Abstract

We examine the effect of interest rates on top wealth inequality. While low rates decrease the average growth rate of existing fortunes, we argue that they increase the growth rate of new fortunes by making it cheaper to raise capital. To understand which effect dominates, we derive a sufficient statistic for the effect of interest rates on the Pareto exponent of the wealth distribution in terms of observable moments, valid for agents with arbitrary exposure to firms with heterogeneous productivity. Using new data on the trajectory of top fortunes in the U.S., we show that the secular decline in discount rates has played an important role in the rise of top wealth inequality.

1 Introduction

Since Wold and Whittle (1957), a widely held view is that high interest rates increase top wealth inequality. As explained in Piketty and Zucman (2015), the intuition is that high rates increase the long-run growth rate of existing fortunes relative to the economy. Yet, this view appears to be at odds with the data: wealth inequality increased substantially in the past forty years in the U.S., a period marked by declining discount rates. Relatedly, a growing body of evidence suggests that the recent rise in top wealth inequality is not driven by the high returns of existing fortunes, but rather by the rise of new fortunes (Bach et al., 2017; Gomez, 2019; Zheng, 2019).

In this paper, we show that lower rates can actually increase top wealth inequality: while lower rates decrease the average growth rate of existing fortunes, they increase the growth rate of new fortunes by making it cheaper to raise capital. Quantitatively, we estimate the effect of lower rates to be large: the secular decline in real interest rate alone can explain most of the rise in “Pareto inequality” since 1985 (i.e., a decline in the Pareto exponent of the wealth distribution by 25%).

To understand our proposed mechanism, consider the typical trajectory of entrepreneurs making it to the top of the wealth distribution. To finance the growth of their firms, they...
have to continuously raise capital by issuing new shares to outside investors. Lower rates, i.e., higher valuations, increase the growth rate of these entrepreneurs, who get less diluted over time. On the other hand, lower rates hurt outside investors, who keep buying shares of growing firms at elevated prices.

The overall effect of lower rates on the Pareto exponent of the wealth distribution depends on the type of individuals making it to the top. If, as in the U.S., individuals at the top of the distribution made their fortunes as entrepreneurs, rather than investors, we show that lower rates increase Pareto inequality.

Our paper has four main parts. First, we formalize our idea in a simple stylized model. Entrepreneurs are born with trees. Trees require a continuous flow of investment to grow. To finance the growth of their tree, entrepreneurs continuously sell shares to outside investors (“rentiers”). With some hazard rate, trees mature, generating a one-time dividend proportional to their size. At maturity, entrepreneurs become rentiers themselves, investing their wealth in a diversified portfolio of trees.

In this stylized economy, we show that Pareto inequality is a U-shaped function of the interest rate. When the interest rate is high, only rentiers make it to the right tail of the distribution. In this case, lower rates decrease Pareto inequality, as in Wold and Whittle (1957) and Piketty (2015). However, as the interest rate continues to decrease, successful entrepreneurs with growing trees start to appear in the right tail of the distribution. When this happens, a decline in rates increase Pareto inequality.

Second, to quantify the effect of lower rates on the Pareto exponent of the wealth distribution, we build a more general model of wealth accumulation. Entrepreneurs are endowed with heterogeneous firms. Firm productivity evolves according to a Markov chain with an arbitrary transition matrix. Firm growth is determined by a q-theory of investment: capital investment is chosen as to maximize the firm’s value subject to adjustment costs.

Despite this arbitrary degree of heterogeneity, we obtain an analytical expression for the effect of the interest rate on the Pareto exponent of the wealth distribution, using insights from large deviation theory. We show that it depends on the effect of the interest rate on the wealth trajectory of individuals making it to the top of the wealth distribution. Put differently, it depends on the effect of the interest rate on the past growth rate of individuals at the top of the wealth distribution, not on their current growth rate.

We then show that this sufficient statistic can be expressed in terms on a few observable moments. In particular, it depends on the average payout yield of the firms owned by these entrepreneurs (i.e., the difference between the dividend yield and the equity issuance yield), as well as these firms’ durations (i.e., the sensitivity of firm value to the interest rate). Intuitively, if top individuals made their fortune holding firms with large equity issuance and a high time to maturity, we expect that lower rates to have a large effect on Pareto inequality. Another important moment is the average leverage of entrepreneurs to their firms. Leverage magnifies the effect of lower rates on Pareto inequality.

Third, we use new data on the trajectory of top fortunes to estimate these moments for the top 100 individuals in the U.S. We find that the lifetime average payout yield of firms owned by top individuals is around −2.2% annually, which means that entrepreneurs at the
top tend to be net equity issuers. The distribution of payout yields is extremely skewed: some entrepreneurs own firms with a lifetime average annual payout yield as low as \(-10\%\) (e.g., Facebook, Tesla, or Uber). Moreover, our data reveals a moderate average leverage of 1.4 (i.e., the market value exceeds the market value of the equity). Leverage is small for firms backed by VC funding, but it is relatively important for private firms that never raise equity over their lifetime.

Plugging these estimates into our sufficient statistic, we find that the effect of interest rates on wealth inequality is large. According to our preferred measures, a 1 percentage point real interest rate decline increases Pareto inequality by 3.6%. Since Pareto inequality has increased by roughly 25% from 1985 to 2015, this suggests that a 5 percentage point decline in discount rates since 1985 can explain roughly two-thirds of the rise in Pareto inequality.

Finally, while our sufficient statistic approach speaks to the long-run effect of lower rates on Pareto inequality, we also examine the dynamics of the wealth distribution along the transition path. In response to a sudden decrease in interest rates, total wealth in the top 1% increases for two reasons. First, due to a valuation effect, there is short-run increase in the wealth of entrepreneurs (i.e., a “level” effect). This increase is short-lived, since they earn lower returns going forward (i.e., a “growth” effect). Second, and more importantly, there is an increase in the flow of entrepreneurs that reach the top percentile. This increases total wealth in the top 1% through a composition effect. In the long-run, total wealth in the top 1% stabilizes at a permanently higher level.

Related Literature. The rise in top wealth inequality in the U.S. over the post-1980 period has documented extensively (see Saez and Zucman, 2016; Batty et al., 2019; Smitt and Zwik, 2020). A growing literature seeks to the factors behind this phenomenon. One strand of the literature focuses on the role of the average return on wealth for top individuals (Piketty, 2015; Kuhn et al., 2017; Moll et al., 2019; Hubmer et al., 2019).\(^1\) Another strand of the literature stresses the role of the dispersion of returns (Benhabib et al., 2011; Bach et al., 2015; Fagereng et al., 2020; Benhabib et al., 2019; Atkeson and Irie, 2020). We point out that these two factors can not be studied in isolation: a decrease in the average return on wealth can actually increase the dispersion of realized returns. As a result, a decrease in average returns can ultimately increase top wealth inequality. Our mechanism is consistent with a growing body of empirical evidence documenting the fact that the rise in top wealth inequality is driven by the rise of new fortunes, rather than high growth rates of existing fortunes (Bach et al., 2017; Campbell et al., 2019; Gărleanu and Panageas, 2017; Gomez, 2019; Zheng, 2019).

Our characterisation of the Pareto exponent of the wealth distribution builds on the literature on random growth processes (Wold and Whittle, 1957; Acemoglu and Robinson, 2015; Jones, 2015). Recently, this literature has moved towards models with persistent growth rate heterogeneity (Luttmer, 2011; Jones and Kim, 2016; Benhabib et al., 2015; Gabaix et al., 2016). In this case, the Pareto exponent can be obtained as the principal eigenvalue of an operator related to the transition matrix between states (see de Saporta, 2005; Beare et al., 2019). Relative

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\(^1\)Hubmer et al., 2019 argue that the decline in tax progressivity has played a key role in increasing the average after-tax return on wealth. Kaymak and Poschke, 2016 also emphasize the importance of the decline in tax progressivity.
to that literature, a theoretical contribution of our paper is to obtain a closed-form expression for the derivative of the Pareto exponent with respect to a parameter (here, the interest rate). We show that the derivative of the Pareto exponent with respect to a parameter depends on its effect on the whole wealth trajectory of individuals reaching the top of the wealth distribution. Remarkably, the effect of interests on the wealth accumulation trajectory of individuals making it to the top depends on a set of moments that can be estimated empirically. This “sufficient statistic” approach allows us to directly quantify the effect of interest rates on Pareto inequality.

Beyond the literature on Pareto tail, several papers examine the redistributive effect of changes in the interest rate. Gârleanu and Panageas (2017), Gârleanu and Panageas (2019) and Kogan et al. (2020) build models in which lower discount rates benefit entrepreneurs at the expense of households. Auclert (2019) stresses the heterogenous exposure of households to transitory changes in the interest rate.

Our model also relates to the literature on entrepreneurial wealth accumulation (Quadrini, 2000, Cagetti and De Nardi, 2006; Moll, 2014; Guvenen et al., 2019; Peter, 2019; Tsiaras, 2019). As in these papers, we assume that entrepreneurs remain exposed to their firms, which plays an important role in shaping the wealth distribution. One key difference is that we consider a model where firms can freely issue equity. In our model, as in the data, the most successful firms continuously raise equity and, therefore, vastly outgrow their founder. This allows us to distinguish two channels by which lower rates can increase top wealth inequality: the “leverage” effect (for entrepreneurs who issue debt) and the “dilution” effect (for entrepreneurs who issue equity). The “dilution” effect turns out to be more important quantitatively, consistent with the fact that firms backed by VC funding tend to be all-equity firms.

