta\phi U}-1}{\zeta} - \left(e^{\phi U}-1\right)\right)1_{U\leq 0}\right].$$

Therefore, taking the limit $p \to 0$ in Proposition 7 gives a simple balance equation for top wealth shares:

$$0 = \underbrace{\mu}_{\text{Intensive}} + \underbrace{\frac{\zeta-1}{2}\nu^2 + \lambda\mathrm{E}_U\left[\frac{e^{\zeta\phi U}-1}{\zeta} - \left(e^{\phi U}-1\right)\right]}_{\text{Displacement}}.$$

Alternatively, we can directly take the limit $p \to 0$ in Corollary 8 to obtain:[58]

$$0 = \underbrace{\mu}_{\text{Intensive}} + \underbrace{\sum_{n\geq 2}\frac{\zeta^{n-1}-1}{n!}\kappa_n}_{\text{Displacement}}. \tag{33}$$

where κ_n denotes the limit of the cumulant of order n in the right tail of the wealth distribution; that is,

$$\kappa_n = \begin{cases} \nu^2 + \lambda\phi^2\mathrm{E}_U\left[U^2\right] & \text{for } n = 2, \\ \lambda\phi^n\mathrm{E}_U\left[U^n\right] & \text{for } n > 2. \end{cases}$$

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[58] Alternatively, we could obtain this equation by rewriting $\xi \to e^{\xi\phi U}$ as a Taylor series around $\xi = 0$ in (9).

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Online Appendix

APPENDIX C: PROOFS FOR EXTENSIONS

PROOF OF PROPOSITION 5: Denote $\boldsymbol{\mu}_t(w) = (\mu_{jt}(w))_{1\leq j\leq J}$ the vector of drifts for each type. Similarly, denote $\boldsymbol{\nu}_t(w) = (\nu_{jt}(w))_{1\leq j\leq J}$ the vector of idiosyncratic volatilities for each type. The Kolmogorov forward equation for $\mathbf{g}_t$ is

$$\partial_t \mathbf{g_t}(w) = -\partial_w(\boldsymbol{\mu}_t(w)w \cdot \mathbf{g}_t(w)) + \frac{1}{2}\partial_w^2(\boldsymbol{\nu}_t(w)w \cdot \boldsymbol{\nu}_t(w)w \cdot \mathbf{g}_t(w)) + \mathbb{T}'\mathbf{g}_t(w),$$

where $\cdot$ denotes the element-wise product. Therefore, the law of motion of the overall wealth density, g_t, is:

$$\partial_t g_t(w) = -\partial_w\left((\boldsymbol{\mu}_t(w)w)'\mathbf{g}_t(w)\right) + \frac{1}{2}\partial_w^2\left((\boldsymbol{\nu}_t(w)w \cdot \boldsymbol{\nu}_t(w)w)'\mathbf{g}_t(w)\right).$$

As in the proof of Proposition 1, we can plug this equation into (22) and integrate by parts to obtain

$$p\partial_t \overline{w}_t = \int_{q_t}^{\infty} (\boldsymbol{\mu}_t(w)w)'\mathbf{g}_t(w)\,\mathrm{d}w + \frac{1}{2}\left(\boldsymbol{\nu}_t(q_t)q_t \cdot \boldsymbol{\nu}_t(q_t)q_t\right)'\mathbf{g}_t(q_t).$$

Dividing by $p\overline{w}_t$ gives the result. *Q.E.D.*

PROOF OF PROPOSITION 6: We follow the same steps as in the proof of Proposition 1, with the key difference that the wealth density is now stochastic. As discussed in the text, the proof is only informal.

We start by rederiving the law of motion of the quantile q_t in the presence of aggregate risk. Applying Ito's lemma on the implicit definition of the quantile (20) gives

$$0 = \int_{q_t}^{\infty} \mathrm{d}g_t(w)\,\mathrm{d}w - g_t(q_t)\,\mathrm{d}q_t - \sigma[\mathrm{d}g_t(q_t)]^2\,\mathrm{d}t,$$

where $\sigma[\mathrm{d}g_t(q_t)]$ denote the exposure of $g_t(q_t)$ to aggregate shocks. Applying Ito's lemma on the definition of the average wealth in the top percentile (21) gives

$$p\,\mathrm{d}\overline{w}_t = \int_{q_t}^{\infty} w\,\mathrm{d}g_t(w)\,\mathrm{d}w - q_t g_t(q_t)\,\mathrm{d}q_t - q_t\sigma[\mathrm{d}g_t(q_t)]^2\,\mathrm{d}t - \frac{1}{2}g_t(q_t)\sigma[\mathrm{d}q_t]^2\,\mathrm{d}t.$$

