

## WEALTH INEQUALITY IN A LOW RATE ENVIRONMENT

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We study the effect of interest rates on wealth inequality. While lower rates decrease the growth rate of rentiers, they also increase the growth rate of entrepreneurs by making it cheaper to raise capital. To understand which effect dominates, we derive a sufficient statistic for the effect of interest rates on the Pareto exponent of the wealth distribution: it depends on the lifetime equity and debt issuance rate of individuals in the right tail of the wealth distribution. We estimate this sufficient statistic using new data on the trajectory of top fortunes in the U.S. Overall, we find that the secular decline in interest rates (or more generally of required rates of returns) can account for about 40% of the rise in Pareto inequality; that is, the degree to which the super rich pulled ahead relative to the rich.

KEYWORDS: Wealth inequality, Pareto distribution, Interest rate, Entrepreneurship.

### 1. INTRODUCTION

Since the seminal contribution of [Wold and Whittle \(1957\)](#), a widespread view is that high interest rates tend to increase top wealth inequality. As summarized by [Piketty and Zucman \(2015\)](#), the intuition is that high rates of return increase the growth rate of existing fortunes. Yet, this view appears to be at odds with recent data: wealth inequality has increased substantially in the past forty years in the U.S., a period marked by declining interest rates.

In this paper, we argue that a lower interest rate can actually *increase* top wealth inequality for two reasons. First, a low interest rate increases the market value of assets owned by existing fortunes (a revaluation channel). Second, a low rate environment also increases the rate of creation of new fortunes, as it decreases the cost of external financing for successful entrepreneurs (a capital accumulation channel).

To be more concrete, consider the trajectory of entrepreneurs making it to the top of the wealth distribution. To finance the growth of their firms, these entrepreneurs typically raise external funding from outside investors. Lower interest rates *increase* the rate of capital accumulation of these entrepreneurs, since it reduces their cost of external financing. On the other hand—as emphasized by the existing literature—lower rates decrease the rate of capital accumulation of outside investors, as they now earn lower returns on their investment. If, as in the U.S., individuals at the top of the wealth distribution built their wealth as entrepreneurs (who issue financial claims) rather than investors (who purchase them), lower rates tend to increase top wealth inequality.

We use a sufficient statistic approach to quantify the long-run effect of lower interest rates (or, more generally, of lower required returns on wealth) on the Pareto exponent of the wealth

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distribution. Our sufficient statistic formula depends on the lifetime equity and debt issuance of individuals reaching the right tail of the wealth distribution. We estimate this sufficient statistic using new data on the trajectory of top fortunes in the U.S. Our preferred estimate is that a permanent one percentage point decline in the required return on wealth generates a decline in the Pareto exponent of the wealth distribution by 4.2 log points. To put this estimate into perspective, this suggests that the 2 pp. decline in the required return on wealth over the 1985–2015 period can account for between a third and half of the fattening of the wealth distribution during this time period.<sup>1</sup>

*Overview of the paper.* In Section 2, we describe our main mechanism in a stylized model of wealth inequality. Entrepreneurs are born with a tree. Trees require a continuous flow of investment to grow. To finance the growth of their tree, entrepreneurs continuously sell equity shares to outside investors (i.e., “rentiers”). With some hazard rate, trees blossom and generate a one-time dividend equal to their size. Afterwards, entrepreneurs become rentiers themselves and invest their wealth in a diversified portfolio of trees.

In this stylized economy, we show that Pareto inequality is a u-shaped function of the interest rate. When the interest rate is sufficiently high, only rentiers make it to the right tail of the wealth distribution. In this case, a decline in the interest rate decreases top wealth inequality since it decreases the growth rate of rentiers, as in [Wold and Whittle \(1957\)](#) and [Piketty and Zucman \(2015\)](#). In contrast, when the interest rate is sufficiently low, entrepreneurs reach the right tail of the wealth distribution. In this case, a decline in rates *increases* top wealth inequality, since it decreases the cost of external financing for these new fortunes.

In Section 3, we develop a sufficient statistic approach to *quantify* the effect of interest rates on Pareto inequality. We first show that, in the stylized model, the effect of a small change in interest rate on Pareto inequality can be expressed in terms of three observable moments: the equity payout yield (i.e., the net payout to equity holders over the market value of equity) times the duration of trees (i.e., the semi-elasticity of their valuations with respect to the interest rate) divided by the growth rate of entrepreneurs. Intuitively, if entrepreneurs rely on a lot of external financing (i.e., the equity payout yield is negative), and/or if equity valuations are very sensitive to interest rates (i.e., the duration of the tree is high), we expect lower rates to have a large effect on Pareto inequality.

We then generalize this formula along three dimensions. First, we consider a production economy extension in which the level of investment is optimally chosen by entrepreneurs (subject to convex adjustment costs). We show that the endogenous response of investment to shifts in interest rates does not alter our sufficient statistic formula. This is due to the envelope theorem: when investments adjust to lower interest rates, entrepreneurs end up owning smaller percentages of larger firms. These two effects counterbalance each other.

We then consider an economy in which entrepreneurs differ in their production and investment productivity, which evolve according to an unrestricted Markov process. We show that the effect of interest rates on Pareto inequality depends on the entire wealth trajectory of individuals making it to the top of the wealth distribution. More precisely, the derivative of (log) Pareto inequality with respect to the interest rate depends on the derivative of the (log) growth rate of individuals making it to the top of the wealth distribution. In turn, this derivative can be expressed in terms of the *lifetime* average equity payout yield and duration of firms owned by these entrepreneurs.

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<sup>1</sup>Pareto inequality is defined as the inverse of the Pareto exponent. A high level of Pareto inequality corresponds to a distribution with a thick right tail.

As a final extension, we consider the case where entrepreneurs can issue both equity and debt to finance investment in their firms. In this case, we show that our sufficient statistic also depends on the leverage of individuals making it to the top of the wealth distribution. This reflects the fact that both equity and debt issuance affect the impact of required rates of return on Pareto inequality.

In Section 4, we use new data to estimate these moments for the wealthiest 100 individuals in the U.S. We find that the lifetime average equity payout yield of firms owned by top individuals is around  $-2.2\%$  annually. The key observation here is that it is negative: these firms have spent more years raising cash from equity holders than distributing cash back to them. This is the case even though the corporate sector, as a whole, has a positive equity payout yield: in the data, as in the stylized model, firms tend to have a negative equity payout yield when they are young (and small) and a positive equity payout yield when they are old (and large). The distribution of equity payout yields is extremely skewed: some entrepreneurs own firms with a lifetime average equity payout yield as low as  $-10\%$ . Moreover, our data reveals an average market leverage of about 1.4 (i.e., the market value of the firm exceeds the market value of the equity).

We use these estimates to quantify our sufficient statistic. According to our preferred measure, a 1 pp. permanent decline in the required return on wealth increases Pareto inequality by 4.2 log points. We use this estimate to quantify the contribution of declining required returns on rising Pareto inequality over the 1985–2015 period. We estimate that Pareto inequality has increased by roughly 22 pp., while the required returns on both debt and equity have declined by roughly 2 pp. A back-of-the-envelope calculation indicates that declining required returns account for roughly 40% of the rise in Pareto inequality during this time period. About two-thirds of our mechanism operates via equity issuance, with the remainder due to debt issuance.

Finally, in Section 5, we build a general equilibrium version of our model with both capital and labor as inputs in the production process. Relative to our sufficient statistic approach, the model allows us to trace out the transition dynamics following a decline in required returns, consider measures of wealth inequality beyond Pareto inequality, and quantify general equilibrium effects. We calibrate the model by targeting the set of micro moments that enter the sufficient statistic, in addition to important macro moments.

To generate a 2 pp. decline in the required return, we feed a sequence of MIT shocks to foreign savings (i.e., a “global savings glut”). We then examine the transition dynamics of the wealth distribution. In the model, as in the data, the top 0.1% wealth share increases more than the top 1%, and the top 0.01% increases more than the top 0.1%, which reflects a rise in Pareto inequality. The model exhibits a relatively high speed of convergence, owing to the presence of high-growth entrepreneurs who reach the top of the wealth distribution quickly.

In our model, the rise in top wealth shares can be decomposed into two terms: a revaluation channel (a relative rise in the valuation of their capital) and a capital accumulation channel (an increase in the quantity of capital owned by top entrepreneurs due to their lower cost of capital). While both channels contribute roughly equally to the rise in the top 1% wealth share, the relative importance of the capital accumulation channel increases sharply in the right tail of the wealth distribution. This reflects the fact that top entrepreneurs disproportionately benefit from a lower cost of capital, as they raise more external financing over their lifetimes.

Finally, we use the model to consider alternative drivers of the decline in required returns (i.e., a domestic savings glut) as well as alternative calibrations of the elasticity of capital (i.e., the degree of capital adjustment costs). While we set the elasticity of capital to zero in the baseline model experiment in order to match a constant return on capital despite falling interest rates (as observed in the U.S. data), we show that the negative effect of interest rates on Pareto inequality persists across a wide range of calibration for the elasticity of capital.

*Related literature.* There is a large body of evidence documenting a rise in top wealth inequality in the U.S. since the 1980s (e.g., [Saez and Zucman, 2016](#); [Batty et al., 2019](#); [Smith et al., 2023](#)). A growing literature seeks to understand the factors behind this phenomenon. One strand of the literature focuses on the role of the return on wealth for top individuals ([Piketty, 2015](#); [Kuhn et al., 2017](#); [Moll et al., 2022](#); [Hubmer, Krusell, and Smith Jr, 2020](#)).<sup>2</sup> Another strand of the literature emphasizes the importance of return dispersion ([Benhabib et al., 2011](#); [Fagereng et al., 2020](#); [Benhabib et al., 2019](#); [Bach, Calvet, and Sodini, 2017](#); [Gomez, 2023a](#); [Atkeson and Irie, 2022](#); [Zheng, 2019](#)). We show that a decline in the required return on wealth increases top wealth inequality through both these components: it increases the realized return of existing fortunes along the transition path (as in the first strand of papers) and the dispersion of their returns in the long-run (as in the second strand of papers).

Our characterization of the Pareto exponent of the wealth distribution builds on the literature on random growth processes ([Wold and Whittle, 1957](#); [Jones, 2015](#)). Recently, this literature has examined more realistic models with *persistent* growth rate heterogeneity ([Luttmer, 2011](#); [Jones and Kim, 2018](#); [Gabaix et al., 2016](#)). In this case, the Pareto exponent can be obtained as the principal eigenvalue of an operator related to the transition matrix between states (see [de Saporta, 2005](#); [Beare et al., 2021](#); [Beare and Toda, 2022](#)). Relative to that literature, a theoretical contribution of our paper is to obtain a closed-form expression for the derivative of the Pareto exponent with respect to a parameter (here, the interest rate). We show that it depends on the derivative of the growth rate of individuals *reaching* the top of the wealth distribution (or, equivalently, on the derivative of the *past* growth rates of individuals currently at the top of the wealth distribution). For the case of the interest rate, this derivative can be expressed in term of a few moments that can be estimated empirically. This sufficient statistic approach allows us to quantify the effect of required returns on Pareto inequality in a transparent manner.

Several papers examine the redistributive effect of changes in the interest rate. [Gârleanu and Panageas \(2017\)](#), [Gârleanu and Panageas \(2021\)](#) and [Kogan et al. \(2020\)](#) build models in which lower discount rates benefit entrepreneurs at the expense of households. [Auclert \(2019\)](#) studies the redistributive effect of transitory changes in the interest rate (due to monetary policy shocks). More recently, [Greenwald et al. \(2021\)](#) argue that a decline in interest rates increases wealth inequality due to the fact that portfolio duration increases with wealth (a “revaluation” channel). Relative to this paper, we emphasize that lower interest rates also increase wealth inequality by decreasing the cost of capital for entrepreneurs (a “capital accumulation” channel) and we quantify the relative importance of both effects.

Our model also relates to the literature on entrepreneurial wealth accumulation (e.g., [Quadrini, 2000](#), [Cagetti and De Nardi, 2006](#); [Moll, 2014](#); [Guvenen et al., 2019](#); [Peter, 2021](#)). The most closely related paper is [İmrohoroğlu and Zhao \(2022\)](#), who use a calibrated model to argue that declining interest rates have contributed to the rise in wealth inequality by lowering the cost of debt for entrepreneurs. Relative to this paper, we use a sufficient statistic approach based on a novel closed-form expression for the effect of interest rates on Pareto inequality. We also emphasize, theoretically and quantitatively, the joint role of debt and equity issuance in determining the effect of lower required returns on top wealth inequality.

*Supplementary Material* The paper is accompanied with an online appendix, a supplemental appendix ([Gomez and Gouin Bonenfant, 2023](#)), and a replication package ([Gomez, Matthieu and Gouin-Bonenfant, Emilien, 2023](#)).

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<sup>2</sup>[Hubmer et al. \(2020\)](#) argue that the decline in tax progressivity has played a key role in increasing the average after-tax return on wealth. [Kaymak and Poschke \(2016\)](#) also emphasize the importance of the decline in tax progressivity.

## 2. STYLIZED MODEL

In this section, we describe our central mechanism in a stylized model of wealth inequality. Our key departure from the standard model from [Wold and Whittle \(1957\)](#) is that newborn agents are endowed with trees which require outside investments to grow. Under this assumption, we show that Pareto inequality is a u-shaped function of the interest rate.

### 2.1. Environment

The economy is populated by infinitely-lived agents. Population grows at rate  $\eta$ . We consider a small open economy equilibrium, where the interest rate  $r$  is taken as given. There are two types of agents: “entrepreneurs” and “rentiers”. All agents are born entrepreneurs and are endowed with a tree. Each tree requires outside investment to grow until it blossoms. Entrepreneurs issue equity shares to rentiers to finance the growth of their tree. When an entrepreneur’s tree blossoms, which happens only once, it produces a harvest of apples (the *numéraire*). The entrepreneur then becomes a rentier, who invests in a diversified portfolio of trees.

*Trees.* A tree starts with a size of one and grows at rate  $g$ . To grow, the tree requires a flow of outside investment  $i > 0$  proportional to its size. With constant hazard rate  $\tau$ , the tree blossoms and returns a one-time positive dividend equal to its size. We assume that  $i < \tau$  (so that trees yield positive cash flows in expectation) and that  $g < \tau + \eta$  (so that the total size of trees does not grow faster than the population).

*Returns.* Because the cash flow of the tree is proportional to its size, the value of a tree is also proportional to its size: we denote  $q$  the ratio of the value of a tree to its size. The instantaneous return of holding a tree during a short period of time  $dt$  is

$$\frac{dR_t}{R_t} = \begin{cases} \left(g - \frac{i}{q}\right) dt & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T \end{cases} \quad (1)$$

where  $T$  denotes the stochastic (idiosyncratic) time at which the tree blossoms.<sup>3</sup> This equation says that, while the tree is still growing (i.e.,  $t < T$ ), the return in a period  $dt$  is the difference between the growth rate of the tree  $g dt$  and the relative amount of new shares  $i/q dt$  that must be sold to outside investors to raise  $i dt$ . This adjustment corresponds to the extent to which existing shareholders get diluted (i.e., the rate at which their ownership share in the tree declines). Finally, when the tree blossoms (i.e.,  $t = T$ ), the instantaneous return is  $1/q - 1$  since the tree (with price  $q$ ) is transformed into apples (with price 1).

The tree price  $q$  is pinned down by the fact that the expected return of holding a tree must equal the (exogenous) interest rate; that is,

$$\underbrace{r}_{\text{Required return}} = \underbrace{g - \frac{i}{q} + \tau \left(\frac{1}{q} - 1\right)}_{\text{Expected return}}. \quad (2)$$

<sup>3</sup>Formally,  $R_t$  denotes the cumulative return of owning the tree up to time  $t$ .

We assume  $r > g - \tau$  to ensure that the price of the tree is finite. In this case, the equation implies that  $q = (\tau - i)/(r + \tau - g)$ ; in particular,  $q$  decreases in  $r$ . While a low interest rate naturally decreases the *average* return of holding a tree, notice that it increases the return of holding a tree *conditional on it not blossoming*, which is  $g - i/q$ . The intuition is that lower rates (i.e., higher valuations) decrease the rate at which existing shareholders get diluted as their trees grow.

*Wealth accumulation.* Agents have log utility and discount the future at rate  $\rho$ , which implies that they optimally consume a constant fraction  $\rho$  of their wealth.<sup>4</sup>

Our maintained assumption is that each entrepreneur must have all of their wealth invested in their tree. Let  $W_t$  be the wealth of an individual. The growth rate of wealth for an entrepreneur is the return of holding a tree minus the consumption rate, which gives

$$\frac{dW_t}{W_t} = \begin{cases} \left(g - \frac{i}{q} - \rho\right) dt & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T \end{cases} \quad (3)$$

where  $T$  denotes the stochastic time at which the tree blossoms.

When the tree blossoms, the entrepreneur becomes a rentier and invests in a diversified portfolio of trees. The wealth of a rentier evolves as:

$$\frac{dW_t}{W_t} = (r - \rho) dt \quad \text{if } t > T. \quad (4)$$

Notice that the interest rate has an opposite effect on the growth rate of wealth of entrepreneurs and rentiers. While a lower interest rate (i.e., higher asset valuations) increases the growth rate of successful entrepreneurs (who issue financial claims), it decreases the growth rate of rentiers (who purchase them). This is shown graphically in Figure 1, which plots the total wealth of an entrepreneur with a tree that blossoms at  $T = 15$ , in a high interest rate economy as well as a low interest rate economy.

*Discussing our assumptions.* We now discuss two key assumptions that we made. The first assumption is that trees initially require outside investment (i.e.,  $i > 0$ ). This assumption captures an important characteristic of young firms: they typically require external financing to grow. As we will discuss in Section 4, this external financing can take the form of equity issuance (venture capital funding, public equity offering, stock-based compensation) or debt issuance. We relax this assumption in Section 3, and allow firms to endogenously have a positive or negative payout yield depending on their current production and investment productivity.