This focus on equity issuance relates our paper to a large literature studying how firms raise capital: this includes VC funding (Cochrane, 2005; Hall and Woodward, 2010; Opp, 2019; Gornall and Strebulaev, 2015; Gornall and Strebulaev, 2020), equity-based compensation (Ofek and Yermack, 2000; Frydman and Jenter, 2010; Ai et al., 2018; Eisfeldt et al., 2019), IPOs (Ritter and Welch, 2002; Pastor and Veronesi, 2005), as well as seasoned equity offering (Fama and French, 2004; Baker and Wurgler, 2006; Boudoukh et al., 2007).

2 Stylized model

In this section, we describe our mechanism in a stylized model of wealth inequality. Our main result is that, in presence of entrepreneurs, Pareto inequality is a U-shaped function of the interest rate.

2.1 Environment

The economy is populated by infinitely-lived agents. Population grows at rate $\eta$. There are two types of agents: “entrepreneurs” and “rentiers”. All agents are born “entrepreneurs” and are endowed with a tree. Trees require outside investment to grow until they blossom. When trees blossom, entrepreneurs cash out and become rentiers.
**Trees.** Each tree starts with a size of one and grows at rate $g$. To grow, the tree requires a flow of investment $i$ proportional to its size. With hazard rate $\delta$, the tree blossoms and returns a one-time positive dividend equal to its size. Formally, the instantaneous cash-flow $dD_t$ of a tree which blossoms at time $T$ is given by

$$dD_t = \begin{cases} -ie^{\delta t} dt & \text{if } t < T \\ e^{\delta t} & \text{if } t = T. \end{cases}$$

We assume that $g - \delta < \eta$ so that trees do not grow faster than the population. We also assume that $i < \delta$ so that trees return a positive amount of dividend in expectation.

**Pricing.** Because the cash-flow of the tree is proportional to its size, the value of a tree is also proportional to its size. Denote $q$ the ratio of the value of a tree to its size. The instantaneous return of holding a tree is given by

$$\frac{dR_t}{R_t} = \begin{cases} \left( -\frac{i}{q} + g \right) dt & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T. \end{cases} \tag{2.1}$$

In words, while the tree is still growing, i.e. $t < T$, the return of holding the tree during a time $dt$ is the sum of a negative payout yield, $-i/q dt$, and a capital gain $g dt$. It is useful to think of investment as being financed by issuing new shares, rather than by requiring existing shareholders to invest more money in the firm. Accordingly, the return can be seen as the growth of the tree $g dt$, minus the relative amount of new shares $i/q dt$ that must be sold to outside investor to raise $i dt$. This adjustment reflects the extent to which existing shareholders get diluted during a time period $dt$. When the tree blossoms, i.e., $t = T$, the instantaneous return is $1/q - 1$, since the tree returns a one-time dividend equal to its size.

Denote $r$ the interest rate, which we take as exogenous for now. We assume that $r > g - \delta$ to ensure that the price of the tree is finite. The price $q$ is pinned down by the fact that the expected return of a tree must equal the interest rate $r$

$$r = -\frac{i}{q} + g + \delta \left( \frac{1}{q} - 1 \right), \tag{2.2}$$

which implies that $q = (\frac{-i}{q} + \delta)/(r - g + \delta)$. In particular, the price $q$ is a decreasing function of the interest rate $r$.

The key point is that, while a low interest rate naturally decreases the average return of holding a tree, it increases the return of the tree conditional on not blossoming, $-i/q + g$. Intuitively, lower rates, i.e., higher valuations, decrease the rate at which new shares $i/q$ must be issued to finance growth. Therefore, existing shareholders get less diluted as the tree grows.

**Wealth accumulation.** Agents have log utility and discount the future at rate $\rho$ which implies that they optimally consume a constant fraction $\rho$ of their wealth. Our key assumption is that entrepreneurs must remain fully exposed to their tree while it is growing.
Let $W_t$ be the wealth of an individual. The wealth growth of an entrepreneur during a time period $dt$ is $dW_t/W_t = dR_t/R_t - \rho dt$, i.e.,

$$
\frac{dW_t}{W_t} = \begin{cases} 
  \left( -\frac{i}{q} + g - \rho \right) dt & \text{if } t < T \\
  \frac{1}{q} - 1 & \text{if } t = T.
\end{cases}
$$

(2.3)

When the tree blossoms, the entrepreneur then becomes a rentier and invests in a diversified portfolio of trees. The wealth of a rentier evolves as

$$
\frac{dW_t}{W_t} = (r - \rho) dt.
$$

(2.4)

The interest rate has opposite effects on the growth rates of entrepreneurs and rentiers. While a lower interest rate decreases the growth rate of rentiers, it increases the growth rate of successful entrepreneurs (i.e., the ones with trees that keep growing). This is shown graphically in Figure 1, which plots the total wealth of an entrepreneur with a tree blossoming at date $T = 15$ in a low vs. high interest rate economy.

![Figure 1: Wealth trajectory of an entrepreneur with a tree that blossoms after 15 years ($T = 15$)](image)

Numerical example with $i = 0.4$, $g = 0.5$, $\delta = 0.5$, $\eta = 0.05$, $\rho = 0.04$

**Discussing our assumptions.** We now discuss the two key assumptions that we make in the model. The first assumption is that trees require outside investment to grow, i.e., $i > 0$. This assumption captures an important characteristic of young firms: they require outside funding to grow. As we will discuss in Section 4, this outside funding is a mix of equity issuance (VC funding or public equity offering) and stock-based compensation. In the general model, we will allow firms to have positive or negative payout yields depending on their current productivity.

The second key assumption is that entrepreneurs must remain fully exposed to their tree. This assumption captures the fact that most of the wealth of entrepreneurs remains invested in their own firms (Quadrini, 2000; Cagetti and De Nardi, 2006; Roussanov, 2010). We take this
as exogenous, but this type of portfolio choices can be derived by moral hazard or asymmetric information problems (He and Krishnamurthy, 2012; Brunnermeier and Sannikov, 2014; di Tella, 2017). In the general model, we will allow entrepreneurs to have an exposure to their firms lower or higher than one.

The key distinction between entrepreneurs and rentiers is that entrepreneurs fully invest in one tree whereas rentiers own a diversified portfolio of trees. More generally, the term “entrepreneur” should be understood as referring to any agent that is disproportionately exposed to a growing tree. This includes founders of firms that use equity financing, but also early employees paid in stock-options or restricted stocks, or even some investors with concentrated portfolios, such as V.C. investors.

2.2 Interest rates and the wealth distribution

We now characterize the Pareto exponent of the wealth distribution in this economy.

**Pareto inequality.** We focus on a measure of wealth inequality that captures the right tail of the distribution.

**Definition 2.1.** We say that the distribution of a random variable \( X \) has a Pareto tail if there exists \( \zeta > 0 \) such that

\[
\lim_{x \to \infty} \frac{\log \mathbb{P}(X > x)}{\log x} = -\zeta.
\]

\( \zeta \) is called the Pareto exponent.

A low \( \zeta \) corresponds to a thick tail (i.e., a density that decays very slowly). Following Jones, 2015, we define Pareto inequality \( \theta \) as the inverse of the Pareto exponent (i.e., \( \theta = 1/\zeta \)). A high level of Pareto inequality corresponds to a thick upper tail. The following proposition characterizes Pareto inequality as a function of the interest rate \( r \).

**Proposition 2.2.** Assume that \( 0 < \rho < g - i \). There exists \( r^* \in (g - \delta, \rho + \eta) \) such that

\[
\theta = \begin{cases} 
-\frac{i}{\eta} + \frac{g - \rho}{\eta + \delta} & \text{for } r \in (g - \delta, r^*) \\
\frac{r - \rho}{\eta} & \text{for } r \in (r^*, \rho + \eta)
\end{cases}
\]

This proposition says that Pareto inequality \( \theta \) is a U-shaped function of the interest rate: it is a strictly decreasing function for \( r \in (g - \delta, r^*) \) and a strictly increasing function for \( r \in (r^*, \rho + \eta) \).

When \( r > r^* \), the upper tail of the wealth distribution is only populated by rentiers. In this case, lower rates decrease Pareto inequality, since these agents benefit from higher interest rates. The formula for the Pareto exponent is the same as in an economy with only one type of agents (Wold and Whittle, 1957; Piketty and Zucman, 2015).

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\(^2\)To be precise, we consider a balanced growth path where growth per capita is zero.
In contrast, when \( r < r^* \), entrepreneurs are present in the upper tail of the wealth distribution.\(^3\) In this case, lower rates increase Pareto inequality. This is because the right tail of the distribution is determined by the growth rate of successful entrepreneurs and those agents benefit from lower interest rates. As explained above, a decline in the interest rate \( r \) leads to an increase of the price of a share \( q \). High valuations imply that entrepreneurs can finance the growth of their tree by issuing fewer shares, thereby leading to a lower dilution rate. Figure 2 plots the relationship between the interest rate and Pareto inequality in a numerical example.

The case \( r < r^* \) is the empirically relevant one. In the data, the wealthiest Americans often have most of their wealth invested in fast-growing firms that they founded. For instance, Cagetti and De Nardi (2006) show that entrepreneurs (i.e., business owners who actively manage their business) represent only 11.5% of the total population, yet they account for 65% of individuals in the top 1% of the wealth distribution.