Combining these two equations gives

$$\begin{aligned} p\,\mathrm{d}\overline{w}_t &= \int_{q_t}^{\infty}(w-q_t)\,\mathrm{d}g_t(w)\,\mathrm{d}w - \frac{1}{2}g_t(q_t)\sigma[\mathrm{d}q_t]^2\,\mathrm{d}t \\ &= \int_{q_t}^{\infty}(w-q_t)\,\mathrm{d}g_t(w)\,\mathrm{d}w - \frac{1}{2}\frac{1}{g_t(q_t)}\left(\int_{q_t}^{\infty}\sigma[\mathrm{d}g_t(w)]\,\mathrm{d}w\right)^2\mathrm{d}t. \end{aligned} \tag{34}$$

Denote $\boldsymbol{\sigma}_t(w) = (\sigma_{jt}(w))_{1\leq j\leq J}$ the vector giving the aggregate exposures of each type. As shown in Kurtz and Xiong (1999) and Carmona and Delarue (2018), in the presence of aggregate risk, the wealth density evolves following the equation:

$$\mathrm{d}\mathbf{g_t}(w) = -\partial_w(\boldsymbol{\mu}_t(w)w \cdot \mathbf{g}_t(w))\,\mathrm{d}t - \partial_w(\boldsymbol{\sigma}_t(w)w \cdot \mathbf{g}_t(w))\,\mathrm{d}Z_t$$

$$+\frac{1}{2}\partial_w^2\left(\left(\boldsymbol{\nu}_t(w)w\cdot\boldsymbol{\nu}_t(w)w+\boldsymbol{\sigma}_t(w)w\cdot\boldsymbol{\sigma}_t(w)w\right)\cdot\mathbf{g}_t(w)\right)\mathrm{d}t+\mathbb{T}'\mathbf{g}_t(w)\,\mathrm{d}t.$$

Therefore, the law of motion of the overall wealth density, g_t, is:

$$\begin{aligned}\mathrm{d}g_t(w)=&-\partial_w\left((\boldsymbol{\mu}_t(w)w)'\mathbf{g}_t(w)\right)\mathrm{d}t-\partial_w\left((\boldsymbol{\sigma}_t(w)w)'\mathbf{g}_t(w)\right)\mathrm{d}Z_t\\&+\frac{1}{2}\partial_w^2\left(\left(\boldsymbol{\nu}_t(w)w\cdot\boldsymbol{\nu}_t(w)w+\boldsymbol{\sigma}_t(w)w\cdot\boldsymbol{\sigma}_t(w)w\right)'\mathbf{g}_t(w)\right)\mathrm{d}t.\end{aligned}$$

Plugging this equation into (34) and integrating by parts gives:

$$\begin{aligned}p\,\mathrm{d}\overline{w}_t=&\int_{q_t}^{\infty}(\boldsymbol{\mu}_t(w)w)'\mathbf{g}_t(w)\,\mathrm{d}w\,\mathrm{d}t+\int_{q_t}^{\infty}(\boldsymbol{\sigma}_t(w)w)'\mathbf{g}_t(w)\,\mathrm{d}w\,\mathrm{d}Z_t\\&+\frac{1}{2}\left(\boldsymbol{\nu}_t(q_t)q_t\cdot\boldsymbol{\nu}_t(q_t)q_t+\boldsymbol{\sigma}_t(q_t)q_t\cdot\boldsymbol{\sigma}_t(q_t)q_t\right)'\mathbf{g}_t(q_t)\,\mathrm{d}t\\&-\frac{1}{2}\frac{1}{g_t(q_t)}\left((\boldsymbol{\sigma}_t(q_t)q_t)'\mathbf{g}_t(q_t)\right)^2\mathrm{d}t.\end{aligned}$$

Dividing by $p\overline{w}_t$ gives the result. *Q.E.D.*

PROOF OF PROPOSITION 7: We follow the same steps as in the proof of Proposition 1. The Kolmogorov-Forward equation associated with the process (29) is:

$$\begin{aligned}\partial_t g_t(w)=&-\partial_w(\mu_t(w)wg_t(w))+\frac{1}{2}\partial_w^2\left(\nu_t^2(w)w^2g_t(w)\right)\\&+\lambda\left(\int_0^{\infty}\frac{1}{\phi_t(w')w}f_U\left(\frac{\log(w/w')}{\phi_t(w')}\right)g_t(w')\,\mathrm{d}w'-g_t(w)\right.\\&\left.+\partial_w\left(\mathrm{E}_U\left[e^{\phi_t(w)U}-1\right]wg_t(w)\right)\right).\end{aligned}$$

Plugging this into (22) and integrating by parts gives

$$\begin{aligned}p\partial_t\overline{w}_t=&\int_{q_t}^{\infty}\mu(w)wg_t(w)\,\mathrm{d}w+\frac{1}{2}g_t(q_t)q_t^2\nu_t(q_t)^2\\&+\lambda\left(\int_{q_t}^{\infty}(w-q_t)\left(\int_0^{\infty}\frac{1}{\phi_t(w')w}f_U\left(\frac{\log(w/w')}{\phi_t(w')}\right)g_t(w')\,\mathrm{d}w'-g_t(w)\right)\mathrm{d}w\right.\\&\left.-\int_{q_t}^{\infty}\mathrm{E}_U\left[e^{\phi_t(w)U}-1\right]wg_t(w)\,\mathrm{d}w\right).\end{aligned}$$