The second key assumption is that entrepreneurs must maintain all of their wealth in their trees. This assumption captures the fact that most of the wealth of entrepreneurs is invested in their own firm (Quadrini, 2000; Cagetti and De Nardi, 2006; Roussanov, 2010). We take this as exogenous, but this type of portfolio choice constraint can be the result of moral hazard or asymmetric information problems (He and Krishnamurthy, 2012; Di Tella, 2017). While our model is very stylized, the term “entrepreneur” should be understood as any individual that is disproportionately exposed to a firm requiring outside financing. This represents a much larger

<sup>4</sup>Note that this is the case even in the presence of idiosyncratic risk (here, the date at which the tree blossoms); intuitively, the income and substitution effect of facing idiosyncratic risk cancel out.

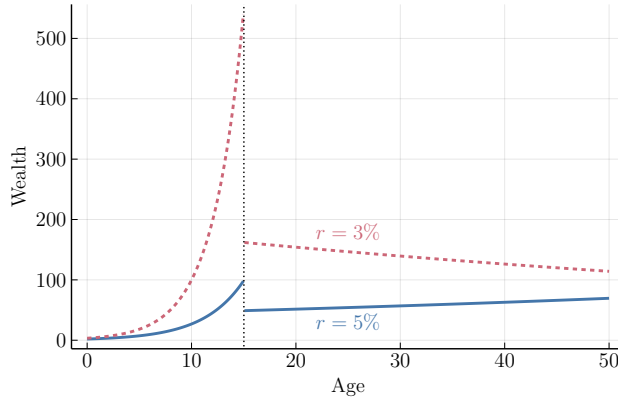


FIGURE 1.—Wealth trajectory of an entrepreneur who becomes a rentier after  $T = 15$  years  
Numerical example with  $g = 0.5$ ,  $i = 0.4$ ,  $\tau = 0.5$ ,  $\eta = 0.05$ ,  $\rho = 0.04$ .

fraction of the population than strictly-defined entrepreneurs. For instance, this includes all the early employees in startups who are paid in stock-options or restricted stocks or investors with concentrated portfolios, such as venture capitalists.<sup>5</sup> Finally, note that, as  $\tau \rightarrow \infty$ , our model reverts back to the benchmark model of [Wold and Whittle \(1957\)](#), as all agents in the economy own a diversified portfolio.

## 2.2. Wealth distribution

We now characterize the wealth distribution in this economy. We focus on a measure of wealth inequality (i.e., Pareto inequality) that captures the thickness of the right tail of the wealth distribution (see [Appendix A.1](#) for a full characterization of the wealth distribution).

**DEFINITION 1**—Pareto tail: *We say that the distribution of wealth has a Pareto tail if there exists a  $\zeta > 0$  such that*

$$\lim_{w \rightarrow \infty} \frac{\log \mathbb{P}(W > w)}{\log w} = -\zeta.$$

*The parameter  $\zeta$  is called the Pareto exponent of the distribution.*

Following [Jones \(2015\)](#), we define Pareto inequality  $\theta$  as the inverse of the Pareto exponent; that is,  $\theta = 1/\zeta$ . Hence, a higher level of Pareto inequality  $\theta$  corresponds to a thicker right tail (i.e., a density that decays more slowly as  $w \rightarrow \infty$ ). We are now ready to state the main result of this section.

<sup>5</sup>[Eisfeldt et al. \(2019\)](#) reports that, in recent years, equity-based compensation accounted for 45% of total compensation to high-skilled labor in the U.S.

PROPOSITION 1: Assume that  $\rho < g - i$ . Then the distribution of wealth of agents in our economy has a Pareto tail, with Pareto inequality

$$\theta = \max \left( \frac{g - \frac{i}{q} - \rho}{\eta + \tau}, \frac{r - \rho}{\eta} \right). \quad (5)$$

The proposition says that Pareto inequality is the maximum of two terms. The first term corresponds to the growth rate of successful entrepreneurs divided by their transition rate (the sum of population growth  $\eta$  and the Poisson rate  $\tau$  at which the tree blossoms). The second term corresponds to the growth rate of rentiers divided by population growth. Intuitively, this expression reflects the fact that Pareto inequality is pinned down by the type of agents with the highest growth rate after accounting for its persistence.<sup>6</sup>

As discussed above, the growth rate of successful entrepreneurs is decreasing in  $r$  while the growth rate of rentiers is increasing in  $r$ . As a result, there is a unique interest rate  $r^* \in (g - \tau, \rho + \eta)$  for which the two terms in (5) are equal.<sup>7</sup> Hence, the expression for Pareto inequality can be rewritten as

$$\theta = \begin{cases} \frac{g - \frac{i}{q} - \rho}{\eta + \tau} & \text{for } r \in (g - \tau, r^*) \\ \frac{r - \rho}{\eta} & \text{for } r \in (r^*, \rho + \eta). \end{cases} \quad (6)$$

This equation implies that Pareto inequality  $\theta$  is a u-shape function of the interest rate. To visualize this relationship, Figure 2 plots Pareto inequality  $\theta$  as a function of the interest rate  $r$ . When  $r > r^*$  (henceforth the rentier regime), lower interest rates decrease Pareto inequality. This comes from the fact that, in this high interest rate environment, the right tail of the wealth distribution is only populated by rentiers,<sup>8</sup> and their growth rates are increasing in the interest rate. This is similar to the standard models described in [Wold and Whittle \(1957\)](#) and [Piketty and Zucman \(2015\)](#).

In contrast, when  $r < r^*$  (henceforth the entrepreneur regime), lower interest rates *increase* Pareto inequality. This is because, when the interest rate is low enough, individuals making it to the top of the wealth distribution are entrepreneurs and those agents benefit from lower interest rates.<sup>9</sup> As explained earlier, this is because lower rates decreases the cost of external financing for entrepreneurs (at the expense of rentiers investing in them). As shown in [Appendix A.1](#), the economy is in the entrepreneur regime as soon as the relative fraction of entrepreneurs does not “vanish” in the right tail. This suggests that the entrepreneur regime is the relevant one empirically. For instance, [Cagetti and De Nardi, 2006](#) stress that most of the wealth in the top 1% of the US population is held by entrepreneurs.

<sup>6</sup>The expression for  $\theta$  is reminiscent of Proposition 4 in [Luttmer \(2011\)](#), who relates the Pareto exponent of the firm size distribution to the growth rate of high-growth firms (often referred to as “Luttmer rockets”).

<sup>7</sup>See the proof of Proposition 1.

<sup>8</sup>To be precise, the relative mass of entrepreneurs at a given level of wealth converges to zero as wealth goes to infinity (see the proof of Proposition 1 in [Appendix A](#)).

<sup>9</sup>A closed form expression for the relative mass of entrepreneurs in the right tail is given in the proof of Proposition 1 in [Appendix A.1](#).



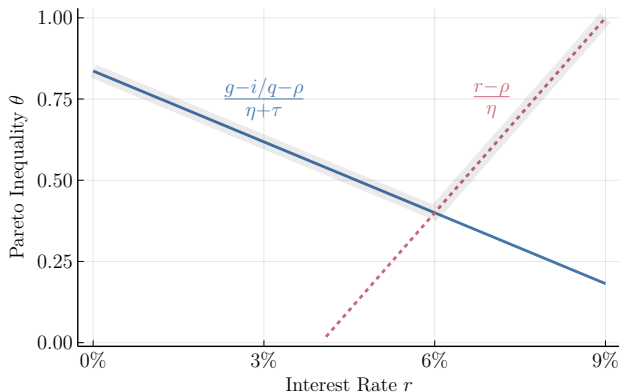


FIGURE 2.—Pareto inequality  $\theta$  as a function of the interest rate.  
 Numerical example with  $g = 0.5$ ,  $i = 0.4$ ,  $\tau = 0.5$ ,  $\eta = 0.05$ ,  $\rho = 0.04$ .

*Closing the model.* For simplicity, we have treated the interest rate  $r$  as an exogenous parameter of the model (small open economy assumption). In Appendix A.2, we study a closed economy version of the stylized model, which incorporates an additional group of agents (i.e., “workers”) that always hold diversified portfolios. We show that, by varying the subjective discount factor of workers from zero to infinity, we can generate the range of values for  $r$  considered in Proposition 1. Because workers never make it to the right tail of the wealth distribution, Pareto inequality remains the same as in the stylized (open-economy) model. Hence, our exogenous changes of the interest rate  $r$  in the stylized model can be interpreted as the result of changes in the demand for savings from this group of outside agents.

*Aggregate growth.* For simplicity, our stylized model does not feature aggregate growth: the aggregate income produced by trees grows at the same rate as the population. In Appendix A.3, we consider an economy with (potentially stochastic) aggregate growth. We show that all our equations above remain valid after deflating  $r$  and  $g$  (the interest rate and the growth rate of trees, respectively) by the growth rate of per-capita income.

### 3. SUFFICIENT STATISTIC

We are interested in *quantifying* the effect of lower interest rates on top wealth inequality. To do so, we now develop a sufficient statistic for the derivative of Pareto inequality with respect to the interest rate. Section 3.1 starts by deriving the sufficient statistic in the stylized model. Section 3.2 consider a number of model extensions and describe their effects on the sufficient statistic. Finally, Section 3.3 discusses the interpretation of our sufficient statistic approach in a general equilibrium context, as well as the effect on the interest rate on other dimensions of wealth inequality (e.g. top wealth shares).

#### 3.1. Sufficient statistic in the stylized model

We start by deriving a simple formula for the effect of the interest rate on Pareto inequality in the stylized model. Here, and in the rest of the paper, we suppose that we are in the entrepreneur regime, which, as discussed above, is the empirically relevant case. In this case, as shown in Proposition 1, Pareto inequality is given by  $\theta = (g - i/q - \rho)/(\eta + \tau)$ . Differentiating with

respect to the interest rate gives says that the log change in Pareto inequality is given by the log change in the growth rate of wealth for entrepreneurs:

$$\partial_r \log \theta = \partial_r \log \left( g - \frac{i}{q} - \rho \right),$$

In turn, the log change in the growth rate of wealth for entrepreneurs can be rewritten as:

$$\partial_r \log \theta = \frac{\left( -\frac{i}{q} \right) (-\partial_r \log q)}{g - \frac{i}{q} - \rho} \quad (7)$$

The numerator corresponds to the derivative of the growth rate of wealth of rentiers with respect to the interest rate,  $\partial_r(g - i/q - \rho) = (-i/q)(-\partial_r \log q)$  while the denominator corresponds to their growth rate of wealth,  $g - i/q - \rho$ . The numerator is the product of two terms:  $-i/q$  corresponds to the payout yield of the tree (the amount of cash returned to equity holders per unit of time divided by the market value of the tree) while  $(-\partial_r \log q) > 0$  corresponds to its duration.<sup>10</sup> Hence, Equation 7 can be seen as a “sufficient statistic” for the sensitivity of Pareto inequality to  $r$ , that says, in words:

$$\partial_r \log \theta = \frac{\text{Payout yield} \times \text{Duration}}{\text{Growth rate of wealth}}. \quad (8)$$

Note that the sign of the sufficient statistic is determined by the sign of the payout yield: as long as the payout yield is negative (i.e., entrepreneurs raise equity), a lower interest rate environment increases Pareto inequality.

While we have derived the expression for  $\partial_r \log \theta$  under the assumption that we are in the entrepreneur regime, we show in Appendix B.1 that the sufficient statistic also holds in the rentier regime, except that what matters in this case is the payout yield and growth rate of wealth of rentiers rather than entrepreneurs. This finding previews a more general result, established below, which is that the effect of interest rates on Pareto inequality is determined by the payout yield and growth rate of wealth of agents *getting to the top of the wealth distribution*.

### 3.2. Extensions

We now show that our “sufficient statistic” expression for  $\partial_r \log \theta$  holds in more general models. In particular, we study three extensions of the stylized model that incorporate, respectively, (i) endogenous investment, (ii) heterogeneous firm dynamics, and (iii) debt issuance. Of

<sup>10</sup>We now briefly explain why  $(-\partial_r \log q)$  is often called duration. Consider an asset with a cash flow stream  $(CF_t)_{t \geq 0}$ . Using a constant required rate of return  $r$ , the market value of the asset at time 0 is

$$V_0 = \mathbb{E}_0 \left[ \int_0^\infty e^{-rt} CF_t dt \right].$$

Differentiating with respect to  $r$  gives

$$-\partial_r \log V_0 = \frac{\mathbb{E}_0 \left[ \int_0^\infty t e^{-rt} CF_t dt \right]}{\mathbb{E}_0 \left[ \int_0^\infty e^{-rt} CF_t dt \right]}.$$

This equation says that  $-\partial_r \log V_0$  can be written as the weighted-average time to maturity of the asset’s cash flows, which justifies the term duration.

these three extensions, only the last one (debt issuance) substantially changes our expression for the sufficient statistic, as one needs to take into account both equity and debt issuance.

*Endogenous investment.* In the stylized model, there is no investment decision: the amount of investment  $i$  and the growth rate of the tree  $g$  are exogenous. We now show that our sufficient statistic is robust to incorporating an endogenous investment decision, as a result of the envelope theorem.

Suppose that entrepreneurs are born with one unit of capital and a technology that, given an amount of capital  $K$  allows them to (i) produce a quantity of good  $aK$  (i.e., a gross return on capital  $a$ ), and (ii) grow capital at rate  $g$  subject to a convex adjustment cost function  $\iota(g)$ . As in the stylized model, at Poisson rate  $\tau$ , the capital of each firm is transformed into the consumption good one-for-one.

Denote by  $q$  the market value of a firm divided by its capital stock, which is the original definition of Tobin's  $q$ . Given a required return  $r$ , the valuation of a firm  $q$  as well as its optimal growth rate  $g$  are now pinned down by the following Hamilton Jacobi Bellman (HJB) equation:

$$rq = \max_g \left\{ a - \iota(g) + gq + \tau(1 - q) \right\}. \quad (9)$$

The usual first-order condition for investment gives  $\iota'(g) = q$ : the marginal cost of investment must equal the marginal value of capital.<sup>11</sup>

All of the model formulas from Section 2 remain unchanged, except that  $i$  is replaced by  $\iota(g) - a$ . However, the key implication of endogenous investment is that a change in  $r$  now has the additional effect of changing the optimal growth rate of capital  $g$ . Replicating the derivation in the stylized model (see Equation 7), under the maintained assumption that we are in the entrepreneur regime, we have that the effect of  $r$  on Pareto inequality is given by

$$\begin{aligned} \partial_r \log \theta &= \partial_r \log \left( \frac{a - \iota(g)}{q} + g - \rho \right), \\ &= \underbrace{\frac{\frac{a - \iota(g)}{q} (-\partial_r \log q)}{\frac{a - \iota(g)}{q} + g - \rho}}_{\text{Baseline sufficient statistic}} + \underbrace{\frac{1 - \frac{\iota'(g)}{q}}{\frac{a - \iota(g)}{q} + g - \rho} \partial_r g}_{=0}. \end{aligned}$$

The first term on the right-hand side maps to the data exactly in the same way as in the stylized model (i.e., payout yield times duration divided by growth rate of wealth). The second term, which accounts for the response of investment to  $r$ , is new. However, given that the optimal growth rate  $g$  ensures that the entrepreneur invests up to the point where the marginal cost of investment  $\iota'(g)$  equals its marginal value  $q$ , the second term is zero. The key takeaway is that, at the first order, the response of investment to lower required returns does not matter for the sensitivity of Pareto inequality to required returns. In particular, in the case where investment reacts a lot to changes in required returns, entrepreneurs simply end up with a smaller fraction of larger firms.<sup>12</sup> Finally, note that this result relies on the fact that investment is optimally

<sup>11</sup>The stylized model can be seen as special case with no production and infinite adjustment costs; that is,  $a = 0$ , and  $\iota(\phi) = i$  if  $\phi = g$  and  $+\infty$  otherwise.

<sup>12</sup>For the same reason, the size of the investment response does not matter for the elasticity of the firm value to the interest rate,  $\partial_r \log q$ , which can still be inferred from the maturity of its cash flows (see Footnote 10).

chosen (i.e.,  $u'(g) = g$ ). In Appendix B.5, we consider a model extension where investment decisions are distorted due to constraints on external financing. We show that binding constraints on external financing amplifies the sensitivity of Pareto inequality to required returns.<sup>13</sup>

Finally, while we allow investment to react to the interest rate, the presence of adjustment cost implies that there is a wedge between the marginal return of capital and the interest rate. We discuss this point in more details in Appendix B.2.

*Heterogeneous firm dynamics.* So far, we have considered a simple environment where all firms operate the same production and investment technologies. We now show that the key insights from the stylized model hold in an general model of firm dynamics, where firms transition stochastically between different states.

We assume that firm heterogeneity is fully summarized by a state  $s \in \{1, \dots, S\}$ , which follows a continuous-time finite-state Markov chain with transition rate matrix  $\mathcal{T}$ .<sup>14</sup> Agents are born entrepreneur with a firm with one unit of capital and initial state drawn from an arbitrary distribution  $\psi$ . At Poisson rate  $\tau$ , entrepreneurs diversify and become rentiers.<sup>15</sup> As in the stylized model, we assume that both the production and investment technology features constant returns to scale, which implies that the value of a firm with capital stock  $K$  in state  $s$  is given by  $q_s K$ , where  $(q_s)_{1 \leq s \leq S}$  is the solution to

$$rq_s = \max_g \left\{ \underbrace{a_s - \iota_s(g)}_{\text{Payout}} + \underbrace{gq_s}_{\text{Growth}} + \underbrace{(\mathcal{T}q)_s}_{\text{Shocks}} \right\}. \quad (10)$$

As in the stylized model, the annuity value of a firm  $rq_s$  is determined by the current payout, the growth rate, and the expected contribution of idiosyncratic shocks. Unlike in the stylized model, we do not make any assumption on the sign of the payout  $a_s - \iota_s(g)$ . Going forward, we assume that there exists a unique, strictly positive solution to (10).<sup>16</sup>

The parameter  $a_s$  denotes the gross return on capital, which is a measure of production efficiency. The term  $\iota_s(g)$  denotes the investment rate. Notice that we allow the adjustment cost function  $\iota_s(\cdot)$  to depend on the state  $s$ , which captures differences in investment efficiency. For instance, a state could be associated with low production efficiency and a high investment efficiency, in which case the payout would be negative (i.e., the firm raises equity). Another state could be associated with a high production efficiency and low production efficiency, in which case the payout would be positive (i.e., the firm pays dividends).