![Figure 2: Pareto Inequality \( \theta \) as a function of the interest rate. Numerical example with \( i = 0.4, g = 0.5, \delta = 0.5, \eta = 0.05, \rho = 0.04 \).](image)

So far, we have considered an exogenous change in interest rates. Throughout the paper, we remain agnostic about the exact source of the change in the interest rate. It could come from a change in savings coming from abroad (global savings glut) or from domestic markets (population aging). As an example, in Appendix A.2, we extend our model to incorporate an additional group of agents (i.e., “workers”). We show that varying the impatience of workers from zero to infinity can generate the range of interest rates considered in Proposition 2.2.

What is the quantitative effect of the interest rate on Pareto inequality? In this stylized model, it depends on the payout yield of the tree \(-i/q\) and the sensitivity of the share price to the interest rate (i.e., the duration). This insight holds in a much more general model, as we show in the next section.

### 3 General model

We now extend the stylized model to make it more realistic. This will allow us to quantify the effect of interest rates on the Pareto exponent of the wealth distribution.

\(^3\)Formally, the relative mass of entrepreneurs does not converge to zero as wealth tends to infinity.
Entrepreneurs now own firms, not trees. We allow for an arbitrary degree of firm heterogeneity, i.e., firm TFP follows a continuous-time Markov Chain. Moreover, firm investment is optimally chosen to maximize the value of the firm. Despite these extensions, the model remains tractable. In particular, we are able to derive a sufficient statistic for the effect of the interest rate on top wealth inequality.

3.1 Set up

Environment. The economy is populated by a continuum of entrepreneurs and rentiers that grows at exogenous rate $\eta$. Agents are born entrepreneurs and are endowed with a firm. The firm has initial capital equal to one.

4 All risk is idiosyncratic. At rate $\delta$, entrepreneurs sell their firm and become rentiers who live forever and hold a diversified portfolio with return $r$.

Firm problem. Firms produce an homogeneous consumption good (the numéraire) and operate an $aK$ technology where $a$ denotes TFP and $K$ denotes the capital stock. TFP evolves over time according to time-reversible Markov Chain with states $s \in \{1, \ldots, S\}$.

The problem of a firm in state $s$ is to choose a growth rate $g$ as to maximize the present value of future payouts discounted at rate $r$. In order to grow its capital stock by $gK$, the firm must invest $i(g)K$ units of the consumption goods, where $i'(\cdot) > 0$ and $i''(\cdot) > 0$. The value of a firm $V_s(K)$ is the solution to the following Hamilton-Jacobi equation (HJB):

$$rV_s(K) = \max_g \left\{ (a_s - i(g))K + V_s'(K)gK + (\mathcal{T}V)_s(K) \right\}, \quad (3.1)$$

where $\mathcal{T} = (\tau_{ss'})$ denotes the transition probability matrix for the states $s$.

Given that the value function is homogeneous in capital $K$, it can be written as $V_s(K) = q_sK$, where $q_s$ is ratio of the market value of the firm to its book value (i.e., Tobin’s $q$). The HJB thus simplifies to

$$rq_s = \max_g \left\{ a_s - i(g) + q_sg + (\mathcal{T}q)_s \right\}. \quad (3.2)$$

From now on, we assume that there exists a solution to (3.2). The HJB implies that the optimal growth rate $g_s$ satisfies the following first-order condition

$$i'(g_s) = q_s. \quad (3.3)$$

Pricing. The return of a firm is

$$\frac{dR_t}{R_t} = a_{g_t} - i(g_t) \frac{dq_t}{q_t} dt + g_t dt + \frac{dq_t}{q_t},$$

\[\frac{dq_t}{q_t}\] payout yield \[\frac{dq_t}{q_t}\] capital gains

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4 We could also allow initial capital to be heterogeneously distributed across entrepreneurs. As long as the right tail of this initial distribution remains thinner than the tail of the wealth distribution, this has no impact on our results.

5 To be precise, $\mathcal{T}$ is the infinitesimal generator for $s$ defined as $\mathbb{E}_s[\frac{dq_t}{dt}] = (\mathcal{T}q)_s$. We abuse notation and treat $\mathcal{T}$ as an $S \times S$ matrix.
where $s_t$ denotes the Markov state of the firm at time $t$. The return is the sum of three terms: the payout yield $(a_s - i(g_s)) / q_s$, the growth rate of the firm $g_s$, and the change in share price $dq_s / q_s$.

As in the stylized model, it is useful to think of a firm that never pays dividends and simply sells and repurchases shares to distribute cash flows to shareholders. When the firm payout is negative (i.e., $i(g_s) > a_s$), the return can be seen as the growth rate of the number of outstanding shares $g_s$, minus the fraction $(i(g_s) - a_s) / q_s$ of shares sold to outside investors, plus the change in the value of a share. Conversely, when the firm payout is positive, the return can be seen as the growth rate of the number of outstanding shares plus the fraction $(a_s - i(g_s)) / q_s$ of shares that are repurchased from existing shareholders, plus the change in the value of a share.

The HJB ensures that the expected return of owning a share equals the interest rate $r$, which pins down the vector of firm prices $q_s$.

### 3.2 Wealth Accumulation

**Market Wealth.** As in the stylized model, agents have log utility and discount the future with rate $\rho$, which implies that they optimally consume a constant fraction $\rho$ of their wealth. We assume that entrepreneurs invest all of their wealth in their firms.

The growth rate of wealth for an entrepreneur is given by

$$\frac{dW_t}{W_t} = \frac{dR_t}{R_t} - \rho \, dt. \quad (3.4)$$

With hazard rate $\delta$, investors sell their firms, and use the proceed to invest in a diversified portfolio of firms. The growth rate of wealth for rentiers is simply given by

$$\frac{dW_t}{W_t} = (r - \rho) \, dt. \quad (3.5)$$

**Book Wealth.** It is useful to define the book wealth of an entrepreneur $B_t$ as wealth divided by the price of the firm they own, i.e. $B_t = W_t / q_s$. It can be seen as the amount of “in-firm” capital owned by the entrepreneur.

The distribution of book wealth and market wealth for entrepreneurs share the same Pareto exponent. This is because the ratio between the two (i.e., the price of a share $q$) is bounded: inequality between wealthy entrepreneurs is mostly driven by differences in the amount of capital that they own, rather than differences in the market value $q$ of that capital.

Compared to market wealth, book wealth is easier to work with because it does not “jump” when the TFP of the firm changes. Using Proposition 3.4, the law of motion of book wealth of

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6See the proof of Proposition 3.1 for a proof.
an entrepreneur holding a firm is given by:

\[
\frac{dB_t}{B_t} = \left( \frac{a_s - i_s}{q_s} + g_s - \rho \right) \mu_s dt
\]  

(3.6)

We now examine the effect of the interest rate on the growth rate of book wealth. Using Equation 3.6, the derivative of the growth rate of book wealth in state \( s \) is given by

\[
\partial_r \mu_s = \frac{a_s - i(g_s)}{q_s} \left| \partial_r \log q_s \right| + \left( 1 - \frac{i'(g_s)}{q_s} \right) \partial_r g_s.
\]  

(3.7)

It is the sum of a term that is due to the effect of \( r \) on the share price, and a term that is due to the effect of \( r \) on the optimal growth rate of the firm. Because the optimal growth rate ensures that the firm invests up to the point where the marginal cost of capital equals its marginal value, the second term is zero (this comes from the envelope theorem). \(^8\)

Therefore, the effect of interest rate on the growth rate of book wealth depends on the payout yield and duration of the firm owned by the entrepreneur. In particular, lower rates increase the growth rate of book wealth as long as the entrepreneur is a net equity issuer (i.e., \( a - i(g) q < 0 \)). Intuitively, when interest rates are low, valuations are high, which means that fewer shares need to be sold to outside investors to raise funds. This increases the rate at which successful entrepreneurs accumulate firm capital.

3.3 Leverage

So far, we have considered the case of all-equity firms. In reality, firms issue a mix of equity and debt claims. We now assume that firms have a ratio of debt to book equity \( \kappa \), and that entrepreneurs invest all of their wealth in the equity of their firms. This creates a supplementary channel by which lower rates can increase wealth accumulation.

Let \( q_{ks} \) be the price of an equity share for a firm in state \( s \) with debt to book equity ratio \( \kappa \):

\[
q_{ks} = q_s + \kappa(q_s - 1).
\]

Denote \( B_t = W_t / q_{ks} \) the book wealth of the entrepreneur, i.e., the amount of book equity owned by the entrepreneur. The growth rate of book wealth for an entrepreneur owning a firm of type

\[\footnote{Alternatively, we can rewrite this equation using the accounting concept of return on assets, i.e. roa_s = a_s - (i(g_s) - g_s): \( \mu_s = \text{roa}_s + (g_s - \text{roa}_s)\left(1 - \frac{1}{q_{ks}}\right) - \rho \).}

\[\footnote{Ozdagli (2018) and Darmouni et al. (2020) use a similar insight to interpret a firm stock market response to monetary policy.}
s is now given by

$$\frac{dB_t}{B_t} = \left( \frac{a_s - i(g_s) + \kappa(a_s - i(g_s)) - (r - g_s)}{\rho_s} + g_s - \rho \right) dt$$

(3.8)

The growth rate of book wealth corresponds to the equity payout yield plus the growth rate of book equity minus the consumption rate $\rho$. To understand the expression for the equity payout yield, note that during a short period of time $dt$, the payout to equity-holders (per unit of equity) is equal to profits net of investment $(1 + \kappa)(a_s - i(g_s))$ minus the payout to debt-holders $\kappa(r - g_s)$.