The last term, due to jumps, can be rewritten as

$$\begin{aligned}&\lambda\mathrm{E}_U\left[\int_0^{\infty}\left(e^{\phi_t(w')U}w'-q_t\right)^+g_t(w')\,\mathrm{d}w'-\int_{q_t}^{\infty}\left(e^{\phi(w)U}w-q_t\right)g_t(w)\,\mathrm{d}w\right]\\&=\lambda\mathrm{E}_U\left[\int_0^{q_t}\left(e^{\phi_t(w)U}w-q_t\right)^+g_t(w)\,\mathrm{d}w+\int_{q_t}^{\infty}\left(q_t-e^{\phi_t(w)U}w\right)^+g_t(w)\,\mathrm{d}w\right].\end{aligned}$$

Dividing by $p\overline{w}_t$ gives the result. *Q.E.D.*

PROOF OF COROLLARY 8: Using the change of variable $J=\phi_t(w)U$, we can rewrite the displacement term due to jump in (30) as

$$\frac{\lambda}{p\overline{w}_t}\int_J\left(\int_0^{q_t}(e^Jw-q_t)^+\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)g(w)\,\mathrm{d}w\right.$$
$$\left.+\int_{q_t}^{\infty}(q_t-e^Jw)^+\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)g(w)\,\mathrm{d}w\right)\mathrm{d}J$$
$$=\frac{\lambda}{p\overline{w}_t}\int_J\int_{q_te^{-J}}^{q_t}(e^Jw-q_t)\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)g_t(w)\,\mathrm{d}w\,\mathrm{d}J$$

Assuming that the function $v\to\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\frac{1}{\phi_t(w)}f_U(\frac{J}{\phi_t(w)})g_t(w)\,\mathrm{d}w$ is analytic, we can rewrite it as

$$\frac{\lambda}{p\overline{w}_t}\int_J\sum_{n\geq2}\frac{1}{n!}\partial_v^n\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}J^n\,\mathrm{d}J$$
$$=\frac{\lambda}{p\overline{w}_t}\sum_{n\geq2}\frac{1}{n!}\partial_v^n\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\left(\int_J J^n\frac{1}{\phi_t(w)}f_U\left(\frac{J}{\phi_t(w)}\right)\mathrm{d}J\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}$$
$$=\frac{\lambda}{p\overline{w}_t}\sum_{n\geq2}\frac{1}{n!}\partial_v^n\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\left(\int_U\phi_t(w)^nU^nf_U(U)\,\mathrm{d}U\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}$$
$$=\frac{\lambda}{p\overline{w}_t}\sum_{n\geq2}\frac{1}{n!}\partial_v^n\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\left(\phi_t(w)^n\mathrm{E}_U[U^n]\right)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}.$$

Plugging this equality into the law of motion of $\overline{w}_t$ (30), and using the definition of cumulants (31), we obtain

$$\frac{\mathrm{d}\overline{w}_t}{\overline{w}_t}=\mathrm{E}^{g_t\cdot}\left[\mu_t(w)|w\geq q_t\right]+\frac{1}{p\overline{w}_t}\sum_{n\geq2}\frac{1}{n!}\partial_v^n\left(\int_{q_te^{-v}}^{q_t}(e^vw-q_t)\kappa_{nt}(w)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}$$
$$=\mathrm{E}^{g_t\cdot}\left[\mu_t(w)|w\geq q_t\right]+\frac{1}{p\overline{w}_t}\sum_{n\geq2}\frac{1}{n!}\partial_v^{n-1}\left(\int_{q_te^{-v}}^{q_t}e^vw\kappa_{nt}(w)g_t(w)\,\mathrm{d}w\right)\Big|_{v=0}$$
$$=\mathrm{E}^{g_t\cdot}\left[\mu_t(w)|w\geq q_t\right]+\frac{q_t}{\overline{w}_t}\sum_{n\geq2}\frac{1}{n!}\sum_{m=0}^{n-2}(-w\partial_w)^m\left(\kappa_{nt}(w)wg_t(w)/p\right)|_{w=q_t}.$$

Q.E.D.