Denote  $\mu_s \equiv \frac{a_s - \iota_s(g_s)}{q_s} + g_s - \rho$  the growth rate of capital for entrepreneurs in state  $s$ . The following proposition, which builds on Beare et al. (2021), characterizes the right tail of the wealth distribution in terms of the vector  $\boldsymbol{\mu} = (\mu_s)_{1 \leq s \leq S}$ .

<sup>13</sup>In our empirical application, we focus on U.S. entrepreneurs reaching the right tail of the wealth distribution. Given the wide availability of external financing available for these successful firms (through V.C. or private equity), we think that it is more realistic to think of the growth rate of these firms as being limited by their investment technology rather than by the availability of external financing. This is a conservative assumption: if the growth of these firms is limited by constraints on external financing, then our results underestimate the effect of  $r$  on Pareto inequality.

<sup>14</sup>More precisely,  $\mathcal{T}$  is a  $S \times S$  matrix: its off-diagonal elements  $\mathcal{T}_{ss'}$  contain the Poisson rates at which firms transition from state  $s$  to state  $s'$  for  $s \neq s'$  while its diagonal elements  $\mathcal{T}_{ss}$  correspond to  $-\sum_{s' \neq s} \mathcal{T}_{ss'}$  for  $s \in \{1, \dots, S\}$ . In particular, note that we have  $\mathbb{E}_t [df(s_t) | s_t = s] = (\mathcal{T}f)(s) dt$  for any function  $f$  defined on the set of states  $\{1, \dots, S\}$ .

<sup>15</sup>In the stylized model, the parameter  $\tau$  governs both the Poisson rate at which the tree blossoms and the Poisson rate at which entrepreneurs diversify. This extension relaxes this assumption and allows the diversification event to be independent from the state of the firm.

<sup>16</sup>This is analogous to our assumptions in the stylized models that (i) trees have a positive payout in expectation ( $i < \tau$ ) and (ii) the value of a tree is finite (i.e.,  $r > g - \tau$ ).

PROPOSITION 2—Pareto tail: *Suppose that there is at least one productivity state  $s$  such that  $\mu_s > 0$ . Then, the distribution of wealth has a Pareto tail with Pareto inequality given by*

$$\theta = \max\left(\theta_E, \frac{r - \rho}{\eta}\right),$$

where  $\theta_E$  denotes the unique positive number such that

$$\varrho\left(\frac{1}{\theta_E}\mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}\right) = \eta + \tau, \quad (11)$$

$\varrho(\cdot)$  denotes the dominant eigenvalue of a matrix and  $\mathcal{D}(\boldsymbol{\mu})$  denotes the diagonal matrix with  $\boldsymbol{\mu}$  as its main diagonal.<sup>17</sup>

Notice that Proposition 2 is a generalization of Proposition 1. Indeed, in the stylized model, we have that  $\mathcal{T} = 0$  (there is only one state) and  $\mathcal{D}(\boldsymbol{\mu}) = g - i/q - \rho$ . Plugging these expressions in (11) gives  $\theta_E = (g - i/q - \rho)/(\eta + \tau)$ , which is the same as the expression in Proposition 1.

We now characterize the effect of interest rates on Pareto inequality  $\partial_r \log \theta$  under the assumption that we are in the entrepreneur regime (i.e.,  $\theta = \theta_E$ ).

PROPOSITION 3—Sufficient statistic: *The semi-elasticity of Pareto inequality  $\theta$  to the required return  $r$  is given by*

$$\partial_r \log \theta = \lim_{W \rightarrow +\infty} \mathbb{E} \left[ \frac{\frac{1}{a_i} \int_0^{a_i} \partial_r \mu_{s_{ia}} da}{\frac{1}{a_i} \int_0^{a_i} \mu_{s_{ia}} da} \middle| W_i = W \right],$$

where  $W_i$  and  $a_i$  denote respectively the current wealth and age of an individual  $i$ , and  $s_{ia}$  denotes the state of individual  $i$  at age  $a$ .

The key takeaway is that the relative effect of a change in the required return on Pareto inequality  $\theta$  is given its relative effect on the *lifetime* average rate of capital accumulation of individuals at the top of the wealth distribution. Using the fact that  $\partial_r \mu_s = \frac{a_s - \iota_s(g_s)}{q_s} (-\partial_r \log q_s)$ , we obtain the following sufficient statistic in words:

$$\partial_r \log \theta = \mathbb{E} \left[ \frac{\text{Lifetime average of Payout yield} \times \text{Duration}}{\text{Lifetime average of Growth rate of wealth}} \middle| \text{Being in top percentile} \right]. \quad (12)$$

As in the stylized model, the sufficient statistic depends on the payout yield, duration, and growth rate of wealth. The key new result is that, in the presence of heterogeneity across entrepreneurs, what matters is the lifetime average of these moments for entrepreneurs at the top of the wealth distribution. One critical point is that the statistic is *backward looking*: what matters is the *backward looking* average of these quantities for individual currently at the top of the wealth distribution. Intuitively, this reflects the fact that the effect of interest rates on the wealth level of these entrepreneurs depends on all financing rounds that happened over their lifetimes.

<sup>17</sup>The dominant eigenvalue is defined as the eigenvalue with the largest real part. When all off-diagonal elements of a matrix are nonnegative (which is the case here), the dominant eigenvalue is real (see, for instance, [Beare and Toda, 2022](#)).

*Debt issuance.* So far, we have assumed that entrepreneurs finance the growth of their trees solely by issuing equity. We now consider a more realistic extension where external financing takes the form of both equity and debt issuance. The key takeaway is that, in this case, both debt and equity issuances matter in determining the effect of lower required returns on Pareto inequality.

For simplicity, we abstract from endogenous investment and we consider an extension of the stylized model where (i) trees must maintain a constant ratio of debt to their size, (ii) debt has zero maturity (i.e., continuous-time equivalent of a one-period bond), and (iii) entrepreneurs hold all of their wealth in the equity of their tree.<sup>18</sup> Define a tree's book equity as its size minus outstanding debt. Let  $\lambda$  be book leverage (i.e., the ratio between size and book equity). Denote by  $i_\lambda$  the flow of investment that equity holders need to pay as a proportion of the tree's book equity:<sup>19</sup>

$$i_\lambda \equiv g - r_f + \lambda(i - (g - r_f)), \quad (13)$$

where  $r_f$  denotes the interest rate on debt. Note that we allow the interest rate on debt  $r_f$  to differ from the required return on unlevered equity  $r$ . This spread could reflect an adjustment for aggregate or idiosyncratic risk, or, alternatively, some market segmentation between debt and equity market, as in [Baker and Wurgler \(2002\)](#).

Similarly, denote  $q_\lambda$  the market value of equity divided by its book value.<sup>20</sup>

$$q_\lambda \equiv 1 + \lambda(q - 1). \quad (14)$$

Note that  $q_\lambda$  is pinned down by  $q$ , which is itself pinned down by the required return on unlevered equity  $r$  via (2), as in the stylized model.

The growth rate of wealth for an entrepreneur whose tree is growing is given by the equity payout yield plus the growth rate of equity minus the consumption rate:

$$\frac{dW_t}{W_t} = \left( -\frac{i_\lambda}{q_\lambda} + g - \rho \right) dt \quad \text{if } t < T.$$

We now consider a joint change in the required return on debt  $dr_f$  and in the required return on unlevered equity  $dr$  on Pareto inequality, holding book leverage  $\lambda$  fixed. We show in [Appendix B.4](#) that the log change in Pareto inequality can be rewritten as:

$$d \log \theta = \frac{dr_f + \lambda_M \left( -\frac{i_\lambda}{q_\lambda} (-\partial_r \log q) dr - dr_f \right)}{-\frac{i_\lambda}{q_\lambda} + g - \rho}, \quad (15)$$

where  $\lambda_M = \lambda q / q_\lambda$  denotes market leverage (i.e., the ratio of the market value of the tree to the market value of its equity). As in the stylized model, the log change in Pareto inequality is given by the ratio between the change in the growth rate of entrepreneurs (numerator) relative to their growth rate (denominator).

<sup>18</sup>We could also consider the case in which entrepreneurs own levered position in the equity of their firms. Our formulas would remain the same, after redefining  $\lambda$  to be the effective leverage of entrepreneurs.

<sup>19</sup>The expression for  $i_\lambda$  corresponds to the total cash required by the tree per unit of book equity,  $\lambda i$ , minus the total cash paid by debt holders per unit of book equity,  $(\lambda - 1)(g - r_f)$ .

<sup>20</sup>The expression for  $q_\lambda$  corresponds to the market value of the tree per unit of book equity,  $\lambda q$ , minus the market value of the debt per unit of book equity,  $(\lambda - 1)$ .

In turn, the change in the growth rate of entrepreneurs (numerator) can be interpreted as the rate of issuance for each security (amount issued in dollars relative to the market value of equity) times the log change in the price of the security. More precisely, a change in the required return on equity increases the growth rate of entrepreneurs by their rate of equity issuance,  $i_\lambda/q_\lambda$ , times the log change in the price of this equity,  $\partial_r \log q_\lambda dr$ . Similarly, a change in the required return on debt increases the growth rate of entrepreneurs by their rate of debt issuance,  $\lambda_M - 1$ , times the log change in the price of this debt,  $-dr_f$ .<sup>21</sup> Note that, consistent with intuition, changes in the required return on equity  $r$  only affect the growth rate of entrepreneurs who issue equity (i.e.,  $i_\lambda \neq 0$ ), while changes in the required return on debt  $r_f$  only affect the growth rate of entrepreneurs who issue debt (i.e.,  $\lambda_M \neq 1$ ).

From now on, we will focus on the effect of a common change in the required returns on debt and equity,  $dr_f = dr$ . This assumption, made for the sake of simplicity, is also motivated by the fact that  $r_f$  and  $r$  have declined by a similar amount in our sample, as shown in Appendix C.2.2. In this case, Equation 15 can be written in words as:

$$\partial_r \log \theta = \frac{1 + \text{Market leverage} \times (\text{Equity payout yield} \times \text{Duration} - 1)}{\text{Growth rate of wealth}}. \quad (16)$$

The key takeaway is that both debt and equity issuances matter for the effect of a change in required returns on Pareto inequality. First, as in the stylized model, a lower required rate of return increases the market value of equity, which benefits entrepreneurs with negative equity payout yield (i.e., who are net equity issuers). Second, a lower required rate of return increases the value of one-period debt, which benefits entrepreneurs with market leverage higher than one (i.e., who are net debt issuers). In particular, even entrepreneurs who does not issue equity (as is often the case for private firms) benefit from lower rates as long as they issue debt.

### 3.3. Discussions

We now discuss two topics related to our sufficient statistic approach: general equilibrium considerations and revaluation gains.

*General equilibrium.* So far, our thought experiment has been to consider a partial equilibrium change in the required return  $r$  (small open economy assumption). A strength of this approach is that it allows us to remain agnostic with regard to the exact source of the change in  $r$ . We now discuss how to interpret our sufficient statistic approach when the required return  $r$  is determined in general equilibrium (i.e., in a closed economy).

Formally, denote  $z$  some structural parameter that affects interest rates (e.g., growth rate of the economy, subjective discount factor, or technology). Consider a small change in this parameter  $dz$ , which changes interest rates by  $dr$  in equilibrium. The resulting change in Pareto inequality can be decomposed into two terms:

$$d \log \theta = \underbrace{\partial_z \log \theta \times dz}_{\text{Direct effect through change in } z} + \underbrace{\partial_r \log \theta \times dr}_{\text{Indirect effect through GE change in } r}. \quad (17)$$

The first term corresponds to the direct effect of  $z$  on Pareto inequality while the second term (captured by our sufficient statistic approach) corresponds to the indirect effect of  $z$  on Pareto inequality through its equilibrium effect on  $r$ .

<sup>21</sup>This is easier to see in a discretized version of the model: the price of a one-period bond is  $\exp(-r_f \Delta t)$ , where  $r_f$  denotes the continuously compounded rate, and so the log change in its price is  $-\Delta t dr_f$ .

It is useful to illustrate this formula with a few examples. Consider first a shift in asset-demand originating outside of the entrepreneurial sector. As this shift does not affect Pareto inequality directly ( $\partial_z \log \theta = 0$ ), our sufficient statistic approach captures the total effect of this demand shift on Pareto inequality. This is the situation described in Appendix A.2, where changes in the subjective discount factor of workers affect the equilibrium interest rate without affecting Pareto inequality directly.

In contrast, consider a decline in the subjective discount factor of entrepreneurs. This decline has a direct effect on Pareto inequality as entrepreneurs now consume a smaller fraction of their wealth every period ( $\partial_z \log \theta \neq 0$ ). In this case, our sufficient statistic approach isolates the indirect effect of the decline on Pareto inequality through the equilibrium change in the interest rate. We quantify both effects in the context of our calibrated model in Online Appendix D.3.<sup>22</sup>

*Revaluation channel.* For now, we have focused on how the level of the required return on wealth  $r$  affects the rate of capital accumulation of each individual, and, therefore, the distribution of capital in the economy. At the same time, a decline in required returns also increases the market value of this capital. If this revaluation effect is heterogeneous across households, this may shift the observed distribution of wealth.

To fix ideas, we examine these two effects in the context of the stylized model. Integrating the law of motion for wealth (3) over time gives us the wealth level of an entrepreneur born  $t$  periods ago

$$W_t = \underbrace{q}_{\text{Tree valuation}} \times \underbrace{e^{(g - \frac{z}{q} - \rho)t}}_{\text{Quantity of tree}}. \quad (18)$$

It is the product of the valuation of the tree  $q$  times the quantity of tree that they own. So far, we have stressed that a lower required return  $r$  increases the quantity of tree owned by an entrepreneur by reducing their dilution rate (a “capital accumulation” channel). An additional effect of lower required returns is that they increase the valuation of trees that they own (a “capital revaluation” channel).

In the stylized model, there is a single asset, which means that all agents experience the same proportional revaluation of their wealth. Hence, the revaluation channel does not affect wealth inequality. In a more general model with multiple types of assets, however, revaluation gains can be heterogeneous across agents, which may affect wealth inequality. In particular, the revaluation channel could increase top wealth shares if agents at the top of the wealth distribution tend to have levered positions in firms (as in the [Debt issuance](#) extension) or if they tend to own firms with a higher duration than the rest (as in the [Heterogeneous firm dynamics](#)).<sup>23</sup>

Still, one key distinction between the accumulation and revaluation channels is that only the capital accumulation channel affects Pareto inequality. This comes from the fact that the capital accumulation channel affects the *growth rate* of wealth of an entrepreneur (i.e., the amount of capital they accumulate per unit of time) while the revaluation channel only affects its *level* (i.e., the market value of this capital). Put differently, the effect of the capital accumulation channel increases exponentially with wealth (as richer entrepreneurs go through a higher number of funding rounds), while the effect of the revaluation channel does not.<sup>24</sup> In Section 5, we will

<sup>22</sup>A simple application of Proposition 3 says that the direct effect, the derivative of Pareto inequality with respect to  $\rho$ , is minus one over the average growth rate of individuals making it to the top of the wealth distribution.

<sup>23</sup>See [Gomez \(2016\)](#) for empirical evidence on heterogeneous leverage across the wealth distribution and [Greenwald et al. \(2021\)](#) for empirical evidence on asset durations across the wealth distribution.

<sup>24</sup>Formally, in our model in which the wealth of individual  $i$  is given by  $q_i K_i$  where  $K_i$  denotes the quantity of capital that they own and  $q_i$  is a bounded variable denoting the valuation of this capital, the distribution of wealth



quantify the relative effect of each channel for the wealth share of the top 1%, 0.1%, and 0.01% in a calibrated model disciplined by existing empirical evidence on leverage and duration across the wealth distribution.<sup>25</sup>

#### 4. TAKING THE SUFFICIENT STATISTIC APPROACH TO THE DATA

In this section, we take our sufficient statistic approach to the data. We first estimate our sufficient statistic for the U.S. using data on top individuals in Section 4.1. We then use this estimate to quantify the effect of the secular decline of the required rate of return  $r$  on Pareto inequality  $\theta$  in Section 4.2.

##### 4.1. Estimating the sufficient statistic

Given a sample of  $i = 1, \dots, N$  individuals currently in the right tail of the wealth distribution in a given reference year, we estimate our sufficient statistic with

$$\widehat{\partial_r \log \theta} = \frac{\frac{1}{N} \sum_{i=1}^N (1 + \text{Market leverage}_i \times (\text{Equity payout yield}_i \times \text{Duration} - 1))}{\frac{1}{N} \sum_{i=1}^N \text{Growth rate}_i}, \quad (19)$$

where Equity payout yield $_i$ , Market leverage $_i$ , and Growth rate $_i$  are lifetime averages for each individual  $i = 1, \dots, N$ . Equation 19 thus corresponds to the empirical analogue of the sufficient statistic in the stylized model (see Equation 8) augmented to account for heterogeneous firm dynamics (see Equation 12) and leverage (see Equation 16).<sup>26</sup> As we will discuss shortly, we do not attempt to estimate firm duration at the individual level, but instead treat it as a parameter to be calibrated.

While the existing literature focuses on the characteristics of individuals at the top of the wealth distribution (e.g., [Cagetti and De Nardi, 2006](#)), relatively little is known regarding the *trajectory* of individuals reaching the top of the wealth distribution. Hence, a contribution of our paper is to construct a database on the growth rate of wealth, equity payout yield, and leverage of individuals reaching the top of the wealth of the wealth distribution.

*Forbes list.* We identify individuals in the right tail of the wealth distribution using the list of the wealthiest 400 Americans produced by Forbes Magazine. The list is created by the staff of the magazine based on a mix of public and private information.<sup>27</sup> For our application, we choose 2015 as the reference year and define the “right tail” as individuals in the top 100, a group for which information is widely available.

Table I contains information on the top 100 individuals included in the Forbes list in 2015. We assign to each individual the main firm that they or their family founded. Out of this set of

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$q_i K_i$  “inherits” the Pareto exponent of the distribution of capital  $K_i$  (see [Gabaix, 2016](#)). To break this result, one would need the distribution of  $q_i$  to have a thicker tail than the distribution of capital  $K_i$ .