There are now two channels by which lower rates can increase wealth accumulation of successful entrepreneurs. First, as in (3.7), a lower $r$ increases the value of the firm, which tends to reduce the dilution of existing shareholders. Second, due to leverage, a lower $r$ increases the amount of cash returned to equity-holders at the expense of debt-holders. Formally, we obtain that the effect of a change in $r$ on the rate of wealth accumulation is now given by

$$\partial_r \mu_s = \frac{a_s - i(g_s) + \kappa(a_s - i(g_s)) - (r - g_s)}{q_{ks}} \frac{\partial \ln q_{ks}}{\partial t} - \frac{\kappa}{q_{ks}}.$$  

(3.9)

Note that, when firms are not levered, i.e. $\kappa = 0$, this reverts to the formula without leverage.

### 3.4 Wealth distribution

**Pareto Tail.** We now characterize the steady-state wealth distribution of wealth for entrepreneurs and rentiers. It is useful to denote the following matrix

$$A(\theta) = \frac{1}{\theta} D(\mu) + \mathcal{T},$$

(3.10)

where $D(\mu)$ denotes the diagonal matrix with $(\mu_1, \ldots, \mu_S)$ on its diagonal. In Hansen and Scheinkman (2009) terminology, $A(\theta)$ corresponds to the infinitesimal generator associated with the multiplicative functional $B_1^\mu$.

The matrix $A(\theta)$ is the sum of a diagonal matrix and of the infinitesimal generator of the Markov process for productivity. In particular, all of its off-diagonal elements are non-negative. Such a matrix has a dominant eigenvalue $\rho_D(A(\theta))$, which is the real eigenvalue such as all other eigenvalues have smaller real parts. The following proposition uses this property to characterize the tail of the wealth distribution in our economy.

**Proposition 3.1.** Suppose that there exists at least one state $s'$ such that $\mu_{s'} > 0$ (i.e., there is a productivity state in which the growth rate of book wealth is positive). Then there is a one and only one positive number $\theta_E$ such that

$$\rho_D(A(\theta_E)) = \eta + \delta.$$
Moreover, the wealth distribution has a Pareto tail with Pareto inequality

\[ \theta = \max \left( \theta_E, \frac{r - \rho}{\eta} \right). \]

If \( \theta_E > \frac{r - \rho}{\eta} \), rentiers “inherit” the Pareto tail of entrepreneurs. Otherwise, the distribution of wealth across rentiers has a thicker tail than the distribution of wealth across entrepreneurs, which implies that the upper tail of the wealth distribution is populated only by rentiers. As discussed in the stylized model, the first case is the empirically relevant one. Therefore, from now on, we assume that \( \theta_E > \frac{r - \rho}{\eta} \) (i.e. \( \theta = \theta_E \)).

This proposition is a generalization of Proposition 2.2. Indeed, in the stylized model, there is only one productivity state. In this case, \( A(\theta_E) = \frac{1}{\theta_E} \left( -\frac{1}{\delta} + g - \rho \right) \), which gives \( \theta_E = \frac{-\frac{1}{\delta} + g - \rho}{\eta + \delta} \).

**Effect of interest rate on Pareto tail.** Compared to the stylized model, there is no closed-form solution for Pareto inequality in terms of the interest rate. However, we can still derive a simple expression for the derivative of Pareto inequality with respect to the interest rate \( \partial_r \theta \). This is the main theoretical result of this section.

**Proposition 3.2.** Denote \( u, v \) the left and right eigenvector associated with the dominant eigenvalue of the matrix \( A(\theta_E) \), normalized so that \( \sum_s u_s = 1 \) and \( \sum_s u_s v_s = 1 \). The derivative of Pareto inequality \( \theta \) with respect to the interest rate is given by:

\[ \partial_r \log \theta = \frac{(u \cdot v)' \partial_r \mu}{(u \cdot v)' \mu}, \quad (3.11) \]

where \( u \cdot v \) denotes the element-wise multiplication of \( u \) and \( v \).

In words, the derivative of the Pareto exponent with respect to \( r \) is proportional to the derivative of the growth rate of book wealth, averaged with respect to the vector \( u \cdot v \).

The following proposition gives a physical interpretation for the vector \( u \cdot v \): it corresponds to the density of past states for individuals in the right tail of the wealth distribution.

**Proposition 3.3 (Sufficient statistic).** Let \( \tau \) denote the age of an entrepreneur, and \( s_t \) the state of her firm at age \( t \). The derivative of Pareto inequality \( \theta \) with respect to the interest rate is given by

\[ \partial_r \log \theta = \lim_{w \to +\infty} \mathbb{E} \left[ \frac{1}{\tau} \int_0^\tau \partial_r \mu_s \, dt \bigg| W = w \right]. \quad (3.12) \]

This equation says that the effect of the interest rate on the Pareto exponent is given by its effect on the past growth rate of entrepreneurs in the right tail. This form of ex-post conditioning is key: what matters is the effect of interest rates on the growth rate of the entrepreneurs that are going to reach the top, not its effect on the growth rate of entrepreneurs already at the top.

Plugging Equation 3.9 into Proposition 3.3, we obtain a sufficient statistic for the effect of
interest rates on the Pareto exponent in terms of observable moments:

\[
\partial_r \log \theta = \lim_{w \to +\infty} \mathbb{E} \left[ \frac{1}{w} \int_0^\infty \left( \frac{a_t - i(g_{st}) + \kappa(a_t - i(g_{st}) - (r-g_{st}))}{q_{st}} \right) \partial_r \log q_{st} \right] \left| \frac{\kappa}{q_{st}} - \frac{\kappa(q_{st})}{d} \right| W = w \right].
\] (3.13)

This is our key sufficient statistic. Note that the right-hand side can be directly measured in the data, by measuring the lifetime average of the payout yield, duration and growth rate of entrepreneurs in the right tail of the distribution.

4 Empirics

In this section, we use data from various sources to estimate our sufficient statistic for the effect of interest rates on Pareto inequality. We then use this estimate to quantify how much of the rise in top wealth inequality in the U.S. over the post-1980 period can be explained by the secular decline in interest rates.

4.1 Measuring the sufficient statistic

We estimate the effect of interest rate on Pareto inequality using our summary statistic (Equation 3.13). To estimate it in the data, we need to measure the lifetime average equity payout yield, duration, market debt-to-equity, and growth rate of wealth for individuals in the right tail of the distribution. Given a sample of \(N\) individuals in the right tail of the wealth distribution, our estimator is:

\[
\hat{\partial_r \log \theta} = \frac{1}{N} \sum_{i=1}^N \left( \text{equity payout yield}_i \times \text{duration}_i - \text{market debt to equity}_i \text{growth rate}_i \right).
\] (4.1)

As we discuss shortly, we calibrate the duration rather than trying to estimate it separately for each individual.

**Forbes list.** We identify individuals in the right tail of the wealth distribution using the list of the wealthiest 400 Americans produced by Forbes Magazine. The list is created by a dedicated staff of the magazine, based on a mix of public and private information. For our application, we focus on the year 2015 and define the “right tail” as individuals in the top 100, a group for which information is more widely available.

Table 1 contains information on the top 100 individuals included in the Forbes list in 2015. Out of this set of individuals, we remove 34 individuals who inherited their wealth (i.e., “heirs”) and 22 individuals who own manage financial firms or have a diversified portfolio of assets, since our framework does not directly apply to them. We are left with 44 individuals for which we have detailed information (age, wealth, source of wealth, firms that they founded).

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9Forbes Magazine reports: “We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don’t pretend to know what is listed on each billionaire’s private balance sheet, although some candidates do provide paperwork to that effect.”
Table 1: Individuals in the top 100 (Forbes list, 2015)

<table>
<thead>
<tr>
<th>Group</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entrepreneurs</td>
<td>44</td>
</tr>
<tr>
<td>Public corporation</td>
<td>29</td>
</tr>
<tr>
<td>Private corporation</td>
<td>15</td>
</tr>
<tr>
<td>Heirs</td>
<td>34</td>
</tr>
<tr>
<td>Others</td>
<td>22</td>
</tr>
</tbody>
</table>

Notes. “Entrepreneurs” are defined as individuals who founded a non-financial firm that they actively manage; “Heirs” are defined as individuals who inherited their wealth; “Others” includes individuals who own or manage financial firms as well as individuals with a diversified portfolio of assets.

Roughly two thirds own public firms (i.e., C corporations) while the rest own private firms. Table 6 in the appendix contains a detailed list of the individuals that we consider.

**Growth rate of wealth.** We approximate the cumulative growth rate of wealth for individuals at the top of the wealth distribution as the log of the ratio between current wealth and an imputed initial wealth of $100,000 in 2015 dollars. We obtain the average lifetime growth rate by dividing the cumulative growth rate by the age minus 18. We also explore two alternative hypotheses regarding the initial wealth ($10,000 and $1,000,000). We report summary statistics on the growth rate of wealth in Table 2. We estimate an average growth rate of 0.30, with large outliers corresponding to Mark Zuckerberg (1.08) and Dustin Moskovitz (0.95), who are cofounders of Facebook.

**Equity Payout yield.** To estimate the equity payout yield, we first focus on the contribution of share issuance and repurchases—i.e., the dilution rate—and then estimate the contribution of dividends. To gain some intuition, consider a firm who sells and repurchases its shares but does not pay dividends. In that case, the equity payout yield is precisely the rate at which equity holders get diluted: when a firm issues shares, the existing equity holders get diluted and the opposite is true when a firm repurchases its own shares.