APPENDIX D: EMPIRICAL APPENDIX

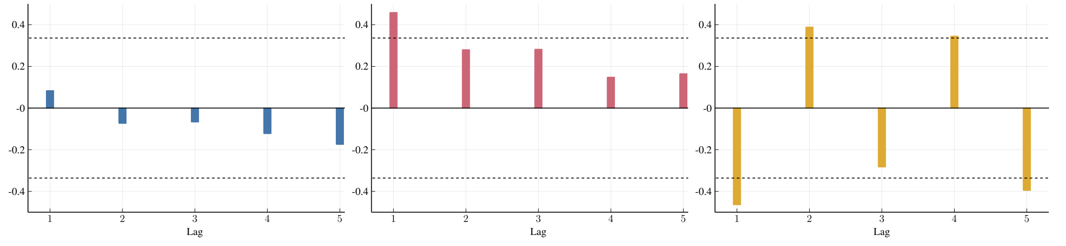
D.1. *Autocorrelogram*

Figure D.1 plots the serial correlation of the intensive, displacement, and demography terms. Each term has different patterns. The intensive term is serially uncorrelated, which reflects the fact that stock market returns are approximately uncorrelated over time. The displacement term is positively correlated, which reflects the fact that the local Pareto exponent and the dispersion of wealth growth among top households tends to be serially correlated over time (as seen in

Figure 4a). Finally, the demography term is negatively correlated at the one year horizon. This comes from the fact that the death rate tends to be negative correlated: a large amount of death in a given year means that the remaining households in the top percentile are younger, which implies fewer death in the following year.

FIGURE D.1.—Autocorrelogram for the Intensive, Displacement and Demography Terms in the Forbes 400 [a]

(a) Intensive (b) Displacement (c) Demography



[a] The figure plots the autocorrelation for the intensive, displacement, and demography terms obtained by applying the logarithmic accounting framework (3) on the growth of the Forbes 400 wealth share. The dashed lines represent the 95% confidence bands corresponding to the null hypothesis of no correlation. Data are from *Forbes* and the Financial Accounts of the United States.

D.2. *Wealth of Drop-Offs from the Forbes 400*

The decomposition in Section 2 requires knowing the wealth of households that drop off the top percentile. However, before 2012, *Forbes* only rarely reported the wealth of individuals who dropped out from the top 400. First, 70% of households that drop off the top percentile actually stay in the Forbes 400. Indeed, the top percentile used in this paper is composed of only 264 households in 1983 (it was chosen so that, with population growth, it includes 400 households in 2017). Because wealth is so concentrated in the top, there is usually a great difference between the last individual in this top percentile and the wealth of the last individual in the top 400. Therefore, most households that drop off this top percentile stay in the top 400.

I now focus on the remaining 30% of households that drop off the Forbes 400. Formally, the problem boils down to estimating the average of a variable (the wealth growth of top households) that is left censored. In this particular setting, the Kaplan and Meier (1958) estimator gives tight bounds to estimate this quantity. The idea is to estimate the average growth rate of drop-offs using the observed negative growth rates of households in the top percentile. The identifying assumption is that the distribution of growth rates is homogeneous for households within the top percentile.

More precisely, the Kaplan and Meier (1958) method is to first estimate the survival function, that is, in my setting the probability that wealth growth is lower than a certain threshold $\mathrm{P}((w_{it+1} - w_{it})/w_{it} \leq x)$. This survival function can then be used to estimate the expectation of wealth growth conditional on being lower than a certain threshold, that is, $\mathrm{E}[(w_{it+1} - w_{it})/w_{it} | (w_{it+1} - w_{it})/w_{it} \leq x]$.

I check the validity of this imputation method by focusing on years where *Forbes* reports the wealth of drop-offs (i.e., 2012-2017). In these years, I compare the result obtained from the estimated method and the result obtained using the real wealth of drop-offs. Table D.I shows the average return of these drop-offs using the imputed method and the actual data reported by wealth. The estimates differ by only 2 percentage points, on average (-26% versus -28%). The fact that the Kaplan-Meier estimator gives such a good result is intuitive: because wealth is so concentrated, households at the very top of the wealth distribution hold ten times more wealth than the households at the percentile threshold, and therefore I actually observe a large part of the distribution of negative wealth shocks.

The last four columns report the estimates for the intensive and displacement terms using imputed and real data. The estimates differ by less than 0.1 percentage points. The bias is small because, as discussed above, the Kaplan-Meier method provides accurate estimates of the wealth growth of imputed households. Moreover, the wealth share represented by the imputed households is small to begin with.

I use the same method to impute the second, third, and fourth order of wealth to obtain the estimate of standard deviation, skewness, and excess kurtosis reported in Table IIb.

TABLE D.I

EFFECT OF USING IMPUTED VERSUS REPORTED WEALTH OF DROP-OFFS ON DECOMPOSITION[a]

Year	Average Wealth Growth of Drop-Offs		Intensive		Displacement	
	Imputed (%)	Actual (%)	Imputed (%)	Actual (%)	Imputed (%)	Actual (%)
2011-2012	−33.56	−36.48	7.46	7.42	1.12	1.17
2012-2013	−27.34	−29.82	6.55	6.51	1.05	1.11
2013-2014	−23.82	−19.35	2.34	2.41	1.63	1.58
2014-2015	−26.44	−30.28	−3.04	−3.13	1.37	1.47
2015-2016	−26.46	−32.53	3.24	3.10	1.28	1.44
2016-2017	−16.97	−19.14	1.47	1.43	1.57	1.63
2011-2017	−25.76	−27.93	3.00	2.96	1.34	1.40

[a]The table compares the intensive and the displacement term (defined in (3)) using imputed data and actual data about the wealth of drop-offs. Starting from 2011, *Forbes* systematically reports the wealth of all drop-offs. For these years, one can compare the results of the accounting framework using this reported data as opposed to the imputation using Kaplan and Meier (1958). Data are from *Forbes* and the Financial Accounts of the United States.