<sup>25</sup>Another distinction between the capital accumulation and the revaluation channels is that an increase in wealth due to revaluation does not have a one-to-one mapping to welfare, as discussed in [Greenwald et al. \(2021\)](#) and [Fagereng et al. \(2022\)](#).

<sup>26</sup>Note that, relative to Equation 12, we calculate a ratio of averages, instead of an average of ratios. We make this choice in order to make the calibration strategy in Section 5—where we calibrate our model by targeting each of the individual moments (market leverage, equity payout yield, and growth rate of wealth) separately—entirely

TABLE I  
INDIVIDUALS IN THE TOP 100 (FORBES LIST, 2015)

Group	Count
Entrepreneurs	71
Public corporations	41
Private corporations	30
Rentiers	4
Financiers	25

*Notes.* “Entrepreneurs” are defined as individuals who are invested in non-financial firms that they (or a family member) founded; “Rentiers” are defined as individuals who are no longer invested in the firm that they (or a family member) founded; “Financiers” are defined as individuals who are invested in a financial firm that they (or a family member) founded. Data are from Forbes.

individuals, we remove 4 “rentiers”, which we define as individuals who are no longer invested in the firm that they or their family founded. As discussed in the context of the stylized model, in the entrepreneur regime, the Pareto exponent of the wealth distribution is entirely determined by the growth rate of wealth of entrepreneurs (see Section 2). We also remove 25 “financiers”, which we define as individuals who own financial firms, as our framework does not directly apply to them.<sup>28</sup> We are left with 71 individuals that we can associate to specific firms (either the firms they founded or the firms they joined early). As reported in Table I, roughly 60% own public firms while the rest own private firms.

*Equity payout yield.* A equity payout yield of a firm is defined as the net cash flows distributed to equity holders over a given period of time (in our case a year), divided by the market value of this equity. These cash flows can be distributed to equity holders through dividends and net share repurchases. As a consequence, the equity payout yield can be written as sum of the dividend yield (i.e., dividends distributed divided by the market value of the firm equity) and the buyback yield (i.e., cash flows distributed through shares repurchases minus the cash flows received by the firm through share issuances, divided by the market value of the firm equity). We now briefly describe our methodology to estimate the dividend and buyback yields (more details are given in Appendix C.1.1).

We first compute the lifetime average dividend yield of each firm owned by our set of entrepreneurs. In years for which a firm is public, we compute its annual dividend yield as the ratio of dividends to the market value of their equity, using data from Compustat ([SP Global Market Intelligence \(2023\)](#)). In years for which a firm is private, we set its annual dividend yield to zero. We then average the dividend yield over all years between the date of incorporation and 2015.

We then turn to the lifetime average buyback yield of each firm in our sample. Remember that, by definition, a firm’s buyback yield is equal to the cash flows distributed through share repurchases minus the cash flows received through share issuances, divided by the market value

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comparable to the sufficient statistic approach that we describe in this section. In Appendix C.1.2, we use estimate an alternative estimator (i.e., the average of ratios) and show that we obtain very similar results.

<sup>27</sup>Forbes Magazine reports: “We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don’t pretend to know what is listed on each billionaire’s private balance sheet, although some candidates do provide paperwork to that effect.”

<sup>28</sup>For a thorough investigation of the importance of financiers at the top of the income distribution, see [Kaplan and Rauh \(2010\)](#).

of the firm. As shown in Appendix C.1.1, this means that the buyback yield of a firm corresponds to the opposite of the growth of its number of shares (after adjusting for stock split). Note that this measure takes into account the wide range of type of equity issuance available to firms: venture capital funding (pre-IPO), IPOs, seasoned equity offering (post-IPO), and stock based compensation.<sup>29</sup>

In years for which a firm is public, we compute its annual buyback yield as (minus) the growth of their number of common shares outstanding, as reported in Compustat. We then back out the annual buyback yield in years leading up to the IPO from the ownership share of founders at the time of the IPO, as reported on their S-1 filings.<sup>30</sup> Finally, we set the buyback yield of firms that are always private to zero. Given that the dividend yield of these firms was also set to zero (as discussed above), this implies that their lifetime equity payout yield is zero. This effectively shuts down the effect of equity valuation for entrepreneurs owning these “always private” firms.

We report summary statistics for the lifetime average dividend yield and buyback yield of each firm in our sample in Table II. We find that firms owned by individuals in the right tail of the wealth distribution in 2015 have had a lifetime average dividend yield of 0.5% and a lifetime average buyback yield of  $-2.8\%$ . Overall, firms owned by individuals in the right tail of the wealth distribution have had a lifetime average equity payout yield of  $-2.2\%$ . The key observation here is that it is negative: these firms have spent more years raising cash from equity holders than distributing cash back to them. This does not contradict the fact that the corporate sector, as a whole, has a positive equity payout yield: in the data, as in the stylized model, firms tend to have a negative equity payout yield when they are young (and small) and a positive equity payout yield when they are old (and large).<sup>31</sup>

TABLE II  
SUMMARY STATISTICS

	Obs.	Average	Percentiles				
			Min	p25	p50	p75	Max
Equity payout yield	71	$-2.2\%$	$-22.1\%$	$-2.3\%$	$-0.0\%$	$0.0\%$	$1.7\%$
Dividend yield	71	$0.5\%$	$0.0\%$	$0.0\%$	$0.0\%$	$0.7\%$	$7.6\%$
Buyback yield	71	$-2.8\%$	$-22.1\%$	$-3.3\%$	$-0.5\%$	$0.0\%$	$0.5\%$
Market leverage	71	1.43	0.91	1.11	1.43	1.43	3.71
Growth rate of wealth	71	0.32	0.06	0.16	0.23	0.39	1.56

*Notes.* This table reports the lifetime average dividend yield, buyback yield, market leverage, and growth rate of the top 100 U.S. individuals in 2015. The construction of each variable is detailed in Appendix C.1.1. Data are from Forbes, Compustat, and SEC S-1 filings.

*Market leverage.* We estimate the market leverage of firms owned by entrepreneurs in our sample as follows. In years for which a firm is public, we compute its market leverage as the ratio between the market value of assets and the market value of equity, using data from Compustat (see Appendix C.1.1). We then construct market leverage in years leading up to the

<sup>29</sup>Economically, paying employees with stock is the same as paying employees with cash and simultaneously issuing equity to cover this cash outflow.

<sup>30</sup>Indeed, this is equal to the total number of shares held by founders when the firm was launched, assuming that founders neither sell or receive shares before the IPO. See Appendix C.1.1 for more details.

<sup>31</sup>See Online Appendix Figure C.1 for a plot of the average equity payout yield as a function of firm age. See also Abel et al. (1989) for theoretical and empirical arguments for a positive aggregate payout yield at the aggregate level.

IPO using the market leverage in the year following the IPO. We obtain that the lifetime average market leverage of firms that are public in 2015 is 1.43. We then use this figure to impute the lifetime average market leverage of firms that remain private.

*Growth rate of wealth.* We estimate the lifetime average growth rate of wealth for each individual as the log ratio between wealth in 2015 and initial wealth, divided by the age of the firm. Formally, we use the formula

$$\text{Lifetime average growth rate} = \frac{\log(W_{2015}/W_{t_0})}{2015 - t_0}, \quad (20)$$

where  $W_t$  denotes the wealth of an individual at time  $t$  normalized by the average wealth in the economy, and  $t_0$  denotes the founding date of the firm.<sup>32</sup> Unfortunately, there is very limited evidence on the wealth of our entrepreneurs at founding date, so, as a baseline, we set  $W_{t_0} = 1$  (i.e., we assume that their initial wealth equals the average wealth in the economy). In Appendix C.1.2, we show that our sufficient statistic does not change much when we set  $W_{t_0} = 1/5$  or  $W_{t_0} = 5$ ; this is because the terminal wealth of our individuals,  $W_{2015}$ , is typically several orders of magnitude larger than the average wealth in the economy, and so their exact starting point does not matter much.

As shown in Table II, we estimate an average growth rate of 32%. The distribution of growth rates is positively skewed, with large outliers corresponding to Facebook and Uber founders. In contrast, heirs of entrepreneurs who founded firms in the distant past have a much lower average growth rates.

*Duration.* The effect of the required return on Pareto inequality depends on the average duration of the firms owned by individuals that reach the top of the wealth distribution, where duration is defined as the semi-elasticity of a firm's market value with respect to the required rate of return, in absolute value.

Ideally, we would measure firm duration as the reaction of a firm's market value to an unexpected and permanent change in the required return on wealth. However, this is hard to do empirically. In particular, unexpected monetary policy shocks correspond mostly to transitory changes in short-term interest rates. For this reason, we do not attempt to measure duration for each firm separately and instead impose a constant duration across firms.

We first start by estimating the duration of the U.S. corporate sector as a whole. For an infinitely-lived representative firm with constant growth, its duration is simply the inverse of its payout yield.<sup>33</sup> This measure averages 3% from 1985 to 2020 (see Equation 21 for the definition of the payout yield of the corporate sector), which implies an average duration of 35 years. This back of the envelope calculation aligns closely with the findings of van Binsbergen (2020), who estimates a similar duration for the U.S. stock market using dividend strips.

We believe that a duration of 35 years is conservative for entrepreneurs in our sample. Indeed, there are reasons to think that firms owned by these entrepreneurs have a higher duration than

<sup>32</sup>As discussed in Appendix A.3, in the presence of aggregate growth, what matters for Pareto inequality is the dynamics of individual wealth *normalized* by the average wealth in the economy.

<sup>33</sup>Consider a firm with a positive cash flow stream  $(CF_t)_{t \geq 0}$  that grows on average at rate  $g$ ; that is,  $\mathbb{E}_0[CF_t] = e^{gt}CF_0$ . Given a constant required rate of return  $r$ , the market value of the firm at time 0 is

$$V_0 = \mathbb{E}_0 \left[ \int_0^\infty e^{-rt} CF_t dt \right] = \mathbb{E}_0 \left[ \int_0^\infty e^{-(r-g)t} dt \right] CF_0 = \frac{CF_0}{r-g}.$$

Differentiating with respect to  $r$  gives  $\partial_r \log V_0 = -1/(r-g) = -V_0/CF_0$ : the duration of a firm with constant average growth is equal to the inverse of its payout yield.

the corporate sector as a whole, since their cash flows are typically negative before turning positive. To account for this fact, we use results from [Gormsen and Lazarus \(2023\)](#), who find that the average duration of the top 20% of the firms in CRSP (sorted according to ex-ante measures of duration) is 46 years. Hence, as a robustness check, we consider an alternative duration calibration of 50 years.

*Results.* We now use our estimator  $\widehat{\partial_r \log \theta}$  (defined in Equation 19) to combine our estimates of the average equity payout yield, market leverage, and growth rate of wealth of individuals reaching the top of the wealth distribution. Table III contains the result. In our preferred calibration, we obtain a value of  $-4.2$ , which means that a permanent and unanticipated one percentage point increase in the required return on wealth changes Pareto inequality by  $-4.2$  log points. To account for sampling uncertainty, we also provide bootstrapped confidence interval for our sufficient statistic. Note that we can statistically reject the hypothesis that the required rate of return does not matter for Pareto inequality at the 5% level.

We also report our sufficient statistic with two alternative duration calibrations: 20 years and 50 years (relative to our baseline of 35 years). Consistent with the intuition, the sufficient statistic declines monotonically with the duration of the firms founded by entrepreneurs: the higher the duration, the bigger the effect of a change in required returns on valuations, and, as a result, the faster the entrepreneurs' wealth grows. In Appendix C.1.2, we also examine the sensitivity of our sufficient statistic to potential biases in our measures of the equity payout yield, leverage, and growth rate of wealth.

Finally, as discussed in our [Debt issuance](#) extension, our sufficient statistic can be seen as the sum of two terms: a term that accounts for equity issuance, Market Leverage  $\times$  Equity payout yield  $\times$  Duration and a term that accounts for debt issuance,  $1 - \text{Market leverage}$  (both divided by the growth rate of wealth). We find that the term due to debt issuance part is  $-1.3$  while the term due to equity issuance is  $-2.9$ . Hence, while both types of financing are important quantitatively, we find that the equity issuance channel dominates: intuitively, while equity issuance is much less frequent than debt issuance, a given change in required returns has a much larger effect on the value of equity than on the value of debt.

TABLE III  
ESTIMATED SUFFICIENT STATISTIC  $\widehat{\partial_r \log \theta}$

	Estimate	95% Confidence interval	
		Lower bound	Upper bound
Duration = 35 years (baseline)	-4.2	-5.4	-3.3
Duration = 20 years	-3.0	-3.9	-2.3
Duration = 50 years	-5.4	-7.0	-4.2

*Notes.* The sufficient statistic is constructed using (19). The 95% confidence-interval is constructed as a percentile bootstrap confidence interval using 1000 replications. Data are from Forbes, Compustat, and S-1 filings.

*Beyond the top 100.* So far, our empirical analysis has focused on the very top of the wealth distribution (i.e., the wealthiest 100 individuals). This is for two reasons. First, it is consistent with the theory: in the case of heterogeneous firm dynamics, our sufficient statistic states that the effect of interest rates on Pareto inequality depends on its effect on the most extreme wealth trajectories (see Equation 12). Second, there is much more data available on individuals at the very top, who tend to own well-known companies, than for the rest of the population.

Despite these data limitations, we argue in Appendix C.4 that our cost of capital mechanism should also be relevant well beyond the top 100. First, using data from the Survey of Consumer Finances, we show that roughly half of individuals the top 1% of the U.S. wealth distribution actively manage a firm that they founded. While we have very little information on how much external financing these U.S. entrepreneurs use, we rely on international evidence from Kochen (2022) to argue that private firms do rely heavily of both debt and, to a lesser extent, equity financing. Second, and as mentioned in Section 2, our mechanism does not only apply to entrepreneurs, but also to households with concentrated exposures in these high-growth firms, such as early investors or workers paid in stock.

#### 4.2. Historical decomposition using the sufficient statistic approach

The required returns on wealth has been declining steadily since the 1980s, a period that saw a rise in wealth inequality. We now use our estimated sufficient statistic to quantify the contribution of declining rates on the rise in Pareto inequality in the U.S. over the 1985–2015 period.

*Decline in required returns.* In our model,  $r$  represents the required return on trees. In the data, a natural counterpart for  $r$  is the required return on business liabilities (i.e., equity and debt claims) issued by the corporate sector. We now propose a simple methodology to estimate this required return using publicly-available macroeconomic data on the nonfinancial corporate sector from the Integrated Macroeconomic Accounts for the United States (Bureau of Economic Activity, 2023).

We measure the required return of owning business liabilities as its expected return, constructed using two assumptions: (i) revaluation gains (i.e., changes in the corporate sector’s  $Q$ ) are zero in expectation and (ii) cash flows are known one period in advance. As detailed in Appendix C.2, these assumptions imply the following formula for the required return on wealth:

$$\text{Required return} = \underbrace{\frac{\text{Return on capital} - \text{Capital formation rate}}{\text{Tobin's } Q}}_{\text{Aggregate payout yield}} + \underbrace{\text{Capital formation rate}}_{\text{Growth rate of cash flows}}, \quad (21)$$

where the first term corresponds to the aggregate payout yield and the second term corresponds to the rate of capital formation.<sup>34</sup> Hence, estimating the required return on wealth requires to estimate (i) the net return on capital (i.e., net operating surplus over the replacement cost of capital), (ii) the net rate of capital formation (i.e., net investment over the replacement cost of capital), and (iii) Tobin’s  $Q$  (i.e., the ratio of the market value of corporations over the replacement cost of capital). In Appendix C.2, we describe the methodology and data in detail. Note that our methodology accounts for inflation, so that everything going forward is in real terms.

One benefit of this methodology is that it allows us to contrast the required return on wealth to the return on capital. In particular, inspecting Equation 21 reveals the fact that the required return on wealth is equal to the return on capital if and only if Tobin’s  $Q$  is equal to one. Intuitively,  $Q$  represents the ratio between how much investors need to pay in order to acquire a claim on one unit of business capital and its replacement cost. When this ratio is one, the return on wealth for investors coincides with the return on capital.

<sup>34</sup>In particular, Equation 21 holds in the stylized model (see Appendix B.1).

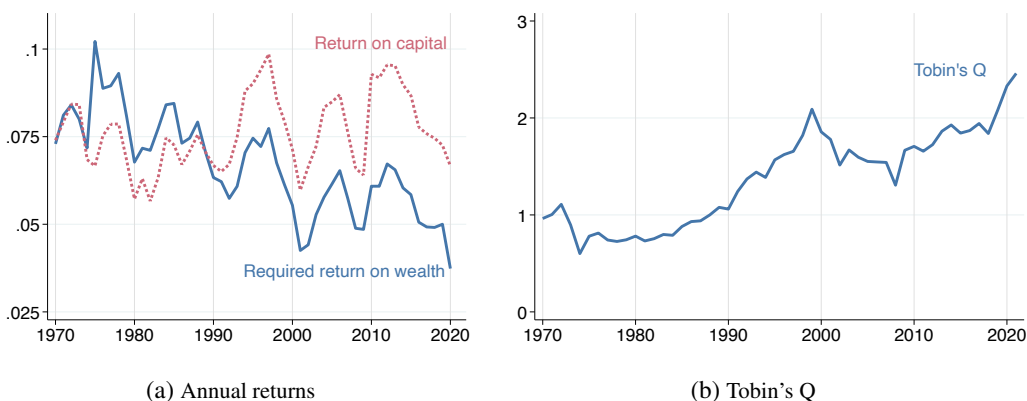


FIGURE 3.—Returns and valuation for the U.S. nonfinancial corporate sector

*Notes.* The figure plots Tobin's Q, the required return on wealth, and the return on capital of the U.S. nonfinancial corporate sector from 1970 to 2020. The construction of each variable is detailed in Appendix C.2. Data are from Bureau of Economic Activity (2023).

The left panel of Figure 3 plots the evolution of the required return on wealth and the return of capital over time over time. We find that required returns have declined over time. Quantitatively, required returns averaged 7.6% at the beginning of the sample (1980–1985) and 4.9% at the end of our sample (2015–2020), implying a  $-2.7$  pp. change in the required return on wealth over the time period. This is quantitatively consistent with the findings in recent papers such as Auclert et al. (2021) and Kuvshinov and Zimmermann (2021), who also use aggregate data to estimate required returns using a similar approach. Our estimate is more conservative (i.e., less negative) than Barkai (2020), who estimates a 5.5 pp. decline, and Mian et al. (2021), who estimate a 3.5 pp. decline.