We now describe how we estimate the (annual) dilution rate for entrepreneurs who founded firms that were public in 2015. For these individuals, we first use 10-K filings to compute the number of outstanding shares in 2015, which we denote $S_2015$ by summing common and reserved shares. (We adjust $S_2015$ to account for stock splits after the IPO). We then use data from the S-1 filing to compute how many shares the founder held at the time of the IPO—which we denote $S^F_{IPO}$—as well as how many shares were granted to the founder in between the founding data and the IPO as a form of labor compensation—which we denote $S^F_{granted}$. Finally, we use the ownership share of the founder when the firm was founded $\omega^F_{t0}$ to estimate the initial number of shares as

$$S_{t0} = \frac{S^F_{IPO} - S^F_{granted}}{\omega^F_{t0}}.$$

Finally, we estimate the average dilution rate between the year when the firm was founded
(i.e., $t_0$) and 2015 as

$$\text{dilution rate} = \frac{\log S_{2015} - \log S_{t_0}}{2015 - t_0 + 1}.$$  

When S-1 filings are not available (mostly for private firms), we gather public information on the ownership share of the founder when the firm was founded as well as in 2015 and then compute the dilution rate as the average log change of the ownership share. For instance, founders of private firms who never issued equity have a dilution rate of zero.

To account for dividend issuance, which contribute positively to the equity payout yield, we use data on the dividend yield. For firms who were public in 2015, we compute their dividend yield using CRSP data for every post-IPO year and make the assumption that the pre-IPO dividend yield was zero. For these firms, we obtain an average dividend yield of 0.81% annually. For firms who were private in 2015, we impute an annual dividend yield of 0.81%.

Finally, we compute the equity payout yield as the dividend yield minus the dilution rate. We report summary statistics on the average payout yield in Table 2. We find that firms owned by individuals in the right tail in 2015 have had an average annual payout yield of -2.2% since they were founded. The distribution of payout yields is highly negatively skewed, with values ranging from -16% for Travis Kalanick (Uber) to 3.7% for Les Wexner (L brands).

Table 2: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>Average</th>
<th>Percentiles</th>
<th>Min</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rate (%)</td>
<td>44</td>
<td>30.2</td>
<td></td>
<td>14.8</td>
<td>19.6</td>
<td>22.4</td>
<td>32.3</td>
<td>107.6</td>
</tr>
<tr>
<td>Equity payout yield (%)</td>
<td>44</td>
<td>−2.2</td>
<td></td>
<td>−16.0</td>
<td>−4.9</td>
<td>0.1</td>
<td>0.8</td>
<td>3.7</td>
</tr>
<tr>
<td>Market Debt-to-Equity</td>
<td>44</td>
<td>0.4</td>
<td></td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>0.4</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Duration. The effect of the interest rate on Pareto inequality depends on the average duration of the firms owned by individuals that reach the top of the wealth distribution. Duration is defined as the present-value weighted time to maturity of the expected cashflows generated by the firm.\(^\text{10}\) In our model, it is equal the derivative of the log price of the firm with respect to the interest rate. Duration is a key determinant of the effect of the interest rate on the dilution rate of entrepreneurs who finance the growth of their firm through equity issuance. The higher the duration of a firm, the more its share price increases in response to a decline in the interest rate, and therefore the lower the dilution rate.

The duration of firm cashflows is hard to measure quantitatively, since it needs to be an ex-ante measure of the cash-flow duration of the firm, rather than an ex-post. In our preferred calibration we assume a duration of 30 years, which we see as conservative. One reference is Gormsen and Lazarus (2019), who finds that the duration of the top 20% of the firms in CRSP is 39 years.\(^\text{11}\) We also explore two alternative hypothesis: a duration of 20 years and a duration

\(^{10}\)For instance, a firm whose expected cashflows are \{ $x_t$ \} has a duration of \(\int_0^\infty \frac{t \cdot e^{-rt} \cdot x_t}{\int_0^\infty e^{-rs} \cdot x_s \, ds} \, dt\).

\(^{11}\)See also Weber (2018) and van Binsbergen (2020).
Market debt-to-equity. For firms in our sample who are public for a subset of years, we estimate their average market to debt equity for those years. For private firms, we impute a market debt-to-equity of 0.4, which corresponds to the average for public firms.

Results. We now use Equation C.1 to combine our estimates of the average equity payout yield, debt-to-equity, and growth rate of wealth of individuals reaching the top of the wealth distribution. Table 3 contains the predicted effect of the interest rate on Pareto inequality \( \partial_r \log \theta \) in our preferred calibration as well as using an alternative hypothesis about duration and initial wealth. To describe the variation in the data, we also compute the sufficient statistic for each individual and report summary statistic. In our favorite calibration, we obtain a value for \( \partial_r \log \theta \) of \(-3.6\). The interpretation is that for a 1 percentage point decrease in the interest rate, Pareto inequality \( \theta \) increases by 3.6%. Alternative calibrations imply values ranging between \(-1.5\)% and \(-5.6\)%.

Table 3: Measuring the effect of interest rate on tail inequality \( \partial_r \log \theta \)

<table>
<thead>
<tr>
<th>Obs.</th>
<th>Average</th>
<th>Percentiles</th>
<th>Min</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>44</td>
<td>-3.6</td>
<td>-19.2</td>
<td>-5.2</td>
<td>-1.4</td>
<td>-0.7</td>
<td>3.2</td>
</tr>
<tr>
<td>Duration = 20 years</td>
<td>44</td>
<td>-3.0</td>
<td>-16.4</td>
<td>-4.2</td>
<td>-1.5</td>
<td>-1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Duration = 40 years</td>
<td>44</td>
<td>-4.2</td>
<td>-22.0</td>
<td>-5.8</td>
<td>-1.8</td>
<td>-0.3</td>
<td>5.1</td>
</tr>
<tr>
<td>Initial wealth = 10,000</td>
<td>44</td>
<td>-3.0</td>
<td>-15.9</td>
<td>-4.4</td>
<td>-1.2</td>
<td>-0.6</td>
<td>2.6</td>
</tr>
<tr>
<td>Initial wealth = 1,000,000</td>
<td>44</td>
<td>-4.5</td>
<td>-24.2</td>
<td>-6.5</td>
<td>-1.7</td>
<td>-0.9</td>
<td>4.0</td>
</tr>
</tbody>
</table>

4.2 Contribution of low interest rates to rising inequality

We now use our sufficient statistic approach to quantify the effect of declining interest rates on top wealth inequality. Before we do so, we briefly present evidence on the rise in top wealth inequality and the decline in the interest rate.

Rising wealth inequality. The wealth inequality concept that we study in this paper is Pareto inequality, which measures the amount of wealth inequality within the rich (i.e., in the right tail of the wealth distribution). Figure 3 plots estimates of Pareto inequality \( \theta \) for the sample of Forbes 400 from 1985 to 2015 using two alternative estimators (i.e., the “top share” and “log rank regression” estimators).

Our preferred estimator—the top share estimator—has an intuitive interpretation. If the wealth distribution past a threshold \( w_{\text{min}} \) obeys a Pareto distribution, then we have that our measure of Pareto inequality \( \theta \) can be expressed as \( \theta = 1 - \frac{w_{\text{min}}}{w_{\text{mean}}} \), where \( w_{\text{mean}} \) is the average wealth of individuals with wealth exceeding \( w_{\text{min}} \). Notice that the top share estimator is in-
variant to a change of units. As a result, our estimate of Pareto inequality is not affected by valuation effects that increase the value of all assets equally.

The 25 log points increase in $\theta$ over the 1985-2015 period can be related to the relative increase in the wealth threshold $w_{\text{min}}$ needed to be included in Forbes list and the average wealth of individuals in the Forbes list $w_{\text{mean}}$. Denoting $\Delta x$ to be the change in a variable $x$ between 1985 to 2015, we have that

$$\Delta \log \theta = \Delta \log (w_{\text{mean}} - w_{\text{min}}) - \Delta \log (w_{\text{mean}}),$$

where the values are in real term. Notice that the average Forbes wealth has increased by 222 log points while the “excess wealth” of individuals in the Forbes list (i.e., their wealth in excess of the including threshold $w_{\text{min}}$) has sustained an even larger increase of 247 log points.

**Declining discount rates.** In our model, the interest rate $r$ represents the discount rate (i.e., expected return) on risky equity. In the data, it should be measured as the sum of the risk-free rate and the equity risk premium (ERP). Real yield on US treasuries have declined steadily since the 1980s. Many explanations have been proposed such as: a shortage of safe assets (Caballero et al., 2008); changing demographics (Carvalho et al., 2016); a shift in monetary policy regime (Lettau et al., 2018); secular stagnation (Eggertsson et al., 2019). For our application, we do not take a stand on the reason for this phenomenon.

One reference for the change in the real interest rate is the real yield on 10-year U.S. bonds. Figure 4 plots the nominal 10-year annual real yield (i.e., net of expected inflation) as estimated by the Cleveland Fed. From 1985 to 2015, the decline is roughly 5 percentage points.

Estimating the equity risk premium (ERP) is notoriously difficult and there is no consensus in the literature regarding the trend properties of the ERP over the period that we study. For example, Campbell and Thompson (2008) and Martin (2017) find that it has been roughly constant, Duarte and Rosa (2015) and Farhi and Gourio (2018) estimate that it has increased while
Greenwald et al. (2019) estimate that it has decreased.