D.3. *Measurement Error and Autocorrelation of Wealth Growth*

How does transitory measurement errors in wealth affect the accounting framework? Suppose that wealth follows a simple difffusion process with drift μ and volatility ν. Moreover, suppose that wealth is only observed at discrete time periods $\Delta t = 1$, and only with some errors; i.e., we only observe $\tilde{w}_{it} = w_{it}e^{\epsilon_{it}}$, where ϵ_{it} is an i.i.d process with $\mathrm{E}[e^{\epsilon_{it}}] = 1$.

The log change in observed wealth between t and $t+1$ can be written as

$$\log\left(\frac{\tilde{w}_{it+1}}{\tilde{w}_{it}}\right) = \mu + \nu(B_{it+1} - B_{it}) + \epsilon_{it+1} - \epsilon_{it}.$$

Denote ρ the slope coefficient in a regression on (logarithmic) observed wealth growth on its lagged value. We can relate ρ to the variance of measurement errors, $\mathrm{Var}(\epsilon_{it})$:

$$\begin{aligned}\rho &= \frac{\mathrm{cov}(\log(\tilde{w}_{it+1}/\tilde{w}_{it}), \log(\tilde{w}_{it}/\tilde{w}_{it-1}))}{\mathrm{Var}(\log(\tilde{w}_{it}/\tilde{w}_{it-1}))} \\ &= \frac{\mathrm{cov}(\mu + \nu(B_{it+1} - B_{it}) + \epsilon_{it+1} - \epsilon_{it}, \mu + \nu(B_{it} - B_{it-1}) + \epsilon_{it} - \epsilon_{it-1})}{\mathrm{Var}(\mu + \nu(B_{it} - B_{it-1}) + \epsilon_{it} - \epsilon_{it-1})} \\ &= -\frac{\mathrm{Var}(\epsilon_{it})}{\nu^2 + 2\mathrm{Var}(\epsilon_{it})}\end{aligned}$$

This equation makes it possible to recover the variance of measurement errors from the serial correlation of log wealth growth. More precisely, Table D.II reports an estimate for $\rho \approx -0.01$, which implies $\widetilde{\text{Var}(}\epsilon_{it})/\nu^2 \approx 1\%$.

Denote $\widetilde{\text{Displacement}}$ the displacement term obtained by applying the accounting framework on $\tilde{w}_{it}$ beween t and $t+1$. Over a short time period, Proposition 1 says that the relative bias in the displacement term is equal to the relative bias in the variance of log wealth growth:

$$\frac{\widetilde{\text{Displacement}} - \text{Displacement}}{\text{Displacement}} \approx 2\frac{\text{Var}(\epsilon_{it})}{\nu^2}$$

This suggest that the relative bias in the displacement term is around 2%, which is very small.

TABLE D.II

SERIAL CORRELATION OF WEALTH GROWTH IN THE FORBES 400[a]

	Wealth Growth
	(1)
Lagged Wealth Growth	–0.01
	(0.01)
Constant	0.05
	(0.00)
R^2	0.18
Period	1983-2016
FE	Individual
N	11,392

[a]The table shows the result of regressing future wealth growth on current wealth growth with individual fixed effects; that is, denoting w_{it} the wealth of household i at time t:

$$\log\left(\frac{w_{it+1}}{w_{it}}\right) = \alpha_i + \beta \log\left(\frac{w_{it}}{w_{it-1}}\right) + \epsilon.$$

Estimation is done via OLS. Standard errors are in parentheses and estimated using Newey-West with three lags. Data are from *Forbes*.

APPENDIX E: DECOMPOSING THE GROWTH OF BILLIONAIRE'S SHARES IN RUSSIA AND CHINA

I now compare the accounting decomposition obtained for the U.S. with other countries. This is possible because *Forbes* also publishes a list of international billionaires starting in 1987.

One difficulty, however, is that the number of billionaires in each country tends to be much smaller than in the U.S. Therefore, I restrict myself to Russia and China, the two countries with the highest count of billionaires in 2010 outside the U.S.[59] I consider the wealth share of the percentile composed of 50 billionaires in 2010. Since China counts less than 50 billionaires before 2010, I only look at the growth of this wealth share in the 2010–2018 period. The evolution of wealth inequality in China and Russia has been discussed in Novokmet et al. (2018) and Piketty et al. (2019).

Another concern is that data quality may be lower for these two countries compared to the U.S. To address this concern, I manually check that each new Chinese or Russian billionaire

[59]For the sake of the exercise, I group together China, Hong Kong, and Taiwan.

can be traced back to a particular event, such as a successful IPO or a high stock market return; otherwise, I remove the household from the sample. Data on per capita household wealth in Russia and China are taken from the World Wealth and Income Database.