In contrast, the return on capital exhibits no secular downward trend, and in fact has slightly increased. This consistent with Gomme et al. (2011), who find that the return on capital is mostly flat over time, and Moll et al. (2022), who emphasize the recent rise in the return on capital. The increasing wedge between the return on capital and the required return on wealth has also been discussed in a different context by Barkai (2020) and Karabarbounis and Neiman (2019).<sup>35</sup> Finally, the right panel of Figure 3 shows the secular rise in Tobin's Q over the time period, which is the mirror image of the growing wedge between the return on capital and the required return on wealth, as seen in Equation 21.

As discussed at the end of Section 2, what matters for Pareto inequality is the decline in required returns *relative* to the growth rate of the economy (in per-capita terms).<sup>36</sup> In Appendix C.2, we estimate the evolution of required returns deflated using various measures of per-capita growth (i.e., the per-capita capital formation rate or the growth rate of TFP). Accounting for the decline in per-capita growth, we estimate a change in required returns *net of per-capita growth* of approximately  $-2$  pp.

Note that we have focused on measuring the required return on the overall corporate sector (i.e.,  $r$  in the model). In Appendix C.2.2, we estimate the required return on corporate debt separately (i.e.,  $r_f$  in the Debt issuance extension) and find that it has declined by a similar amount.

<sup>35</sup>They focus on documenting the rise in “pure profits” as a share of GDP (i.e., capital income that cannot be accounted for by the stock of capital and the required return on wealth). By definition, this share equals the difference between the return on capital and the required return on wealth times capital divided by GDP.

<sup>36</sup>See also Appendix A.3.

This justifies our focus on considering the effect of an homogeneous decline in required rates of return on wealth inequality.

*Rise in Pareto inequality.* Figure 4a plots the evolution of top wealth shares in the U.S. using data from [Smith, Zidar, and Zwick \(2023\)](#), who construct wealth estimates based on the capitalization approach. A clear pattern stands out: the top 0.001% wealth share has grown faster than the top 0.01% share, which itself has grown more than the top 0.1% share, and so on. This pattern is a signature of a thickening of the right tail of the wealth distribution (i.e., an increase in Pareto inequality). As discussed in [Jones and Kim \(2018\)](#), if a distribution has a Pareto tail, then Pareto inequality is directly related to the ratio of top shares. Denoting  $S(p)$  to be the share of wealth owned by individuals in the top  $p \in (0, 1)$ , an estimator for Pareto inequality is

$$\hat{\theta}(p) \equiv 1 + \frac{\log(S(p)/S(10p))}{\log 10}. \quad (22)$$

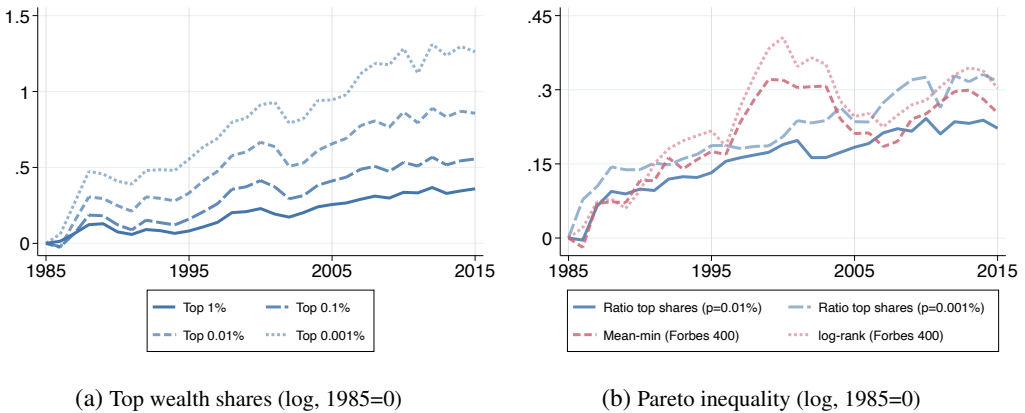


FIGURE 4.—Rise in top wealth inequality (1985-2015)

*Notes.* Panel (a) plots the evolution of top wealth shares, expressed in logarithm difference from their 1985 level. Panel (b) plots four estimates for Pareto inequality, expressed in logarithm difference from their 1985 level. Data from [Smith Matthew \(2022\)](#) and Forbes.

Figure 4b plots the evolution of Pareto inequality using Equation 22 with  $p = 0.01\%$  and  $p = 0.001\%$ , as well as two alternative estimates using Forbes 400 data (i.e., the “mean-min” estimator and the “log rank” estimator, see Appendix C.3 for more details). In each case, we report the log change in Pareto inequality since 1985. The four estimates agree on the broad trend: Pareto inequality has increased substantially since 1985. The fact we obtain a similar trend is reassuring given that the two alternative estimators (i.e., log-rank and mean-min) are purely based on cross-sectional data from the Forbes list, and therefore do not rely on indirect measures of wealth based on the capitalization approach. Taking a simple average of the log change of each estimate implies that Pareto inequality has increased by approximately 22 log points between 1985 and 2015. (See Table C.V in Appendix C.3 for related summary statistics.)

*Sufficient statistic approach.* Given our baseline estimate of a 2 pp. decline in the required rate of return net of per-capita growth, our sufficient statistic implies that the contribution of



declining required returns on Pareto inequality is:

$$\widehat{\partial_r \log \theta} \times (r_{2015} - r_{1985}) \approx -4.2 \times -2 \text{ pp.} = 8.4 \text{ log points.} \quad (23)$$

Since the overall change in Pareto inequality was roughly 22 log points over the time period, we conclude that the decline in the required rate of return on wealth accounts for roughly 40% of the rise in Pareto inequality.

*Robustness checks.* We conduct two robustness checks. First, we assess the importance of differential declines in the required return on wealth and interest rates on corporate debt. However, as discussed earlier, we find that both returns declined by almost the same amount. Hence, we obtain results that are very similar to (23). See Appendix C.2.2 for the details.

Second, we assess the importance of higher-order effects. Note that Equation 23 represents a first-order approximation for the effect of a non-infinitesimal change in the required return  $r$  on Pareto inequality. One may be concerned that this approximation does not capture well the higher-order effects of a 2 pp. change in the required rate of return. To examine this point, we estimate our sufficient statistic using 1985 as a reference year (i.e., focusing on individuals that were at the top of the wealth distribution in 1985). Under certain conditions, the average of the sufficient statistic for the reference years 1985 and 2015 constitutes a second-order approximation for the effect of  $r$  on Pareto inequality. We describe this robustness check in details in Supplemental Appendix E in [Gomez and Gouin Bonenfant \(2023\)](#): overall, we do not find much evidence in favor of higher-order effects, which suggests that our first-order approximation is also accurate at the second-order.

## 5. CALIBRATED MODEL

We now simulate the effect of a decline in the required return  $r$  on top wealth inequality  $\theta$  in a calibrated, general equilibrium model. The goal of this section is fourfold. First, while our sufficient statistic approach quantifies the effect of an infinitesimal decline in required returns, the model allows us to compute the effect of a non-infinitesimal decline in required returns of 2 pp., as in the data. Second, while our sufficient statistic approach is a comparative static on steady-states of the model, and therefore corresponds to the long-run effect of required returns on Pareto inequality, the calibrated model allows us to characterize the transition dynamics of the wealth distribution. Third, while our theoretical framework focuses on the change in Pareto inequality, the calibrated model allows us to compute the full change in the wealth distribution. In particular, we report the relative importance of the capital accumulation and the revaluation channels for the share of aggregate wealth owned by different percentiles of the wealth distribution. Fourth, the model allows us to clarify which assumptions are needed to generate a decline in the required return without a corresponding decline in the return on capital, as in the data.

*Modelling choices.* We study an extended version of the stylized model which combines endogenous investment, adjustable input, leverage, and heterogeneous firm dynamics (see Section 3). In particular, the fact that entrepreneurs produce goods with a combination of capital and labor, the supply of which is fixed in the economy, allows us to incorporate an equilibrium link between the required return  $r$  and the return on capital  $r_{ok}$ . We keep the firm side of the model simple, yet rich enough to match the micro evidence from Section 4. In addition to entrepreneurs, we add two other groups of agents: workers and foreigners. The addition of workers is mainly for accounting: it allows us to match the higher duration of entrepreneur wealth relative to aggregate wealth. The addition of foreigners allows us to generate a rise in the demand for domestic assets that originates from abroad.

### 5.1. Environment

As in the stylized model, there is no aggregate risk, agents are infinitely-lived, and population grows at rate  $\eta$ . There are two types of domestic agents: a fraction  $\pi$  of newborns are entrepreneurs endowed with a firm while the remaining  $1 - \pi$  are workers endowed with human capital. In addition, there is a foreign sector that purchases an exogenous amount of domestic financial assets.

*Firm problem.* Firms are born in a “growth” state,  $s = 0$ , and then transitions to a “mature” state,  $s = 1$ , at Poisson rate  $\tau$ . When the firm transitions to the mature state, its capital jumps by a factor of  $\psi$ . The firm problem is to choose the amount of labor  $L$  to hire and the growth rate of capital  $g$ . The value functions are the solutions to

$$\begin{aligned} r_t V_{0,t}(K) &= \max_{g,L} \left\{ F(K, L) - w_t L - \iota_0(g)K + V'_{0,t}(K)gK + \tau(V_{1,t}(\psi K) - V_{0,t}(K)) \right\} \\ &\quad + \dot{V}_{0,t}(K) \\ r_t V_{1,t}(K) &= \max_{g,L} \left\{ F(K, L) - w_t L - \iota_1(g)K + V'_{1,t}(K)gK \right\} + \dot{V}_{1,t}(K), \end{aligned}$$

where  $V_{s,t}(K)$  denotes the value of a firm in state  $s$  with capital  $K$ , and  $r_t$  is the required return. In terms of the production and investment technology, notice that growth and mature firms only differ in their investment adjustment cost function  $\iota_s(g)$ .

We assume the following functional forms:

$$F(K, L) = K^\alpha L^{1-\alpha}, \quad \iota_s(g) = g + \frac{\chi}{2}(g - \underline{g}_s)^2.$$

The production function is a standard Cobb-Douglas function and the adjustment cost function is quadratic. Solving for the optimal investment, we obtain  $g_s = \underline{g}_s + \frac{1}{\chi}(q_s - 1)$ , where  $q_s \equiv V'_s(K)/K$ . The parameter  $\chi > 0$  thus governs the elasticity of capital with respect to  $q$ . We will refer to the limit  $\chi \rightarrow \infty$  as the “inelastic capital” case, where the investment rate does not respond to the required return. The state-specific shifters  $\underline{g}_s$  allow for “Luttmer-rocket” dynamics, where firm growth is initially high and then stabilizes, which allows the model to match the fact that some firms reach the top of the size distribution very fast (Luttmer, 2011).

*Household problem.* At birth, entrepreneurs are endowed with the equity in a new growth firm of size  $\bar{K}$ , which is worth  $V_0(\bar{K}) - (1 - \lambda^{-1})\bar{K}$ . (We describe the structure of financial markets and the meaning of the book leverage parameter  $\lambda$  shortly.) They have log utility and their subjective discount factor is  $\rho$ . They are required to maintain all of their wealth invested in the equity of their firm. Their optimal consumption rule is to consume a fixed fraction  $\rho$  of their financial wealth.

Workers inelastically supply a unit flow of labor services and earn the equilibrium flow wage  $w$ . Their subjective discount factor is  $\rho_L$  and they invest in a diversified portfolio of financial assets. Their optimal consumption rule is to consume a fixed fraction  $\rho_L$  of their total wealth (i.e., financial wealth plus human wealth).

We assume that foreigners invest in a diversified portfolio of domestic financial assets. Let  $S_{F,t}e^{\eta t}$  denote the flow of savings from from abroad at time  $t$ , which we treat as exogenous. Our baseline model experiment will consist of perturbing the path of savings by foreigners in order to generate an equilibrium decline in the required return.

*Financial markets.* There are three assets available for trading: a floating rate bond, a levered equity share in the growth firm, and a levered equity share in the mature firm. We assume that firms issue both debt and equity and maintain a fixed book leverage  $\lambda$  (see Section 3). Since there is no aggregate risk, all assets have the same expected return in equilibrium. However, they have different duration profiles (i.e., the bond has zero duration while the equity shares have a positive duration).

Having these three assets allows us to parsimoniously match the higher duration of entrepreneur wealth relative to aggregate wealth, which will discipline the importance of the revaluation channel. Since workers and foreigners are indifferent between investing in any of the assets, we assume that they all hold the same diversified portfolio that comprises all of the assets not held by entrepreneurs.

## 5.2. Equilibrium

We consider a detrended economy where variables are defined in per-capita terms (i.e., multiplied by  $e^{-\eta t}$ ). A perfect foresight equilibrium is a sequence of consumption for workers and entrepreneurs  $(C_{L,t}, C_{E,t})_{t \geq 0}$ , a growth rate of capital for both types of firms  $(g_{0,t}, g_{1,t})_{t \geq 0}$ , a labor demand for both types of firms  $(L_{0,t}, L_{1,t})_{t \geq 0}$ , and a level of capital for both types of firms  $(K_{0,t}, K_{1,t})_{t \geq 0}$ , such that (i) consumption, labor demand, and capital growth solves the worker, entrepreneur, and firm problems and (ii) the labor and product markets clear:

$$\sum_{s \in \{0,1\}} L_{s,t} = 1 - \pi, \quad (24)$$

$$\sum_{s \in \{0,1\}} F(K_{s,t}, L_{s,t}) = C_{L,t} + C_{E,t} - S_{F,t} + \sum_{s \in \{0,1\}} K_{s,t} \iota_s(g_{s,t}). \quad (25)$$

In Appendix D.1, we provide an analytical characterization of the equilibrium.

*Neoclassical growth model as a limiting case.* In Appendix D.2 we show that the model nests the neoclassical growth model in the special case where capital is fully elastic ( $\chi = 0$ ), there is no firm heterogeneity ( $\psi = 0$ ), all agents are workers ( $\pi = 1$ ), and there is no population renewal ( $\eta = 0$ ). However, in order to match the empirical evidence from Section 5.3 (i.e., the micro moments related to wealth dynamics as well as the aggregate wedge between the return on capital and the required return on wealth), our calibration strategy will select a set of parameters that differs starkly from the “neoclassical growth calibration”.

## 5.3. Calibration

We calibrate the model by targeting moments for the US economy over the 1985–2015 period. We target a 7% required at the initial steady-state (i.e., roughly the expected return in 1985 net of trend growth, see Appendix Table C.III). Our model experiment will be to feed an exogenous rise in foreign savings that implies an equilibrium decline in the required return to 5%. Depending on the moment we want to match, we use the steady-state of the model associated with a required return of  $r = 7\%$  (i.e., 1985 moments) or the steady-state of the model associated with a required return of  $r = 6\%$  (i.e., 1985–2015 average moments).

First, we set the values for  $(\alpha, \pi, \chi)$  externally. The capital share is set to its standard value of  $\alpha = 1/3$  and the share of entrepreneurs in the economy is set to  $\pi = 0.15$ , which is roughly equal to the share of business owners in the US (see Table 1 of Cagetti and De Nardi, 2006). We set the capital adjustment cost parameter to  $\chi = +\infty$  to make aggregate capital inelastic.

We do so in order to match the fact that investment and the return on capital does not appear to have declined in the US, despite a large rise in Tobin's Q. This will serve as our baseline, but we also consider alternative calibrations with finite values for  $\chi$  in Section 5.5.

TABLE IV  
TARGETED MOMENTS

Moment	Period	Model	Data
Conditional micro moments			
Equity payout yield	1985-2015	-0.022	-0.022
Growth rate of wealth	1985-2015	0.32	0.32
Market leverage	1985-2015	1.4	1.4
Duration	1985-2015	35	35
Macro moments			
Return on capital	1985	0.071	0.07
Depreciation rate	1985-2015	0.08	0.08
Pareto inequality	1985-2015	0.6	0.6
Aggregate duration	1985-2015	20	20
NFA to domestic wealth	1985-2015	-0.05	-0.05

$$(\underline{g}_0, \underline{g}_1, \tau, \psi, \lambda, \bar{K}, \eta, \rho_L, \rho_E) = (0.389, -0.027, 0.429, 0.509, 1.679, 9.718, 0.107, 0.041, 0.046)$$

Second, we use the remaining 9 parameters  $(\underline{g}_0, \underline{g}_1, \tau, \psi, \lambda, \bar{K}, \eta, \rho_L, \rho)$  to match (i) the four conditional micro moments that enter the sufficient statistic (i.e., equity payout yield, growth rate of wealth, duration, and market leverage) and (ii) five macro moments (Pareto inequality, net return on capital, depreciation rate, aggregate duration, and the net foreign asset position). The conditional micro moments are taken directly from Table II. Those are important moments to match: they fully determine the long-run response of Pareto inequality to the required return. While these moments are measured for individuals at the top of the wealth distribution in 2015, they are backward-looking lifetime averages, so we use the model's  $r = 6\%$  steady-state as the theoretical counterpart (1985–2015 moments).

For the macro moments, we first target a net return on capital of 0.07 in the  $r = 7\%$  steady-state of the model (1985 moment), so that the model initially matches the fact that the expected return on wealth is equal to the return on capital at the beginning of the sample (see Appendix Table C.III). All of the other macro moments are associated with the  $r = 6\%$  steady-state of the model (1985–2015 moments). We target a depreciation rate of 8% and a level of Pareto inequality of 0.6 (see Appendix Table C.III). For the duration of aggregate wealth, we rely on evidence from Greenwald et al. (2021), who estimate a duration of roughly 20 years.<sup>37</sup> Finally, we target a net foreign asset position to domestic wealth of  $-5\%$ .<sup>38</sup> Table IV reports the targeted moments in the data and in the model.