**Quantitative result.** Our baseline assumption is that the discount rate \( r \) has declined by 5 percentage points. To quantify the contribution of a (permanent and unanticipated) decline in discount rates on rising wealth inequality, we use our sufficient statistic approach. Let \( \log \theta_{2015} \) be the predicted level of Pareto inequality in 2015 given the initial level of inequality in \( \log \theta_{2015} \), our sufficient statistic \( \partial_r \log \theta \) and an hypothesis regarding the discount rate decline \( r_{1985} - r_{2015} \):

\[
\hat{\log \theta}_{2015} = \log \theta_{1985} + \partial_r \log \theta \times (r_{2015} - r_{1985}).
\] (4.2)

Our baseline assumption (5 percentage point discount rate decline) implies an increase in Pareto inequality of 18 log points which corresponds to a bit more than two-thirds of the 25 log points increase that we estimated. Declining discount rates thus appear to have been an important contributor to the rise in top wealth inequality.

### 4.3 Empirical evidence beyond the top 100

Our empirical analysis has thus far focused on the very top of the wealth distribution (the top 100), which is dominated by entrepreneurs who founded firms that relied heavily on external financing. Is our mechanism relevant for entrepreneurs who founded smaller firms that typically do not have access to public capital markets? To shed light on this issue, we now provide empirical evidence on the prevalence of entrepreneurship as well as the use of external financing in the top 1% of the wealth distribution. We use data from the 2016 wave of the *Survey of Consumer Finances* (SCF).

Table 4 presents summary statistics related to the importance of entrepreneurship in the right tail of the wealth distribution. First, notice that entrepreneurs are over-represented at the top. As in Cagetti and De Nardi (2006), we find that wealthier individuals are much more likely to be entrepreneurs. In the full population, 11% of individuals are entrepreneurs, while in the top 0.01% the fraction increases to 66%. Second, notice that the businesses owned by
wealthy entrepreneurs tend to be pass-through entities, which is consistent with the evidence in Cooper et al. (2016). For instance, 93% of businesses owned by households in the top 0.01% are partnerships or S corporations. This is in a sharp contrast with the fact that roughly two-thirds of entrepreneurs in the top 100 own public firms (i.e., C corporations).

A necessary condition for low discount rates to increase inequality in our framework is that entrepreneurs use external financing (either equity financing or debt financing) to grow their firms. We now provide evidence that entrepreneurs in the top 1% employ both debt and equity financing. The SCF contains a question regarding the use of external financing (i.e., “What external sources of money were used to finance the ongoing operations or improvements in this business during the past year?”).

Table 5 contains summary statistics on the use external financing. The key takeaway is that entrepreneurs use both equity and debt financing to grow their firm. In a given year, 27% of entrepreneurs in the top 1% use debt financing while 0.4% use equity financing. Notice that the use of equity financing increases in the right tail of the distribution. For instance, entrepreneurs in the top 0.01% are much more likely to use equity financing (4.6%) and less likely to use debt financing (18%). One way to interpret these numbers is that entrepreneurs in the top 0.01% raise debt once every 5 years and raise equity once every 20 years.

5 Transition dynamics

For now, we have focused on the effect of interest rates on Pareto inequality between two steady states. This corresponds to the long-run effect of rates on Pareto inequality. To understand the mechanism, it is useful to also study the transitory dynamics of the wealth distribution as the interest rate decreases.
We now examine the transition dynamics of total wealth in top percentile in the stylized model presented in Section 2. Our starting point is an economy that is in steady state with an interest rate equals to 5%, using the same calibration as in Figure 2. We consider the effect of a unanticipated and permanent decline in the interest rate by 3 percentage points. We assume that the decline is gradual, taking place over the course of fifteen years (see Figure 5a). We plot the evolution of total wealth owned by the top 1% during this time in Figure 5b. We obtain it by computing the evolution of the wealth density using the Kolmogorov Forward equation.\footnote{We time discretize the model at the monthly frequency and solve for the transition dynamics over a finite grid using “Pareto extrapolation” (see Gouin-Bonenfant and Toda, 2019).}

The growth rate of the total wealth in the top 1% is initially zero since the initial distribution is in a steady state. It then becomes positive during the transition period, consistent with the fact that Pareto inequality increases when the interest rate decreases. In the long run, it converges back to zero, reflecting the fact that the wealth distribution reaches its new steady state.

\begin{align*}
\text{Numerical example with } i &= 0.4, \quad g = 0.5, \quad \delta = 0.5, \quad \eta = 0.05, \quad \rho = 0.04
\end{align*}
We use the accounting decomposition of Gomez (2019) to decompose the growth rate of wealth held by the top 1% in three terms: a “within” term that accounts for the average wealth growth of existing agents in the top 1%, a “displacement” term that arises due to the flow of new agents in the top percentile, and a “demography” term that accounts for population growth. The within term simply corresponds to \((r_t - \rho) dt + \frac{d q_t}{q_t}\).

As reported in Figure 5b, the within term decreases by 3 percentage points from one steady state to the other. This reflects the fact that the interest rate decreases by 3 percentage points during the period. Note, however, that the within term initially increases during the transition period. The reason is that the average realized return on wealth increases due to the unexpected increase in the value of the tree \(q_t\). However, these transitory fluctuations have no effect on the long run level of top wealth shares, which only depends on the long-run value of the interest rate.

The displacement term increases from one steady state to the other. This reflects the fact that there is a larger flow of new fortunes entering the top percentile. As discussed above, a lower interest rate increases the value of the tree \(q_t\), which increases the growth rate of entrepreneurs whose tree is still growing. This is consistent with the importance of displacement in the recent rise of top wealth shares, as documented in Gomez (2019), Gárleanu and Panageas (2017), and Zheng (2019).

Due to the simultaneous increase of the within and the displacement term, total wealth in the top 1% increases along the transitory path. In the new steady state, the within, displacement, and demography still sum up to zero.

6 Conclusion

In this paper, we study the effect of interest rates on wealth inequality in an economy where successful entrepreneurs continuously displace existing fortunes. In this economy, lower interest rates tend to increase top wealth inequality, since they reduce the rate at which top entrepreneurs get diluted over time. Moreover, it changes the composition of households in the right tail, since lower rates benefit new fortunes at the expense of old fortunes.

We develop a sufficient statistic to quantify the effect of discount rates on inequality. It depends on three key moments: the average growth rate of individuals reaching the top, the average payout yield of the firms that they own, as well as their leverage. We estimate these moments in the data using a combination of CRSP data and SEC filings and conclude that the effect of discount rates on inequality is large. According to our preferred estimates, the 5% decline in discount rates from 1985 to 2015 can explain more than three quarter of the rise in top wealth inequality (i.e., Pareto inequality). In general, the effect of the interest rate on top wealth inequality depends on how reliant productive firms are on external financing. In a world in which the most promising firms require a lot of investment to grow, as in the U.S., we argue that low rates have large effects on inequality.

Our analysis takes the interest rate path as given and therefore abstracts from general equilibrium effects. A growing literature focuses on these general equilibrium effects and argues that high wealth inequality in fact causes interest rates to be low (see for instance, Mian et
al., 2019 for interest rates or Gollier, 2001; Toda and Walsh, 2019; Gomez, 2016 for the equity premium). In the Wold and Whittle (1957) model of top wealth inequality—where wealth inequality is decreasing in the interest rate—such general equilibrium effects would imply that inequality is self-correcting. In contrast, in our model, declining interest rates and rising top wealth inequality would be mutually reinforcing. We leave this idea for future research.
A Appendix for Section 2

A.1 Proof for Proposition 2.2

Proof. Denote

$$\theta_R(r) = \frac{r - \rho}{\eta},$$

$$\theta_E(r) = -\frac{i}{q(r)} - g - \frac{\rho}{\eta + \delta}.$$

Pareto inequality of the wealth distribution is given by the maximum between $\theta_R$ and $\theta_E$ (see, e.g., Jones and Kim (2016)).

$\theta_R$ is increasing in $r$ while $\theta_E$ is decreasing in $r$. Note that we can rewrite $\theta_E$ in terms of the interest rate $r$:

$$\theta_E(r) = \frac{r + \delta \left(1 - \frac{1}{q(r)}\right) - \rho}{\eta + \delta}.$$

Using this formula, we get:

$$\theta_E(\rho + \eta) < \theta_R(\rho + \eta)$$

$$\theta_R(g - \delta) < \theta_E(g - \delta)$$

Therefore, there is a unique $r^*$ at which the two functions intersect, which is given by

$$\theta_E(r^*) = \theta_R(r^*)$$

$$\Leftrightarrow 1 - \frac{1}{q(r^*)} = (r^* - \rho) \frac{\delta}{\eta}$$

$$\Leftrightarrow \eta(-i - r^* - g) = (r^* - \rho)(-i + \delta)$$

$$\Leftrightarrow r^* = \frac{\eta}{-i + \delta + \eta}(g - \delta) + \frac{-i + \delta}{-i + \delta + \eta}(\rho + \eta)$$

$r^*$ is between the lower bound $g - \delta$ and the upper bound $\rho + \eta$. \qed

A.2 Closing the model

Agents. The full economy now also includes workers. Workers have log utility with a subjective discount factor $\bar{\rho}$. Like entrepreneurs, workers are also born with firms, but they can immediately sell them to the market. Denote $\pi$ the proportion of agents that are born workers.