Table E.I shows the result of the accounting decomposition for these two countries as well as the results of the U.S. for the same time period. Figure E.1a plots the average intensive, displacement, and demography terms in each country over time as well as their standard errors. Wealth inequality increased in both countries: the yearly growth rate of the top share is 4.1% in China and 5.4% in Russia. The average intensive term has a large standard error, which reflects the short time window for the decomposition as well as the small number of households in each group. In contrast, the displacement term is much more precisely estimated. In the rest of the analysis, I focus on comparing the displacement term across countries.

The displacement term averages to 4.2% in China, which is three times higher than the average displacement term in the U.S. To understand why, I estimate a model-implied displacement term ${}^1/{}_2 \cdot g_t(q_t)q_t^2/(p\overline{w}_t) \cdot \nu_t(q_t)^2$ in China, using the same methodology as the U.S. (see Figure E.1b). Using the same decomposition as above, this allows one to decompose the change between displacement in China and in the U.S. as a sum of two factors:

$$\underbrace{\Delta\left\langle \frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\nu_t(q_t)^2\right\rangle}_{3.0\%} = \underbrace{\left\langle \frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\right\rangle \Delta\left(\nu_t(q_t)^2\right)}_{1.6\%} + \underbrace{\langle \nu_t(q_t)^2\rangle \Delta\left(\frac{1}{2}\frac{g_t(q_t)q_t^2}{p\overline{w}_t}\right)}_{1.4\%},$$

where Δx (resp. $\langle x\rangle$) denotes the difference (resp. average) of a variable x between China and the U.S. The result of this decomposition indicates that the high level of the displacement term in China relative to the U.S. is a combination of two factors. First, the standard deviation of (log) wealth growth among top households is much higher in China compared to the U.S. Second, wealth inequality in China is much lower to begin with, making it easier for households with high realized growth rates to displace existing fortunes at the top.

Finally, the displacement term averages to 1.0% in Russia over the last ten years, which is roughly similar to the average displacement term in the U.S. over the same time period.

APPENDIX F: DECOMPOSING THE GROWTH OF TOP INCOME SHARES

I now discuss how to apply the framework to decompose the rise in top income shares. However, one key difference between wealth and income is that income is a flow, not a stock. In particular, in comparison with wealth, transitory shocks play a much larger role in driving the dynamics of income in the short run, although they have little impact on long-run inequality.[60] Models such as the ones introduced in Section 3 better describe the permanent component of income (most recently by Jones and Kim (2018) and Gabaix et al. (2016)).

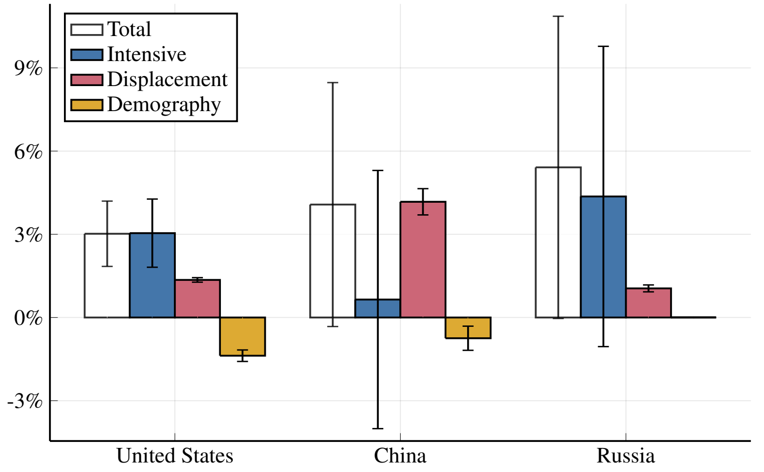
I use the IRS public use panel files created by the Statistics of Income Division from 1979 to 1990. This dataset includes a random sub-sample of taxpayers who can be followed over time between 1979 and 1990. Following Piketty and Saez (2003), I construct a comprehensive measure of earnings, which includes wages, salary, and entrepreneurial income.[61] I focus on tax filers who file jointly. In this context, birth and death represent the fact that a taxpayer starts or stops filing jointly. The sample contains 5,523 distinct taxpayers.

[60] See Heathcote et al. (2010) and Blundell et al. (2008).