<sup>37</sup>Unlike in our model, household wealth in the U.S. is not only composed of corporate liabilities (i.e., corporate equities and debts), but also of real estate and government liabilities. In the context of our model experiment, however, what is important is to match the duration of the wealth held at the top of the wealth distribution and the duration of aggregate wealth, as the difference between the two determines the size of the revaluation channel.

<sup>38</sup>In the model, domestic households do not own foreign assets. Hence, the net foreign asset position (in absolute term) is the value of domestic assets held by foreigners. Using data from the *Integrated Macroeconomic Accounts*, we find that this ratio went from nearly zero in 1985 to  $-10\%$  in 2015, with a midpoint of  $-5\%$  over the sample. See Appendix C for details.

### 5.4. Model experiment

So far, our analysis has remained agnostic on the causes of declining required returns. The existing literature has emphasized the importance of demand shocks such as increased demand for US assets from abroad (i.e., “global savings glut”) as well as an ageing of the US population and a rise in permanent labor income inequality that increases the domestic demand for assets (i.e., “domestic savings glut”).<sup>39</sup> For our baseline model experiment, we generate an exogenous asset demand shock driven by foreigners. As a robustness check, we also simulate in Appendix D.3 a domestic savings glut, which we operationalize via a decline in the subjective discount factor of domestic agents (i.e., entrepreneurs and workers).

More precisely, our baseline model experiment consist of feeding, a every time  $t \geq 0$ , a sequence of MIT shocks to the path of foreign savings  $(dS_{F,t+s})_{s \geq 0}$  in order to generate a smooth decline in the required return from an initial steady-state value of  $r = 7\%$  to a long-run value of  $r = 5\%$ .<sup>40</sup> We construct the path of foreign saving shocks so that, for every  $t \geq 0$ , the instantaneous change in the required return  $dr_t$  is both fully unexpected and believed to be permanent.<sup>41</sup> This is roughly consistent with the empirical evidence. For instance, Farmer et al. (2021) shows that professional forecasters have been consistently forecasting a flat path for short-term interest rates over the 1985–2015 period, despite the fact that short-term rates were a continuous downward path.

The model experiment thus consists of simulating the response of our model economy to a global savings glut (i.e., a rise in savings that originates outside of the domestic economy). We study a time period of 40 years following the beginning of the shock sequence. In Appendix D.1, we provide a detailed description of the equilibrium construction as well as the numerical algorithms used to solve for the equilibrium path.

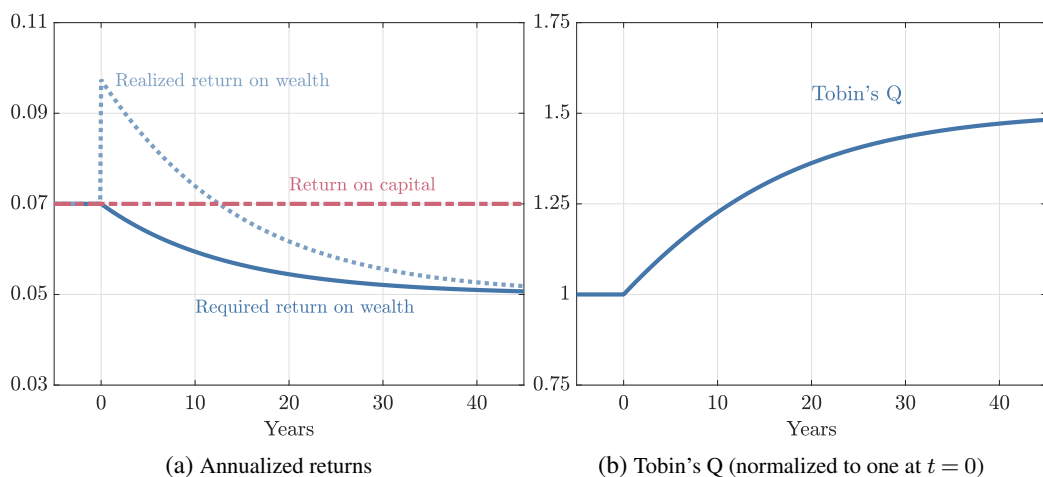


FIGURE 5.—Returns and valuations (model experiment)

<sup>39</sup>See Mian et al., 2020 for a recent review of the evidence. In particular, a recurrent finding in the literature is that declining economic growth is not sufficient to explain the decline in  $r$ .

<sup>40</sup>In practice, we target a path for the required return given by  $r_t = 0.07e^{-\phi t} + 0.05(1 - e^{-\phi t})$ , with  $\phi = 7.5\%$ .

<sup>41</sup>To be precise, at every time  $t \geq 0$ , the contemporaneous consumption, investment, and hiring decisions  $(C_{L,t}, C_{E,t}, g_{0,t}, g_{1,t}, L_{0,t}, L_{1,t})$  are part of a perfect foresight equilibrium with constant required return going forward:  $r_{t+s} = r_t$  for all  $s \geq 0$ .

The left panel of Figure 5 shows the evolution of the required return over time. The sequence of shocks starts at  $t = 0$ , after which the required return on wealth declines monotonically. In contrast, the aggregate return on capital remains constant as capital is fully inelastic in this calibration (this assumption will be relaxed in Section 5.5). By construction, the paths of the required return on wealth and the return on capital over the period  $t \in [0, 30]$  follow closely their empirical counterparts over the 1985–2015 period (see Figure 3).

Figure 5 also plots the evolution of the realized return on aggregate wealth. Note that it initially *increases*. To understand why, it is useful to express the instantaneous realized return as the sum of the required return and the unexpected revaluation of assets. In the model experiment, we have that:

$$\underbrace{\frac{dR_t}{R_t}}_{\text{Realized return}} = \underbrace{r_t dt}_{\text{Required return}} + \underbrace{\frac{dQ_t}{Q_t} - \mathbb{E}_t \left[ \frac{dQ_t}{Q_t} \right]}_{\text{Revaluation}}, \quad (26)$$

where  $Q_t$  denotes Tobin's Q in the economy (the capital-weighted average of firms' individual  $q_s$ ). Since Tobin's Q increases over the transition (see Figure 5b), the revaluation term is positive. Note that the magnitude of this term is empirically disciplined by the fact our model targets the duration of aggregate wealth in the data.<sup>42</sup>

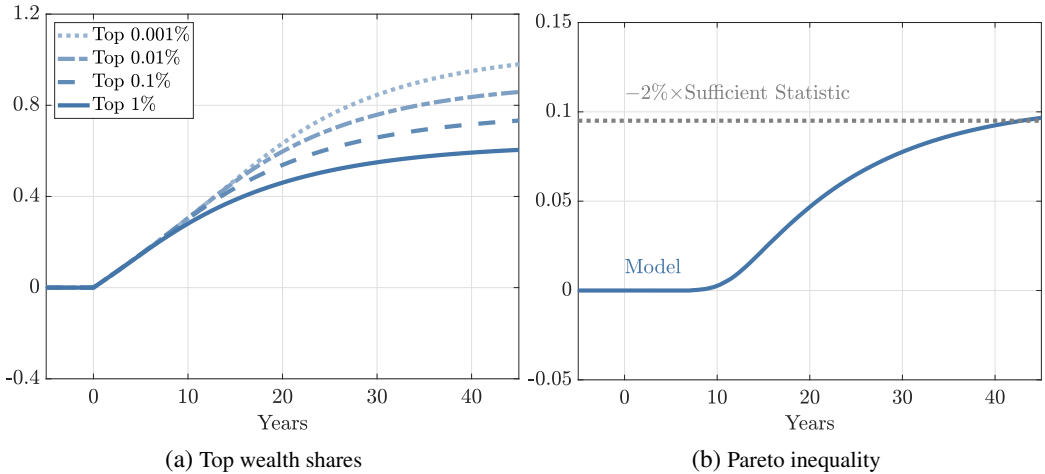


FIGURE 6.—Top wealth inequality (model experiment)

Figure 6a shows the evolution of top wealth shares, all normalized to one at  $t = 0$ . The top shares are calculated numerically and expressed as shares of aggregate wealth. For simplicity,

<sup>42</sup>Our calibration matches a duration of aggregate wealth of 20. During a short time period  $dt$ , the innovation in the required return  $dr_t$  is believed to be permanent. As a result, in a calibration with inelastic capital, the revaluation term is given by

$$\frac{dQ_t}{Q_t} - \mathbb{E}_t \left[ \frac{dQ_t}{Q_t} \right] \approx -20 \times dr_t.$$

See Footnote 10 for a definition of duration.

we assume that all workers are identical, and, as a result, only entrepreneurs are in the top 1%.<sup>43</sup> Notice that the top 0.1% increases more than the top 1%, and the top 0.01% increases more than the top 0.1%, and so on, which we precisely what we observe in the data (see Figure 4). To better visualize this effect, Figure 6b shows the evolution of Pareto inequality (calculated using the top share estimator with  $p = 0.1\%$ , see Equation 22), expressed in log and normalized to zero at  $t = 0$ . Pareto inequality increases steadily and roughly converges to its long-run value after 40 years. Compared to the data, the model roughly matches the rise in the top 1% wealth share, but undershoots the rise in the top 0.001% (see Figure 4a), which is consistent with the fact that our proposed mechanism explains roughly half of the rise in Pareto inequality (see Section 4).

We also plot a dashed grey line showing the long-run level of Pareto inequality predicted by the sufficient statistic approach. This line coincides almost exactly with the long-run limit of Pareto inequality in the model. This suggests that, at least in this model, our sufficient statistic constitutes a very good approximation of a non-infinitesimal change in the required return on wealth on Pareto inequality. More precisely, while the sufficient statistic approach holds exactly in the model for small changes in  $r$  (i.e., it relies on a first-order approximation), we obtain a slightly higher response in the model in the long-run due to higher-order effects.<sup>44</sup>

Finally, the calibrated model generates a relatively fast convergence of Pareto inequality to its long-run steady-state. As discussed in Gabaix et al. (2016), this comes from the presence of high-growth types in our model: indeed, in our model, some agents (i.e., entrepreneurs owning growth firms) reach the right tail of the wealth distribution quickly, as their wealth grows at an annual rate of 32% (see Table IV). Still, note that our estimate of Pareto inequality (measured as the ratio between the wealth share of the top 0.01% relative to the top 0.1%) remains constant in the first ten years. This reflects the fact that, initially, all percentiles benefit similarly from a decline in the required return on wealth. It is only when the new generation of entrepreneurs, born in the new low interest rate environment, reach the top 0.1% that Pareto inequality start increasing.

*Capital accumulation versus revaluation channel.* As discussed in Section 3.3, a lower required return tends to increase the share of aggregate wealth owned in a top percentile through two distinct channels. First, it increases the relative quantity of capital owned by entrepreneurs who raise external financing, as they now face a lower cost of capital (a “capital accumulation” channel). Second, it can increase the valuation of the capital owned by entrepreneurs relative to the valuation of the aggregate capital (a “revaluation” channel).

We now assess the relative contribution of each channel in our model economy by decomposing the cumulative growth of the share of wealth owned by a top percentile. Formally, the average wealth owned by a top percentile  $p$  at time  $t$  can be written  $q_{\lambda pt} E_{pt}$  where  $E_{pt}$  is the average quantity of book equity owned by the top percentile and  $q_{\lambda pt}$  is the (book equity weighted) average valuation of this equity. The average wealth in the economy, can be written  $Q_t K$  where  $K$  is aggregate capital per capita and  $Q_t$  is the capital-weighted valuation of capital in the economy. Given these notations, we can decompose the cumulative growth of the share of aggregate wealth owned by a top percentile  $p$  between 0 and  $t$  into a “capital accumulation”

<sup>43</sup>Given that a fraction  $\pi = 15\%$  of the population are workers, our top 1% group corresponds to the top  $1/15 \approx 6.7\%$  of entrepreneurs.

<sup>44</sup>Formally, the sufficient statistic changes over the time period, as the lifetime average equity payout yield, leverage, duration, and growth rate of entrepreneurs in the top changes in response to the decline in the required return. See Supplemental Appendix E in Gomez and Gouin Bonenfant (2023) for a discussion of higher-order effects.

and a “revaluation” channel:

$$\log \left( \frac{q_{\lambda pt} E_{pt} / (Q_t K)}{q_{\lambda p0} E_{p0} / (Q_0 K)} \right) = \underbrace{\log \left( \frac{E_{pt}}{E_{p0}} \right)}_{\text{Capital accumulation}} + \underbrace{\log \left( \frac{q_{\lambda pt}}{q_{\lambda p0}} \right) - \log \left( \frac{Q_t}{Q_0} \right)}_{\text{Revaluation}}.$$

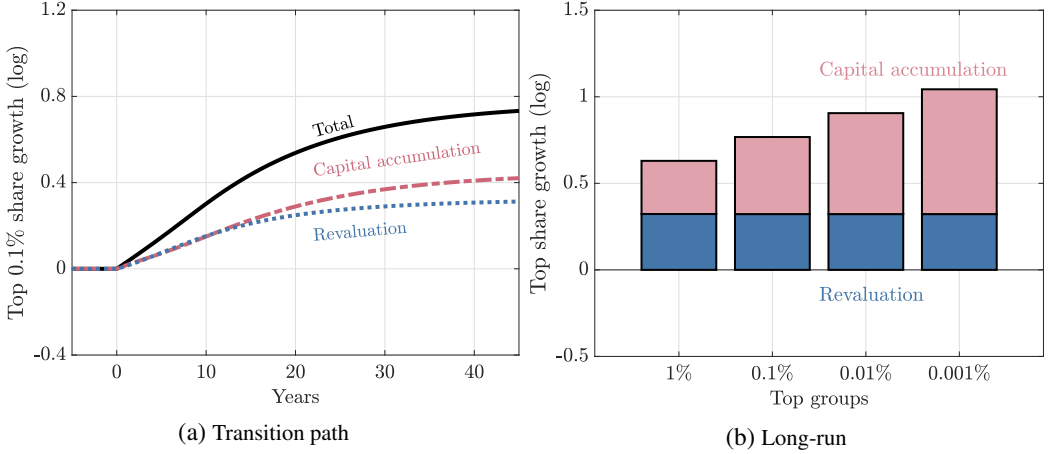


FIGURE 7.—Disentangling capital accumulation and revaluation (model experiment)

Figure 7a plots the cumulative growth of the top 0.1% in the model experiment, as well as the cumulative contribution of the capital accumulation and of the revaluation channels. We find that the revaluation channel is positive. The reason is twofold: (i) entrepreneurs at the top of the wealth distribution hold levered positions in firms, and (ii) they tend to hold firms with a higher duration than the average firm in the economy. Quantitatively, in the  $r = 6\%$  steady-state of the model, the levered duration (i.e., duration times market leverage) of the assets held by the top 0.1% of agents  $-\partial_r \log q_{\lambda pt}$  is 33 while the duration of aggregate wealth  $-\partial_r \log Q_t$  is 21. Hence, a first-order approximation for the long-run contribution of the revaluation channel is  $(-\partial_r \log q_{\lambda pt} + \partial_r \log Q_t) \times \Delta r = (-37 + 20) \times (-2 \text{ pp}) = 34 \text{ log points}$ , which is very close to the 32 log points that we report in Figure 7a. Note that this number is tightly disciplined by our calibration strategy, where we match the duration of aggregate wealth as well as the duration and market leverage of individuals reaching the top of the wealth distribution (see Table IV).

We also find that the contribution of the revaluation channel is builds up more quickly, relative to the capital accumulation channel. The reason is that revaluation gains are immediate: when the required return declines, the market value of capital jumps up, which affects the distribution of wealth on impact. In contrast, the capital accumulation channel builds up slowly over time, as it takes time for existing entrepreneurs to raise new outside financing via equity and debt issuance.

Figure 7b plots the long-run contribution of each channel for the top 1%, 0.1%, 0.01%, and 0.001% wealth shares (i.e., from the initial steady-state to the terminal steady-state). The contribution of the revaluation channel is approximately the same for all top percentiles. The reason is that the composition of individuals in each of these top groups is approximately the same (i.e., the proportion of entrepreneurs owning growth firms versus mature firms), and,



therefore, they experience similar revaluation gains.<sup>45</sup> In contrast, the contribution of the capital accumulation channel increases with the top percentile (i.e., it is higher for the top 0.001% than for the top 0.01%, and so on). As discussed in Section 3.3, this reflects the fact that wealthier individuals spend more time in the high growth state over their lifetime, and, as a result, they benefit more from a lower cost of capital as they go through a higher number of funding rounds.

To sum up, we find that both the revaluation and the capital accumulation channels contribute to the rise in top wealth shares. One key difference, however, is that the relative importance of the capital accumulation channel increases the thickness of the right tail of the wealth distribution. As discussed formally in Section 3.3, this is because the rate of capital accumulation is the only thing that matters for Pareto inequality.

Our finding that the revaluation channel only explains part of the rise in top wealth shares in our model is consistent with Saez and Zucman (2016) and Mian et al. (2020), who argue that the mechanical effect of the revaluation of assets owned by the rich cannot fully explain the rise in top wealth shares over the past four decades. They impute this difference to a rise in the “synthetic” saving rate of individuals in top percentiles. Instead, in our model, the consumption rate of entrepreneurs is fixed: the difference is driven by an increase in the flow of entrepreneurs reaching the top of the wealth distribution. We stress this distinction in more details in Appendix D.4.

### 5.5. Model experiment with elastic capital

In general, the return on capital is an endogenous variable that depends on aggregate capital, labor, and investment. In the baseline calibration of the model, we hardwired a constant return on capital by making capital completely inelastic (i.e., we set the investment adjustment cost to  $\chi = +\infty$ ). We now consider three alternative calibrations where we use the parameter  $\chi$  to match three targets for the decline of the return on capital (i.e., 0.5, 1, and 1.5 pp.). We refer to these calibrations as “low-elasticity”, “medium-elasticity”, and “high-elasticity” calibrations. These targets provide a range of intermediate value between the baseline model (i.e., no decline in the return on capital) and the neoclassical growth model (i.e., 2 pp. decline in the return on capital). The calibration strategy remains otherwise identical, but we have one additional model parameter (i.e.,  $\chi$ ) and one additional moment (i.e., the decline in the aggregate return on capital). See Appendix D.5 for the calibration table.