Stationary equilibrium. Denote $S$ the total quantity of tree divided by total population. It is pinned down by the fact that its growth rate is zero in equilibrium:

$$0 = (g - \delta)S + \eta(1 - S). \quad (A.1)$$

Denote $x$ the steady-state share of aggregate wealth owned by workers. It is pinned down by the fact that the growth rate of the wealth per capita of workers must be zero in equilibrium:

$$0 = (r - \bar{\rho})x + \eta(\pi \frac{1}{S} - x) \quad (A.2)$$
Joining the two equations gives $x$ as a function of the interest rate $r$:

$$(r - \bar{p} - \eta)x = \pi(g - \delta - \eta) \quad (A.3)$$

**Market clearing.** Market clearing for goods requires the amount of goods consumed and invested to be equal to the output of maturing trees:

$$(x\bar{p} + (1 - x)\rho)q + i = \delta. \quad (A.4)$$

The left-hand side is aggregate demand (consumption and investment), while the right-hand side is aggregate supply. Substituting out $q$ in terms of $r$ (Equation 2.2), we obtain an equation that gives the market clearing interest rate $r$ as a function of $x$:

$$r = x\bar{p} + (1 - x)\rho + g - \delta \quad (A.5)$$

Combining Equation A.3 and Equation A.5 gives a system of two equations and two unknowns $x$ and $r$. Therefore, we can solve for the equilibrium interest rate.

The next proposition shows that, when $\pi$ is close enough to one, changes in $\bar{p}$ can generate the full spectrum of interest rates considered in Proposition 2.2.

**Proposition A.1.** Denote $r_\pi(\bar{p})$ the interest rate as a function of the subjective discount factor of workers:

1. $r_\pi(.)$ is an increasing function of $\bar{p}$. Moreover, as $\pi$ tends to one, $r_\pi(.)$ spans the interval $(g - \delta, \rho + \eta)$, i.e.

   $$\lim_{\pi \to 1} \lim_{\bar{p} \to 0} r_\pi(\bar{p}) = g - \delta$$

   $$\lim_{\pi \to 1} \lim_{\bar{p} \to +\infty} r_\pi(\bar{p}) = \rho + \eta$$

2. As long as $i < \delta - \frac{\rho}{1 - \pi}$, the distribution of workers always has a thinner tail than the distribution of entrepreneurs or rentiers. Therefore, tail inequality $\theta$ is given by Proposition 2.2.

Therefore, the proposition says that, when the proportion of workers is high enough in the economy, changes in their subjective discount factor generate changes in the interest rate in the economy.

**Proof of A.1.** There exists one and only one solution $x_\pi(\bar{p}) \in (0, 1)$ that solves the system given by Equation A.3 and A.5:

$$x_\pi(\bar{p}) = \begin{cases} 
\frac{1 + \theta(\bar{p}) - \sqrt{(1 + \theta(\bar{p}))^2 - 4\theta(\bar{p})(1 - \pi)}}{2} & \text{if } 0 < \bar{p} < \rho \\
\pi & \text{if } \bar{p} = \rho \\
\frac{1 + \theta(\bar{p}) + \sqrt{(1 + \theta(\bar{p}))^2 - 4\theta(\bar{p})(1 - \pi)}}{2} & \text{if } \bar{p} > \rho
\end{cases}$$

where $\theta(\bar{p}) = (\eta - (g - \delta)) / (\rho - \bar{p})$.

Moreover, we have

$$\lim_{\bar{p} \to 0} r_\pi(\bar{p}) = g - \delta + \rho \frac{1 + \theta(0) - \sqrt{(1 + \theta(0))^2 - 4\theta(0)(1 - \pi)}}{2}$$

$$\lim_{\bar{p} \to +\infty} r_\pi(\bar{p}) = \rho + \eta - (1 - \pi)(\eta - (g - \delta))$$
Therefore
\[
\lim_{\pi \to 1} \lim_{\rho \to 0} r_{\pi}(\rho) = g - \delta
\]
\[
\lim_{\pi \to 1} \lim_{\rho \to +\infty} r_{\pi}(\rho) = \rho + \eta
\]

Finally, denote \( \bar{\theta}(r) = \frac{r-g}{\eta} \). To ensure that the tail of the wealth distribution is not dominated by workers, a sufficient condition is that

\[
\theta_E(r_{\pi}(\rho)) > \bar{\theta}(r_{\pi}(\rho))
\]

Since \( r_{\pi}(\rho) = \rho + g - \delta \), the condition can be rewritten as

\[
\frac{g - \delta + \delta (1 - \frac{1}{\eta})}{\delta + \eta} > \frac{g - \delta}{\eta}
\]

\[
\Rightarrow i < \delta - \frac{\rho}{1 - \frac{g-\delta}{\eta}}
\]

This concludes the proof.

\[\square\]

B Appendix for Section 3

B.1 Mutual fund balance sheet

Let \( K = (K_1, \ldots, K_S)^t \) be the (normalized) stock of capital in firm of state \( s = 1, \ldots, S \). The law of motion for \( K \) is given by

\[
\dot{K} = (D(g) + T)K + \eta (\psi_0 - K).
\]

Let \( W_{Bs} \) be the be the book wealth of entrepreneurs in states \( s = 1, \ldots, S \) and \( W^R_B \) is the book wealth of rentiers.

\[
\dot{W}_B = (D(\mu) + T)W_B + \eta (\psi_0 - W_B) - \delta W_B
\]

\[
W^R_B = (r - \rho)W^R_B + \delta \sum_{s=1}^S \frac{q_s}{\eta} W_{Bs} - \eta W^R_B
\]

Let \( W^M_{Bs} \) be the book wealth in state \( s = 1, \ldots, S \) that is owned by the mutual fund

\[
W^M_B = (D(g) + T)W^M_B + (D(g) - D(\mu))W^M_B + \delta W_B - \eta W^M_B
\]

Notice that \( W^M_B + W_B = K \implies W^M_B + W^E_B = K \). The mutual fund \( q \) is given by

\[
q \equiv \frac{\sum_s q_s W^M_{Bs}}{\sum_s W^M_{Bs}}
\]

The market clearing interest rate ensures that the market value of the capital held by the mutual fund is equal to the wealth of rentiers

\[
\sum_s q_s W^M_{Bs} = \bar{q} W^R_B.
\]
B.2 Proof of Proposition 3.1

The following lemma shows that the distribution of book wealth and market wealth have the same Pareto exponent. Intuitively, this happens because the ratio of market to book wealth, \( q \), is a bounded random variable.

**Lemma B.1.** If book wealth \( B \) has a Pareto tail with exponent \( \zeta \), then market wealth \( W \) also has a Pareto tail with exponent \( \zeta \).

**Proof of Lemma B.1.** Assume that \( B \) has a Pareto tail, which implies that
\[
\lim_{x \to \infty} \frac{\log P(B \geq x)}{\log x} = -\zeta. \tag{B.7}
\]

Since the state space is finite and we assumed that there exists a solution to the HJB (3.2), then there exists \( 0 < a < b < \infty \) such that
\[
a < q_s < b \tag{B.8}
\]
for all \( s = 1, \ldots, S \). We thus have that
\[
P(B \geq b^{-1}x) \leq P(W \geq x) \leq P(B \geq a^{-1}x). \tag{B.9}
\]
Therefore, this gives
\[
\lim_{x \to \infty} \frac{\log P(B \geq x)}{\log x} \leq \lim_{x \to \infty} \frac{\log P(W \geq x)}{\log x} \leq \lim_{x \to \infty} \frac{\log P(B \geq x)}{\log x}, \tag{B.10}
\]
which implies
\[
\lim_{x \to \infty} \frac{\log P(W \geq x)}{\log x} = -\zeta. \tag{B.11}
\]

**Proof of Proposition 3.1.** The distribution of book wealth for entrepreneurs has a Pareto tail, and its tail inequality \( \theta > 0 \) is the unique solution to
\[
\rho_D(\mathcal{A}(\theta)) = \eta + \delta \tag{B.12}
\]
This is a direct application of Theorem 4.1 in Beare et al. (2019). The only difference is that their theorem does not ensure existence of a solution. Our assumption that \( \mu_{s'} > 0 \) for at least one \( s' \in \{1, \ldots, S\} \) guarantees existence, as shown in Remark 3.5 of Beare and Toda (2017).

From Lemma B.1, we know that tail inequality of market wealth is the same as book wealth. Since the transition from entrepreneurs to rentiers is irreversible, we obtain that tail inequality for the whole distribution of entrepreneurs and rentiers is \( \max \{ \frac{1}{\zeta}, \frac{\eta}{\delta} \} \).

For the sake of intuition, we provide an heuristic derivativation for result. Consider the Kolmogorov Forward Equation (KFE) for the (normalized) book wealth \( p(x) \equiv (p_0(x), \ldots, p_S(x))' \) across entrepreneurs
\[
0 = -\mathcal{D}(\mu) \partial_x p(x) x + T' p(x) + (\eta + \delta) \left( \mathcal{D}(\psi_0) - p(x) \right). \tag{B.13}
\]
Guess that the stationary distribution satisfies \( p(x) \sim ux^{u-1} \) as \( x \to +\infty \), where \( u = (u_1, \ldots, u_S)' \) is the asymptotic distribution of types \( s \). Plugging this guess into the KFE, we obtain

\[
A(\theta)'u = (\eta + \delta)u
\]  

(B.14)

In other words, the vector \( u \) corresponds to the left eigenvector of the matrix \( A(\theta) \) associated with eigenvalue \( \eta + \delta \). Since \( u \) is non-negative elementwise, it corresponds to its principal eigenvector. Therefore, tail inequality \( \theta \) is such that the spectral radius of \( A(\theta) \) is \( \eta + \delta \), which gives the result.