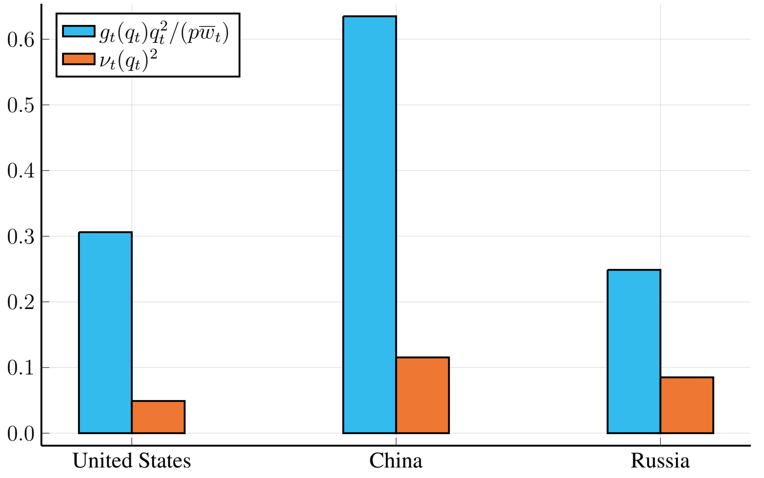
[61] More precisely, I construct earnings as adjusted growth income minus capital gain, dividend income, interest income, rental income, and royalties.

FIGURE E.1.—Decomposing the Annual Growth of Billionaires' Shares (2010-2017)[a]

(a) Intensive, Displacement, and Demography Terms



(b) Estimates of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$



[a]Figure E.1a plots the average annual (logarithmic) growth of the billionaires' shares in the U.S., Russia, and China, as well as the intensive, displacement, and demography terms defined in the logarithmic accounting framework (3). Figure E.1b plots the average estimates of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$ for these three countries. Data are from *Forbes* and the World Wealth and Income Database.

As mentioned earlier, I want to focus on the permanent component of income.[62] To do so, I average income over three-year periods before applying the accounting on the sample. I then annualize each term to obtain yearly terms.[63]

[62]The transitory component of income can be seen as a measurement error, which is discussed in Appendix D.

[63]I find similar results using a larger time period such as six years. This suggests that three years is enough to focus on the permanent component of income.

TABLE E.I

DECOMPOSING THE GROWTH OF BILLIONAIRES' SHARES (2010-2017)[a]

(a) Summary

	Total (%)	Intensive (%)			Displacement (%)	Demography (%)
		Total	Top	$-$Country		
United States	3.0	3.0	9.2	–6.2	1.4	–1.4
China	4.1	0.6	11.1	–10.4	4.2	–0.7
Russia	5.4	4.4	3.1	1.3	1.0	0.0

(b) Displacement (Details)

	Inflow (%)			Outflow (%)		
	Total	$n_{\mathcal{E}}$	$\frac{\overline{w}_{\mathcal{E},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{X}}$	$\frac{q_1-\overline{w}_{\mathcal{X},1}}{\overline{w}_{\mathcal{P}_0,0}}$
United States	1.1	8.2	14	0.2	5.9	4
China	2.9	17.8	17	1.4	16.6	9
Russia	0.6	10.7	5	0.5	10.7	5

(c) Demography (Details)

	Birth (%)			Death (%)			Pop. Growth (%)		
	Total	$n_{\mathcal{B}}$	$\frac{\overline{w}_{\mathcal{B},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{D}}$	$\frac{q_1-\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{D},0}}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$1-n_{\mathcal{P}_0}$	$\frac{q_1-\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0}}{\overline{w}_{\mathcal{P}_0,0}}$
United States	0.3	0.3	78	–0.9	1.6	–56	–0.8	1.0	–74
China	0.1	0.5	22	–0.7	1.2	–54	–0.3	0.5	–54
Russia	0.0	0.0	—	0.0	0.0	—	0.0	0.0	–78

[a] Table E.Ia reports the average of the annual (logarithmitic) growthfor Billionaires' shares as well as the intensive, displacement, and demography terms defined in the (logarithmic) accounting framework (3). Table E.Ib and Table E.Ic report the average of the inflow, outflow, birth, death, and population growth terms defined (2). Data are from *Forbes* and the World Wealth and Income Database.

Table F.I shows the result of the accounting decomposition for the top 100%, top 10%, and top 2% (I stop at the top 2% since it is only composed of 100 households). I find that the annual growth rate of top income shares during the time period for the top 2% is 3.9%. This number can be decomposed as follows: a intensive term of 0.3%, a displacement term of 4.0%, and a demography term of -0.4%. In other words, the displacement term accounts for the whole increase in the top income share.

Figure F.1 plots the result of the decomposition over all percentiles. A striking finding is that the intensive term and the demography term are relatively stable across the income distribution, while the displacement term gradually increases with top percentiles. Moreover, the slope of the displacement term for top percentiles exactly lines up with the slope of the growth of the top share, suggesting the displacement term has been a key driver of the rise in top income shares during the time period.

To understand why the displacement term increases with top percentiles, Figure F.1a plots the contributions of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$ over all percentiles. The standard deviation of log income follows a U-shape, as in Guvenen et al. (2015). Moreover, $g_t(q_t)q_t^2/(p\overline{w}_t)$ increases almost monotonically with the top percentile, except at the very top.