In calibrations with elastic capital, there are two forces that generate a decline in the return on capital. Applying our definition of the return on capital (37) for a firm in state  $s$ , and substituting the expression for optimal labor, we have that

$$\text{rok}_s \equiv \alpha (K_s/L_s)^{\alpha-1} - \frac{\chi}{2} (g_s - \underline{g}_s)^2.$$

The first term is production efficiency, which in this model is the marginal product of capital, and the second term is investment efficiency. The direct effect of a decline in the required return  $r$  is that it increases Tobin’s Q and leads to a rise in investment (recall that the policy function is  $g_s = \underline{g}_s + \frac{1}{\chi}(q_s - 1)$ ). As result, the aggregate capital stock increases. But since labor supply is fixed at the aggregate level, this implies a rise in the capital to labor ratio, which puts downward pressure on the return on capital.<sup>46</sup> The second force that depresses the return on capital is that

<sup>45</sup>The fact that the relative proportion of each type converges in the right tail is a general result in random growth models (see, for instance, Proposition 4.1 in Guin-Bonenfant and Toda, 2022).

<sup>46</sup>Since both types of firms have the same production function, they both employ as much labor per unit of capital. As a result, they both have the same production efficiency.

investment efficiency declines. Indeed, as firm-level investment  $g$  increases, adjustment costs become more severe due to the convexity of  $\iota_s(g)$ , which decreases the amount of net capital income per unit of capital.

TABLE V  
MODEL EXPERIMENT WITH ELASTIC CAPITAL (LONG-RUN, PERCENTAGE POINTS)

Model	$\Delta r$	$\Delta \text{rok}$	$\Delta \log \theta$
Baseline	-2.0	0.0	11
Low-elasticity	-2.0	-0.5	9.3
Medium-elasticity	-2.0	-1.0	7.9
High-elasticity	-2.0	-1.5	6.7
Neoclassical growth model	-2.0	-2.0	—

Table V reports the long-run response of the aggregate return on capital ( $\Delta \text{rok}$ ) as well as the long-run increase in (log) Pareto inequality (i.e.,  $\Delta \log \theta$ ) in each model calibration. The key takeaway is that, the more elastic capital is, the lower the rise in Pareto inequality. For instance, in the baseline model, we have that Pareto inequality increases by roughly 11 log points. In contrast, in the high-elasticity calibration, the corresponding value is 7 log points, which is about one-third lower. The reason is that, for a given cost of capital  $r$ , a lower return on capital reduces the growth rate of wealth for successful entrepreneurs (i.e., those who own growth firms for a long period of time).

Overall, we conclude that, for reasonable model calibrations (i.e., calibrations that imply a moderate decline in the return on capital), a decline in  $r$  increases Pareto inequality materially. In Appendix D.5, we compare the implied values of  $\chi$  in the elastic capital calibrations to existing evidence from investment regressions.

## 6. CONCLUSION

This paper studies the long-run relationship between Pareto inequality in wealth (i.e., inequality between wealthy individuals) and the required rate of return on wealth. We make three distinct contributions. First, we show theoretically that low rates *increase* top wealth inequality whenever individuals reaching the top of the wealth distribution are net issuers of assets (“net borrowers” rather than “net lenders”). Second, we derive a sufficient statistic for the effect of lower rates on top wealth inequality (as measured by the Pareto exponent of the wealth distribution). It depends on three key moments: the average growth rate of wealth for individuals reaching the top of the wealth distribution as well as the average payout yield and leverage of the firms that they own. Third, we collect new data on the wealth trajectory of the top 100 wealthiest individuals in the U.S., which we use to estimate our sufficient statistics.

Overall, our results indicate that the direct effect of lower rates on top wealth inequality is large: the 2% decline in required returns that we estimate from 1985 to 2015 accounts for between a third and half of the rise in top wealth inequality. This finding is guided by the observation that, in the U.S., entrepreneurs reaching the top of the wealth distribution rely heavily on external financing. Technology and institutions presumably affect the extent to which successful firms rely on external financing. In particular, the effect of interest rates on top wealth inequality may be drastically different across countries and time periods. We view our sufficient statistic approach as a first step in understanding this heterogeneous effect.

Taking a step back, one important message of our paper is that the right tail of the U.S. wealth distribution is determined by the wealth dynamics of new fortunes (entrepreneurs), rather than by the wealth dynamics of existing ones (rentiers). We develop a simple model as well as analytical methods that allow us to quantify the effect of changes in entrepreneur wealth dynamics on Pareto inequality. We believe that this set of tools could prove useful to shed light on other factors that might be driving the recent rise in top wealth inequality such as changes in the nature of technology (e.g., [Kaplan and Rauh, 2010](#); [Jones and Kim, 2018](#); [Moll et al., 2022](#)) and changes in corporate taxation (e.g., [Kaymak and Poschke, 2016](#); [Hubmer et al., 2020](#)).

Finally, the idea that low interest rates increase top wealth inequality complements a growing literature which argues that high inequality puts downward pressure on required rates of return in equilibrium (see, for instance, [Straub, 2019](#) for the interest rate or [Gollier, 2001](#); [Toda and Walsh, 2019](#); [Gomez, 2016](#) for the equity risk premium). Taken together, this suggests that high wealth inequality and low required rates of return can be mutually reinforcing in the long-run, an idea we leave for future research.

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## APPENDIX A: APPENDIX FOR SECTION 2

## A.1. Proofs

PROOF OF PROPOSITION 1: Denote  $p_E(\cdot)$  and  $p_R(\cdot)$  the wealth densities of entrepreneurs and rentiers; that is,  $p_E(W) dW$  and  $p_R(W) dW$  denote the mass of rentiers and entrepreneurs with wealth between  $W$  and  $W + dW$  relative to the total population. The Kolmogorov Forward Equation says that these densities solve the following system of ODEs

$$0 = -\partial_W \left( \left( g - \frac{i}{q} - \rho \right) W p_E(W) \right) + \eta \delta(W - q) - (\eta + \tau) p_E(W), \quad (27)$$

$$0 = -\partial_W ((r - \rho) W p_R(W)) + \tau p_E(qW) q - \eta p_R(W), \quad (28)$$

where  $\delta(\cdot)$  denotes the Dirac function.

One approach would be to follow the same steps as in the proof Proposition 2 to characterize the right tail of the wealth distribution. However, the model is stylized enough that we can actually solve for the wealth density in closed form. To do so, it is useful to use the following notations

$$\theta_R(r) = \frac{r - \rho}{\eta}, \quad \theta_E(r) = \frac{g - \frac{i}{q} - \rho}{\tau + \eta}.$$

In words,  $\theta_R$  is the ratio between the growth rate of rentiers and the population growth rate  $\eta$  while  $\theta_E$  is the ratio between the growth rate of entrepreneurs and their “death” rate (the sum of the population growth rate  $\eta$  and their transition rate  $\tau$ ). Moreover, the function  $\theta_E(\cdot)$  is decreasing in  $r$  (through  $q(r)$ ) while the function  $\theta_R(\cdot)$  is increasing in  $r$ . There exists a (unique)  $r^* \in (g - \tau, \rho + \eta)$  such that  $\theta_E(\cdot)$  and  $\theta_R(\cdot)$  intersect, which is given by<sup>47</sup>

$$r^* = \frac{\eta}{\eta + \tau - i} (g - \tau) + \frac{\tau - i}{\eta + \tau - i} (\rho + \eta).$$

The value of  $\theta_E$  and  $\theta_R$  at this intersecting point is  $\theta_E(r^*) = \theta_R(r^*) = (g - i - \rho)/(\eta + \tau - i)$ , which is positive given our assumptions that  $\rho < g - i$  and  $i < \tau$ .

Solving (27) gives that the wealth density of entrepreneurs is given by:

$$p_E(W) = \frac{\eta}{\eta + \tau} \begin{cases} \frac{1}{\theta_E} q^{\frac{1}{\theta_E}} W^{-\frac{1}{\theta_E} - 1} 1_{W \geq q} & \text{if } r \in (g - \tau, r_E) \text{ (i.e., } \theta_E > 0) \\ \delta(W - q) & \text{if } r = r_E \text{ (i.e., } \theta_E = 0) \\ -\frac{1}{\theta_E} q^{\frac{1}{\theta_E}} W^{-\frac{1}{\theta_E} - 1} 1_{W \leq q} & \text{if } r \in (r_E, \rho + \eta) \text{ (i.e., } \theta_E < 0). \end{cases}$$

<sup>47</sup>Indeed,  $\theta_E(r^*) = \theta_R(r^*)$  is equivalent to

$$\frac{r^* + \tau(1 - 1/q(r^*)) - \rho}{\eta + \tau} = \frac{r^* - \rho}{\eta} \iff 1 - \frac{1}{q(r^*)} = \frac{r^* - \rho}{\eta} \iff 1 - \frac{r^* + \tau - g}{\tau - i} = \frac{r^* - \rho}{\eta}.$$

where  $r_E$  denotes the point at which  $\theta_E(\cdot)$  crosses zero; that is,  $r_E \equiv g - \tau + (\tau/i - 1)(g - \rho)$ .<sup>48</sup> Note that the shape of the wealth distribution of entrepreneurs depends critically on whether their growth rate is positive ( $\theta_E > 0$ ) or negative ( $\theta_E < 0$ ).

We now turn to the wealth density among rentiers. Solving (28) gives:

$$p_R(W) = \frac{\tau}{\eta + \tau} \begin{cases} \frac{1}{\theta_E - \theta_R} \left( W^{-\frac{1}{\theta_E} - 1} 1_{W \geq 1} + W^{-\frac{1}{\theta_R} - 1} 1_{W \leq 1} \right) & \text{if } r \in (g - \tau, \rho) \quad (\text{i.e., } \theta_R < 0 < \theta_E) \\ \frac{1}{\theta_E} W^{-\frac{1}{\theta_E} - 1} 1_{W \geq 1} & \text{if } r = \rho \quad (\text{i.e., } 0 = \theta_R < \theta_E) \\ \frac{1}{\theta_E - \theta_R} \left( W^{-\frac{1}{\theta_E} - 1} 1_{W \geq 1} - W^{-\frac{1}{\theta_R} - 1} 1_{W \geq 1} \right) & \text{if } r \in (\rho, r^*) \quad (\text{i.e., } 0 < \theta_R < \theta_E) \\ \frac{1}{\theta_E^2} \log(W) W^{-\frac{1}{\theta_E} - 1} 1_{W \geq 1} & \text{if } r = r^* \quad (\text{i.e., } 0 < \theta_R = \theta_E) \\ \frac{1}{\theta_R - \theta_E} \left( W^{-\frac{1}{\theta_R} - 1} 1_{W \geq 1} - W^{-\frac{1}{\theta_E} - 1} 1_{W \geq 1} \right) & \text{if } r \in (r^*, r_E) \quad (\text{i.e., } 0 < \theta_E < \theta_R) \\ \frac{1}{\theta_R} W^{-\frac{1}{\theta_R} - 1} 1_{W \geq 1} & \text{if } r = r_E \quad (\text{i.e., } 0 = \theta_E < \theta_R) \\ \frac{1}{\theta_R - \theta_E} \left( W^{-\frac{1}{\theta_E} - 1} 1_{W \leq 1} + W^{-\frac{1}{\theta_R} - 1} 1_{W \geq 1} \right) & \text{if } r \in (r_E, \rho + \eta) \quad (\text{i.e., } \theta_E < 0 < \theta_R). \end{cases}$$

We are interested in the total proportion of agents with wealth higher than  $W$ ; that is,

$$\mathbb{P}(\text{Wealth} > W) = \int_W^\infty (p_E(W') + p_R(W')) dW'.$$

The expressions for  $p_E(\cdot)$  and  $p_R(\cdot)$  obtained above imply:

$$\mathbb{P}(\text{Wealth} > W) \sim \begin{cases} \left( \frac{\eta}{\tau + \eta} q^{\frac{1}{\theta_E}} + \frac{\tau}{\tau + \eta} \frac{\theta_E}{\theta_E - \theta_R} \right) W^{-\frac{1}{\theta_E}} & \text{if } r \in (g - \tau, r^*) \quad (\text{i.e., } \theta_R < \theta_E) \\ \frac{\tau}{\tau + \eta} \frac{1}{\theta_E} \log(W) W^{-\frac{1}{\theta_E}} & \text{if } r = r^* \quad (\text{i.e., } \theta_R = \theta_E) \\ \frac{\tau}{\tau + \eta} \frac{\theta_R}{\theta_R - \theta_E} W^{-\frac{1}{\theta_R}} & \text{if } r \in (r^*, \rho + \eta) \quad (\text{i.e., } \theta_R > \theta_E). \end{cases}$$

as  $W \rightarrow \infty$ .<sup>49</sup> Taking logarithms, this simplifies to:

$$\lim_{W \rightarrow \infty} \frac{\log \mathbb{P}(\text{Wealth} > W)}{\log W} = -1 / \max(\theta_E, \theta_R).$$

By definition, this means that the distribution of wealth has a Pareto tail with Pareto inequality  $\theta = \max(\theta_E, \theta_R)$ .

Finally, note that the relative proportion of entrepreneurs among agents with wealth  $W$  is

$$\lim_{W \rightarrow +\infty} \frac{p_E(W)}{p_E(W) + p_R(W)} = \begin{cases} \left( 1 + \frac{\tau}{\eta} q^{-\frac{1}{\theta_E}} \frac{\theta_E}{\theta_E - \theta_R} \right)^{-1} & \text{if } r < r^* \\ 0 & \text{if } r \geq r^*. \end{cases}$$

Hence, the relative mass of entrepreneurs in the right tail converges to a positive number in the right tail if and only if we are in the entrepreneur regime. *Q.E.D.*

<sup>48</sup>Indeed,  $\theta_E(r_E) = 0$  is equivalent to

$$\frac{g - i/q(r_E) - \rho}{\eta + \tau} = 0 \iff g - i \frac{r_E - (g - \tau)}{\tau - i} - \rho = 0.$$

Note that, depending on parameter values,  $r_E$  may be below or above  $\rho + \eta$ . Our notation  $(a, b)$  should be understood as the empty set when  $a \geq b$ .

<sup>49</sup>Here, and in the rest of the paper,  $f(x) \sim g(x)$  as  $x \rightarrow \infty$  for two functions  $f$  and  $g$  means  $\lim_{x \rightarrow \infty} f(x)/g(x) = 1$ .



## A.2. Closing the economy

*Agents.* Suppose that the economy now also includes “workers”. Workers have log utility with a subjective discount factor  $\rho_L$ . Like entrepreneurs, workers are also born with trees, but they immediately sell them and instead hold a diversified portfolio of trees. Denote by  $\pi$  the proportion of newborn that are entrepreneurs.

*State variables.* Denote by  $K$  the average size of a tree in the economy. The law of motion for  $K$  is

$$\dot{K} = (g - \tau)K + \eta(1 - K). \quad (29)$$

Denote  $W_{ER}$  to be the per-capita wealth of entrepreneurs and rentiers,  $W_L$  to be the per-capita wealth of workers, and  $x = \pi W_{ER} / (\pi W_{ER} + (1 - \pi)W_L)$  to be the fraction of aggregate wealth owned by entrepreneurs and rentiers (as opposed to workers). The law of motion of  $x$  is given by:

$$\begin{aligned} \dot{x} &= x(1 - x) \left( \frac{\dot{W}_{ER}}{W_{ER}} - \frac{\dot{W}_L}{W_L} \right), \\ &= x(1 - x) \left( \left( r - \rho + \eta \left( \frac{\pi}{xK} - 1 \right) \right) - \left( r - \rho_L + \eta \left( \frac{1 - \pi}{(1 - x)K} - 1 \right) \right) \right), \\ &= x(1 - x) \left( \rho_L - \rho + \eta \frac{1}{K} \left( \frac{\pi}{x} - \frac{1 - \pi}{1 - x} \right) \right). \end{aligned} \quad (30)$$

Intuitively, the change in  $x$  depends on the difference in subjective discount factors between the two groups as well the difference in the wealth of their newborns. The steady-state is characterized by  $\dot{x} = \dot{K} = 0$ , which gives, after combining (29) and (30),

$$\rho_L - \rho + (\eta + \tau - g) \left( \frac{\pi}{x} - \frac{1 - \pi}{1 - x} \right) = 0. \quad (31)$$

This is a quadratic equation in  $x$  which has one and only one solution  $x \in (0, 1)$ , which pins down the steady-state wealth share  $x$  corresponding to a given value  $\rho_L$ . Product market clearing requires that aggregate consumption equals output minus investment:

$$(x\rho + (1 - x)\rho_L)q = \tau - i. \quad (32)$$

This equation pins down  $q$  as a function of  $x$ . Finally, in steady-state, Equation 2 holds, which also gives the steady-state interest rate  $r$  as a function of  $x$ :

$$r = x\rho + (1 - x)\rho_L + g - \tau. \quad (33)$$

The next proposition shows that, when  $\pi$  is close enough to zero (i.e., entrepreneurs account for a small share of the total population), changes in  $\rho_L$  can generate the full spectrum of interest rates considered in Proposition 1.

**PROPOSITION 4:** *Denote by  $r_\pi(\rho_L)$  the equilibrium interest rate as a function of the subjective discount factor of workers. The following are true:*

1.  $r_\pi(\cdot)$  is an increasing function of  $\rho_L$ ;

2. As  $\pi$  tends to zero,  $r_\pi(\cdot)$  spans the interval  $(g - \tau, \rho + \eta)$ :

$$\begin{aligned}\lim_{\pi \rightarrow 0} \lim_{\rho_L \rightarrow 0} r_\pi(\rho_L) &= g - \tau, \\ \lim_{\pi \rightarrow 0} \lim_{\rho_L \rightarrow +\infty} r_\pi(\rho_L) &= \rho + \eta;\end{aligned}$$

3. As long as  $\rho < (\tau - i) \frac{\eta - (g - \tau)}{\eta}$ , there exists  $\pi$  small enough that the distribution of workers always has a thinner tail than the distribution of entrepreneurs or rentiers. In this case, Proposition 1 gives Pareto inequality  $\theta$  for the full distribution of entrepreneurs, rentiers, and workers.