### B.3 Proof of Proposition 3.3

**Proof for 3.2.** Denote \( u(\theta), v(\theta) \) the left and right eigenvector associated with the dominant eigenvalue of the matrix \( A(\theta) \), normalized so that \( \sum_s u_s = 1 \) and \( \sum_s u_s v_s = 1 \). Pareto inequality \( \theta \) is characterized by the fact that the dominant eigenvalue of \( A(\theta) \) is zero:

\[
A(\theta) v(\theta) = 0.
\]  

(B.15)

Differentiating this equation with respect to \( r \), we obtain

\[
(\partial_r A + \frac{\partial \mu}{\partial \theta} A) v + A(\partial_r v + \frac{\partial \mu}{\partial \theta} v) = 0.
\]  

(B.16)

We can left-multiply by the left eigenvector \( u' \) to obtain an expression for \( \partial_r \theta \):

\[
\partial_r \theta = \frac{u'(\partial_r A) v}{u'(\partial_r \mu) A v}.
\]  

(B.17)

Using the fact that \( \partial_r A = \frac{1}{\theta} D(\partial_r \mu) \) and \( \partial_r \mu = -\frac{1}{\theta^2} D(\mu) \), we obtain

\[
\frac{\partial_r \theta}{\theta} = \frac{u' D(\partial_r \mu) v}{u' D(\mu) v}.
\]  

(B.18)

QED.

The following lemma shows that \( uv \) corresponds to the density of past states for individuals in the right tail of the wealth distribution.

**Lemma B.2.** Take a vector \( f \in \mathbb{R}^S \), and denote, for an agent born at time \( t_0 \), the process \( \bar{f}_t \) defined as:

\[
\bar{f}_{t_0} = 0 \quad \text{and} \quad d\bar{f}_t = f_s dt \quad \text{for} \quad t \geq t_0,
\]  

(B.19)

We have, as \( b \to +\infty \),

\[
E \left[ \int f \log W = w \right] \sim \frac{(uv)' f}{(uv)' \mu} w
\]  

(B.21)

**Proof for B.2.** Denote \( p(\bar{f}, b) \) (i.e., a \( S \times 1 \) vector) the stationary density of \( (\bar{f}, \log B, s) \). Kolmogorov Forward Equation (KFE) gives:

\[
0 = -D(\bar{f}) \partial_{\bar{f}} p(\bar{f}, b) - D(\mu) \partial_{\mu} p(\bar{f}, b) + \bar{T}' p(\bar{f}, b) + (\eta + \delta) (p(\bar{f}, b) - p(\bar{f}, b)),
\]  

(B.22)

where \( p(\bar{f}, b) \) is the distribution of \( (\bar{f}, b) \) at birth. Let \( m(\lambda, b) \equiv \int_{\mathbb{R}} e^{\lambda \bar{f}} p(\bar{f}, b) d\bar{f} \) be the Laplace trans-
form of $p(\mathcal{J},b)$. Multiplying the KFE by $e^{\lambda \mathcal{J}}$ and integrating over $\mathcal{J}$, we obtain
\begin{equation}
0 = -\mathcal{D}(f) \int_{\mathbb{R}} e^{\lambda \mathcal{J}} \partial_{\mathcal{J}} p(\mathcal{J},b) \, d\mathcal{J} - \mathcal{D}(\mu) \partial_{\mathcal{J}} m(\lambda, b) + \mathcal{T}' m(\lambda, b) + (\eta + \delta) (m_0(\lambda, b) - m(\lambda, b)), \quad (B.23)
\end{equation}
where $m_0(\lambda, b) \equiv \int_{\mathbb{R}} e^{\lambda \mathcal{J}} p_0(\mathcal{J},b) \, d\mathcal{J}$. Using integration by part on the first term implies that:
\begin{equation}
0 = \lambda \mathcal{D}(f) m(\lambda, b) - \mathcal{D}(\mu) \partial_{\mathcal{J}} m(\lambda, b) + \mathcal{T}' m(\lambda, b) + (\eta + \delta) (m_0(\lambda, b) - m(\lambda, b)). \quad (B.24)
\end{equation}
Let us now guess that $m(\lambda, b) \sim \tilde{u}(\lambda) e^{-B_0^\lambda(b)}$ as $b \to +\infty$, where $\tilde{u}(\lambda)$ is an $S \times 1$ vector. Plugging this guess into the KFE gives
\begin{equation}
0 = \lambda \mathcal{D}(f) \tilde{u}(\lambda) + \xi(\lambda) \mathcal{D}(\mu) \tilde{u}(\lambda) + \mathcal{T}' \tilde{u}(\lambda) - (\eta + \delta) \tilde{u}(\lambda). \quad (B.25)
\end{equation}
In other words, defining the operator $\hat{A}(\lambda) \equiv \mathcal{D}(f + \xi(\lambda) \mu) + \mathcal{T}$, we obtain that $(\xi(\lambda), \tilde{u}(\lambda))$ is the solution of the following eigenproblem:
\begin{equation}
\hat{A}'(\lambda) \tilde{u}(\lambda) = (\eta + \delta) \tilde{u}(\lambda) \quad (B.26)
\end{equation}
We have that
\begin{align*}
E \left[ f \right| \log B = b, s = s \bigg] &= \partial_{\mathcal{J}} E \left[ e^{\lambda \mathcal{J}} \right| \log B = b, s = s \bigg] |_{f=0} \\
&= \frac{\partial_{\mathcal{J}} m_\mathcal{J}(0,b)}{m_\mathcal{J}(0,b)} \\
&\sim \xi'(0) b
\end{align*}
as $b \to +\infty$.

Since $\log W = \log q + \log B$, we obtain
\begin{equation}
E \left[ \mathcal{J} \right| \log W = w, s = s \bigg] \sim \xi'(0) w \quad (B.28)
\end{equation}
as $w \to +\infty$.

Using the law of iterated expectations, we get
\begin{equation}
E \left[ \mathcal{J} \right| \log W = w \bigg] \sim \xi'(0) w. \quad (B.29)
\end{equation}

Define $\tilde{\theta}(\lambda)$ the right principal eigenvector of $\hat{A}(\lambda)$. Following the same steps as the ones used to get Equation 3.11, we obtain:
\begin{equation}
\xi'(0) = \frac{(\tilde{u}(0) \tilde{\theta}(0))'f}{(\tilde{u}(0) \tilde{\theta}(0))'\mu} \quad (B.30)
\end{equation}
Finally, note that $\hat{A}(0) = A(r), \tilde{u}(0) = u(r)$, and $\tilde{\theta}(0) = v(r)$. We conclude that
\begin{equation}
E \left[ \mathcal{J} \right| \log W = w \bigg] \sim \frac{(uv)'f}{(uv)'\mu} w \quad (B.31)
\end{equation}
as $w \to +\infty$.

Armed with this lemma, we can now prove Proposition 3.3.
Proof for Proposition 3.3. Combining Proposition 3.2 with Lemma B.2, we obtain

\[ \partial_r \log \theta = \frac{(uv)' \partial_r \mu}{(uv)' \mu} \]  

(B.32)

\[ = \lim_{w \to +\infty} \mathbb{E} \left[ \frac{\partial_r \mu | \log W = w}{w} \right] \]

\[ = \lim_{w \to +\infty} \mathbb{E} \left[ \frac{\partial_r \mu}{w} | \log W = w \right] \]

\[ = \lim_{w \to +\infty} \mathbb{E} \left[ \frac{\partial_r \mu}{w} | \log W = w \right] . \]  

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### C EIS

So far we have assumed that entrepreneurs have log utility. This implies that the consumption rate of entrepreneur does not react to the interest rate. In this extension, we allow agents’ saving rates to react to changes in interest rates, which happens when the elasticity of intertemporal substitution is different from one.

Instead of log-utility, entrepreneurs now have Epstein-Zin utilities with arbitrary relative risk aversion RRA and arbitrary elasticity of intertemporal substitution EIS. The effect of interest rate on the
consumption rate is given by the difference between 1 and the EIS. If EIS > 1, the substitution effect is more important than the income effect: households react to a decrease in the interest rate by increasing their consumption rate. If EIS < 1, the income effect is more important than the substitution effect: households react to an decrease in the interest rate by decreasing their consumption rate.

The estimate for the sufficient statistic becomes

$$\hat{\partial} \log \theta = \frac{1}{N} \sum_{i=1}^{N} \frac{\text{equity payout yield}_i \times \text{duration} - \text{market debt to equity}_i + \text{EIS} - 1}{\text{growth rate}_i}.$$  

A EIS higher than one would reduce the effect of lower rates on inequality: Due to the relative importance of the substitution effect, entrepreneurs consume more as a proportion of their wealth. In contrast, a EIS lower than one would amplify the effect of lower rates on inequality: due to the relative importance of the income effect, entrepreneurs will start consuming less as a proportion of their wealth.

Work from Vissing-Jørgensen (2002) suggests that the EIS of stockholders tends to be close to one. Still, for robustness, we explore different calibrations with EIS = 0.5 and EIS = 1.5. As shown in Table 7, even with a high EIS, a lower r still tends to increase wealth inequality.

Table 7: Effect of interest rate on tail inequality when EIS $\neq$ 1

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References