TABLE F.I

DECOMPOSING THE GROWTH OF TOP INCOME SHARES (1979-1990)[a]

(a) Summary

	Total (%)	Intensive (%)			Displacement (%)	Demography (%)
		Total	Top	$-$U.S.		
Top 100%	0.0	–0.1	2.6	–2.7	0.0	0.1
Top 10%	2.2	0.0	2.7	–2.7	2.3	–0.1
Top 2%	3.9	0.3	3.0	–2.7	4.0	–0.4

(b) Displacement (Details)

	Inflow (%)			Outflow (%)		
	Total	$n_{\mathcal{E}}$	$\frac{\overline{w}_{\mathcal{E},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{X}}$	$\frac{q_1-\overline{w}_{\mathcal{X},1}}{\overline{w}_{\mathcal{P}_0,0}}$
Top 100%	0.0	0.0	—	0.0	0.0	—
Top 10%	1.2	2.4	15	1.2	2.5	15
Top 2%	2.5	3.1	24	1.8	2.9	19

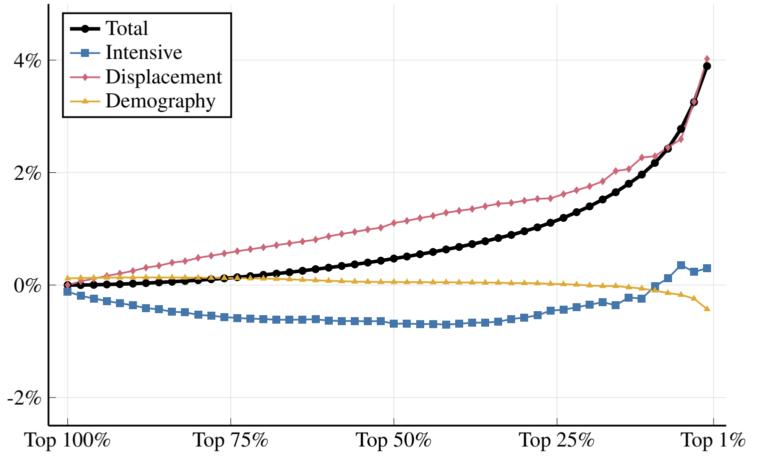
(c) Demography (Details)

	Birth (%)			Death (%)			Pop. Growth (%)		
	Total	$n_{\mathcal{B}}$	$\frac{\overline{w}_{\mathcal{B},1}-q_1}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{D}}$	$\frac{q_1-\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{D},0}}{\overline{w}_{\mathcal{P}_0,0}}$	**Total**	$1-n_{\mathcal{P}_0}$	$\frac{q_1-\frac{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{w}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{w}_{\mathcal{P}_0,0}}{\overline{w}_{\mathcal{P}_0,0}}$
Top 100%	3.8	1.7	74	–3.0	1.4	–67	–0.9	0.3	–91
Top 10%	1.1	1.2	32	–1.0	0.8	–36	–0.3	0.3	–35
Top 2%	0.9	1.0	32	–1.2	0.9	–35	–0.3	0.3	–30

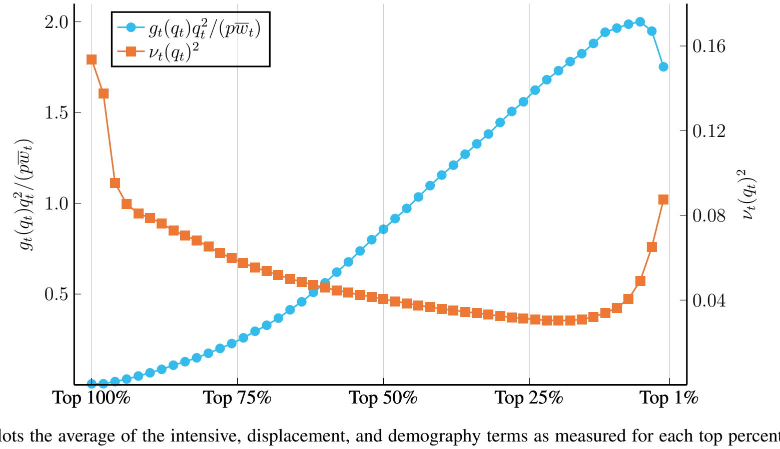
[a] Table F.Ia reports the average of the annual (logarithmitic) growth for the top 100%, 10%, and 2% income shares, as well as the intensive, displacement, and demography terms defined in the (logarithmic) accounting framework (3). Table F.Ib and Table F.Ic report the average of the inflow, outflow, birth, death, and population growth terms defined (2). Data are from the IRS public use panel files.

FIGURE F.1.—Decomposing the Growth of Top Income Shares (1979-1990)[a]

(a) Intensive, Displacement, and Demography Terms across Top Income Percentiles



(b) Estimates of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$ across Top Income Percentiles



[a]Figure F.1a plots the average of the intensive, displacement, and demography terms as measured for each top percentile of the income distribution. Figure F.1b plots the average estimates of $g_t(q_t)q_t^2/(p\overline{w}_t)$ and $\nu_t(q_t)^2$ for each top percentile of the income distribution. Data are from the IRS public use panel files.