PROOF OF PROPOSITION 4: Denote by  $x_\pi(\rho_L)$  the steady state share of wealth owned by entrepreneurs and rentiers. Equation (31) implies that  $x_\pi(\cdot)$  is increasing in  $\rho_L$ . The fact that the growth of the average wealth in the group of entrepreneurs and rentiers must be zero means that  $r_\pi(\cdot)$  is increasing in  $x_\pi(\cdot)$ :

$$0 = r_\pi(\rho_L) - \rho + \eta \left( \frac{\pi}{x_\pi(\rho_L)K} - 1 \right). \quad (34)$$

Combining both results gives us that  $r_\pi(\cdot)$  increases in  $\rho_L$ .

To prove the second part of the proposition, note that Equation 31 implies the following expression for  $x$  as a function of  $\rho_L$ :

$$x_\pi(\rho_L) = \begin{cases} \frac{1}{2} \left( 1 + \vartheta(\rho_L) - \sqrt{(1 + \vartheta(\rho_L))^2 - 4\vartheta(\rho_L)\pi} \right) & \text{if } 0 < \rho_L < \rho \\ \pi & \text{if } \rho_L = \rho \\ \frac{1}{2} \left( 1 + \vartheta(\rho_L) + \sqrt{(1 + \vartheta(\rho_L))^2 - 4\vartheta(\rho_L)\pi} \right) & \text{if } \rho_L > \rho, \end{cases}$$

where  $\vartheta(\rho_L) \equiv (\eta - (g - \tau)) / (\rho - \rho_L)$ . Taking the limit with respect to  $\rho_L$  gives:

$$\begin{aligned}\lim_{\rho_L \rightarrow 0} x_\pi(\rho_L) &= \frac{1}{2} \left( 1 + \vartheta(0) - \sqrt{(1 + \vartheta(0))^2 - 4\vartheta(0)\pi} \right), \\ \lim_{\rho_L \rightarrow +\infty} x_\pi(\rho_L) &= 1.\end{aligned}$$

In turn, taking the limit  $\pi \rightarrow 0$  gives us

$$\lim_{\pi \rightarrow 0} \lim_{\rho_L \rightarrow 0} x_\pi(\rho_L) = 0, \quad \lim_{\pi \rightarrow 0} \lim_{\rho_L \rightarrow +\infty} x_\pi(\rho_L) = 1.$$

Therefore, in terms of the interest rate, we obtain

$$\lim_{\pi \rightarrow 0} \lim_{\rho_L \rightarrow 0} r_\pi(\rho_L) = g - \tau, \quad \lim_{\pi \rightarrow 0} \lim_{\rho_L \rightarrow +\infty} r_\pi(\rho_L) = \rho + \eta,$$

where the first limit uses (33) while the second limit uses (34).

To conclude the proof, it remains to show that, if  $\rho < (\tau - i) \frac{\eta - (g - \tau)}{\eta}$ , workers never dominate the right tail if  $\pi$  is small enough. To see this, denote  $\theta_{L,\pi}(\rho_L)$  to be Pareto inequality for workers as a function of  $\rho_L$ :

$$\theta_{L,\pi}(\rho_L) = \frac{r_\pi(\rho_L) - \rho_L}{\eta} = 1 - \frac{1 - \pi}{1 - x_\pi(\rho_L)} \frac{1}{\eta + \tau - g}.$$

Since  $x_\pi(\cdot)$  is increasing in  $\rho_L$ ,  $\theta_{L,\pi}(\cdot)$  is decreasing in  $\rho_L$ , and, therefore, it is bounded by its limit as  $\rho_L \rightarrow 0$ . We can express this upper bound in terms of exogenous parameters:

$$\lim_{\rho_L \rightarrow 0} \theta_{L,\pi}(\rho_L) = \frac{\lim_{\rho_L \rightarrow 0} r_\pi(\rho_L)}{\eta} = \frac{g - \tau + \rho \frac{1 + \vartheta(0) - \sqrt{(1 + \vartheta(0))^2 - 4\vartheta(0)\pi}}{2}}{\eta},$$

where the equality line uses (33)

On the other hand, as seen in the proof of Proposition 1, Pareto inequality for entrepreneurs/rentiers,  $\max(\theta_{E\pi}(\cdot), \theta_{R\pi}(\cdot))$ , reaches its minimum for  $r = r^*$  at  $\frac{g-i-\rho}{\eta+\tau-i}$ . Putting the two results together, we get that a sufficient condition for entrepreneurs and rentiers to always dominate the right tail (i.e., for  $\max(\theta_{E\pi}(\cdot), \theta_{R\pi}(\cdot))$  to be higher than  $\theta_{L,\pi}$ ) is

$$\frac{g - i - \rho}{\eta + \tau - i} \geq \frac{g - \tau + \rho \frac{1 + \vartheta(0) - \sqrt{(1 + \vartheta(0))^2 - 4\vartheta(0)\pi}}{2}}{\eta}. \quad (35)$$

Taking the limit  $\pi \rightarrow 0$ , this inequality converges to:

$$\frac{g - i - \rho}{\eta + \tau - i} \geq \frac{g - \tau}{\eta} \iff \rho \leq (\tau - i) \frac{\eta - (g - \tau)}{\eta}.$$

Therefore, as long as  $\rho < (\tau - i) \frac{\eta - (g - \tau)}{\eta}$ , Equation 35 is satisfied for  $\pi$  small enough, which implies that entrepreneurs and rentiers dominate the right tail.

*Q.E.D.*

### A.3. Aggregate growth

In the stylized model, the balanced growth path features zero per-capita output growth. We now consider an extension with stochastic aggregate growth. The key takeaway is that the main equations remain the same after redefining  $r$  to be the required return on wealth *net of expected growth per-capita* and  $g$  to be the growth of trees *net of growth per-capita*.

We now describe the environment and state the main result. Consider a “productivity process”  $A_t$  that follows a geometric Brownian motion:

$$\frac{dA_t}{A_t} = g dt + \sigma dZ_t,$$

where  $g$  denotes the drift,  $\sigma$  denotes the volatility, and  $Z_t$  is an aggregate Brownian motion. We assume that output per-capita scales with  $A_t$ : more precisely, we assume that trees born at time  $t$  have an initial size  $A_t$ , and that trees grow by  $g dt + \frac{dA_t}{A_t}$  between  $t$  and  $t + dt$  (i.e.,  $g$  corresponds to the growth rate of the tree relative to the growth rate of  $A_t$ ). Note that, in this economy, the average size of a tree in the economy scales with  $A_t$ .<sup>50</sup>

We assume the existence of an exogenous stochastic discount factor, denoted by  $\Lambda_t$ , that follows a geometric Brownian motion:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_f dt - \kappa dZ_t,$$

<sup>50</sup>Note that this satisfies Kaldor’s fact that the aggregate quantity of capital (i.e., the aggregate quantity of trees) grows at the same rate as output.

where  $r_f$  corresponds to the risk-free rate and  $\kappa$  corresponds to the market price of aggregate risk. Note that we assume that only aggregate risk is priced, which reflects the fact that idiosyncratic risk (i.e., the random time at which the tree matures) can be diversified away by investors.

The instantaneous return of holding a tree is given by

$$\frac{dR_t}{R_t} = \begin{cases} \left(g - \frac{i}{q}\right) dt + \frac{dA_t}{A_t} & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T, \end{cases}$$

where  $T$  denotes the stochastic time at which the tree blossoms. The market pricing equation in continuous-time is  $\mathbb{E}_t[d(\Lambda_t R_t)] = 0$ , which gives

$$\underbrace{r_f + \kappa\sigma - g}_{\text{Required return net of agg. growth per capita}} = \underbrace{g - \frac{i}{q} + \tau \left(\frac{1}{q} - 1\right)}_{\text{Expected return net of agg. growth per capita}}.$$

As in the stylized model, the evolution of wealth is given by

$$\frac{dW_t}{W_t} = \begin{cases} \left(g - \frac{i}{q} - \rho\right) dt + \frac{dA_t}{A_t} & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T \\ (r_f + \kappa\sigma - \rho) dt + \sigma dZ_t & \text{if } t > T. \end{cases}$$

In the presence of aggregate growth, we are interested in the distribution of wealth normalized by the average wealth in the economy. The dynamics of this normalized wealth is:

$$\frac{dW_t}{W_t} - \frac{dA_t}{A_t} = \begin{cases} \left(g - \frac{i}{q} - \rho\right) dt & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T \\ (r_f + \kappa\sigma - g - \rho) dt & \text{if } t > T. \end{cases}$$

Hence, the equations for asset prices and wealth dynamics correspond exactly to the respective equations in the baseline model after defining  $r = r_f + \kappa\sigma - g$ . In particular, the expression for Pareto inequality remains the same as in Proposition 1. Overall, we conclude that the stylized model can be interpreted as a growing and stochastic economy: the only change is that  $r$  should be interpreted as the required return on trees net of expected per-capita growth, and  $g$  should be interpreted as the growth rate of trees relative to per-capita growth.

## Online Appendix

### APPENDIX B: APPENDIX FOR SECTION 3

#### B.1. *Sufficient statistic in the rentier regime*

So far, we have derived our sufficient statistic under the assumption that we are in the entrepreneur regime. We now show that that the sufficient statistic in words (i.e., Equation 8) also holds in the rentier regime. Rentiers own a diversified portfolio. Their portfolio can be seen as a “representative” tree with payout  $\tau$  (the cash flow due to the fraction of trees that blossom every period) minus  $i$  (the negative cash flow due to the investment in existing trees) that grows at rate  $g - \tau$  (the growth rate of trees that keep growing minus the fraction of trees that blossom). This implies a payout yield  $(\tau - i)/q$  and a growth rate  $g - \tau$ . Note that the return on wealth  $r$  can be written as

$$r = \underbrace{\frac{\tau - i}{q}}_{\text{Payout yield}} + \underbrace{g - \tau}_{\text{Growth rate of cash flows}}$$

Plugging this expression for  $r$  in the expression for Pareto inequality (see Proposition 1), we obtain

$$\theta = \max \left( \frac{g - \frac{i}{q} - \rho}{\eta + \tau}, \frac{g - \tau + \frac{\tau - i}{q} - \rho}{\eta} \right).$$

Differentiating with respect to  $r$  gives us

$$\partial_r \log \theta = \begin{cases} \frac{-\frac{i}{q}(-\partial_r \log q)}{g - \frac{i}{q} - \rho} & \text{for } r < r^* \text{ (entrepreneur regime)} \\ \frac{\frac{\tau - i}{q}(-\partial_r \log q)}{r - \rho} & \text{for } r > r^* \text{ (rentier regime)}. \end{cases}$$

The key takeaway is that the sufficient statistic (8) holds regardless of whether we are in the entrepreneur or rentier regime. In other words, the effect of  $r$  on Pareto inequality can be written as

$$\partial_r \log \theta = \frac{\text{Payout yield} \times \text{Duration}}{\text{Growth rate of wealth}} \quad (36)$$

for households reaching the right tail of the wealth distribution (see Appendix B.3 for a generalization of this insight). The only special thing about rentiers is that, because the “representative tree” they own has a constant growth rate, the numerator in the sufficient statistic is equal to one. Mathematically, this comes from the fact that the payout yield of the representative tree is exactly equal to the inverse of the duration (see Footnote 33).

#### B.2. *Return on wealth versus return on capital*

Following the notations in the main text, the definition of the net return on capital in the endogenous investment extension is

$$\text{rok} \equiv \underbrace{a}_{\text{Production efficiency}} + \underbrace{g - \iota(g)}_{\text{Investment efficiency}}. \quad (37)$$

The net return on capital is the sum of production efficiency  $a$  (i.e., how much gross output is produced per unit of capital) and investment efficiency  $g - \iota(g)$  (i.e., the difference between the growth rate of capital and the investment rate). This definition of the net return on capital fully summarizes the technological contribution of a firm to aggregate net value-added, and is consistent with the System of National Accounts.<sup>51</sup>

Plugging this definition into the expression for  $q$  given in (9), we can write:

$$q = 1 + \frac{\text{rok} - r}{r + \tau - g}. \quad (38)$$

The second term in (38) represents the present value of rents. Indeed, from the perspective of a firm owner, the *average* return on investment is  $\text{rok}$  while the *marginal* return on investment is  $r$ , which means that investment generates Ricardian rents that accrue to firm owners.<sup>52</sup> Note that the presence of a convex investment adjustment cost function allows  $\text{rok} > r$  to be sustained in equilibrium. The reason is that convex adjustment costs generate decreasing returns to scale in investment. Hence, rents accrue to firm owners due to the fact that their technology can not be costlessly scaled to accommodate the supply of savings. We make this point quantitatively in Section 5, where we simulate a rise in savings in a general equilibrium model.

Note that we can write the return on wealth for entrepreneurs, while their firms keep growing, as

$$\frac{dR_t}{R_t} = \text{rok} dt + (g - \text{rok}) \left(1 - \frac{1}{q}\right) dt. \quad (39)$$

When entrepreneurs do not use external financing (i.e.,  $g = \text{rok}$ , or equivalently  $a = \iota(g)$ ), their return on wealth is equal to the return on capital. However, when they use external financing (i.e.,  $g > \text{rok}$ , or equivalently  $a < \iota(g)$ ), their return on wealth differs from the return on capital. In particular, their return on wealth *exceeds* the return on capital whenever  $q > 1$ . This comes from the fact that part of an entrepreneur's return comes from selling shares to outsiders. This is the key insight of our paper: for a given return on capital, a low cost of capital  $r$  benefits entrepreneurs who raise external financing.

### B.3. Extension: heterogeneous firm dynamics

**PROOF OF PROPOSITION 2:** Denote  $\omega$  log wealth,  $p_{Et}(\omega)$  the joint density of log wealth and productivity state for entrepreneurs (an  $S \times 1$  vector), and  $p_{Rt}(\omega)$  the density of log wealth for rentiers. Moreover, denote  $m_{Et}(\xi)$ ,  $m_{Rt}(\xi)$  the corresponding moment generating function

<sup>51</sup>The net return on capital is defined as net capital income over capital. In the National Accounts, net capital income is the sum of gross profits minus capital depreciation. Using our notation, gross profits are  $aK$  and the identity that implicitly defines the depreciation rate is

$$K_{t+1} - K_t = -\text{depreciation rate} \times K_t + I_t,$$

where  $K_t$  is capital and  $I_t$  is investment. Hence, the “depreciation rate” in our model is  $\iota(g) - g$ . Putting together, we have that the net return on capital in our model is  $a - (\iota(g) - g)$ , which coincides with (37).

<sup>52</sup>See Cochrane (1991) for a formal proof that the first order condition for investment in q-theory models (i.e.,  $\iota'(g) = q$  using our notation) implies that the marginal return on investment is equal to the discount rate (i.e.,  $r$  using our notation). Note that, at the aggregate level,  $(\text{rok} - r)K$  is the total value of “pure profits” that accrue to firm owners (e.g., Barkai, 2020; Karabarbounis and Neiman, 2019, and Guin-Bonenfant, 2022). In our model, the pure profits are due to Ricardian rents, but, in general, they could also be due to market power.

for wealth for each type of agent:

$$\mathbf{m}_{Et}(\xi) \equiv \int_{\mathbb{R}} e^{\xi\omega} \mathbf{p}_{Et}(\omega) d\omega, \quad m_{Rt}(\xi) \equiv \int_{\mathbb{R}} e^{\xi\omega} p_{Rt}(\omega) d\omega.$$

Applying the Laplace transform on the Kolmogorov forward equation gives the dynamics of these functions over time:

$$\partial_t \mathbf{m}_{Et}(\xi) = \mathcal{D}(\mathbf{q})^\xi (\xi \mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}' - (\eta + \tau)\mathcal{I}) \mathcal{D}(\mathbf{q})^{-\xi} \mathbf{m}_{Et}(\xi) + \eta \mathcal{D}(\mathbf{q})^\xi \boldsymbol{\psi}, \quad (40)$$

$$\partial_t m_{Rt}(\xi) = (\xi(r - \rho) - \eta) m_{Rt}(\xi) + \tau \mathbf{1}' \mathbf{m}_{Et}(\xi), \quad (41)$$

where  $\mathbf{q} \equiv (q_1, \dots, q_S)'$  is the vector of prices (i.e., the solution to the HJB, see Equation 10),  $\boldsymbol{\psi} = (\psi_1, \dots, \psi_S)'$  is the distribution of firm types at birth,  $\mathcal{D}(\boldsymbol{\mu})$  is the diagonal matrix with diagonal elements given by the vector  $\boldsymbol{\mu}$ , and  $\mathcal{I}$  is the identity matrix.

Denote  $p_t(\omega)$  the overall density of log wealth; that is,  $p_t(\omega) = \mathbf{1}' \mathbf{p}_{Et}(\omega) + p_{Rt}(\omega)$ . Hence, the moment generating function for wealth is given by:

$$m_t(\xi) = \mathbf{1}' \mathbf{m}_{Et}(\xi) + m_{Rt}(\xi).$$

We are interested in characterizing the limit of  $m_t(\xi)$  as  $t \rightarrow \infty$  (stationary economy). Our assumption that there exists at least one state  $s$  such that the rate of capital accumulation is positive (i.e.,  $\mu_s > 0$ ) implies the existence of a unique  $\theta_E > 0$  such that  $\rho \left( \frac{1}{\theta_E} \mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}' \right) = \eta + \tau$  (see Proposition 2 in Beare and Toda, 2022). This allows us to characterize the limit  $m_t(\xi)$  as time tends to infinity:

$$m(\xi) \equiv \lim_{t \rightarrow +\infty} m_t(\xi) = \left( 1 + \frac{\tau}{\eta - \xi(r - \rho)} \right) \mathbf{1}' \mathcal{D}(\mathbf{q})^\xi \left( (\eta + \tau)\mathcal{I} - \xi \mathcal{D}(\boldsymbol{\mu}) - \mathcal{T}' \right)^{-1} \eta \boldsymbol{\psi}$$

if  $0 \leq \xi < \min(\frac{\eta}{r - \rho}, \frac{1}{\theta_E})$ , and infinity if  $\xi \geq \min(\frac{\eta}{r - \rho}, \frac{1}{\theta_E})$ . That is,  $m$  has a pole at  $\min(\frac{\eta}{r - \rho}, \frac{1}{\theta_E})$ . Using Theorem 3.1 in Beare et al. (2021), this implies that the long-run wealth distribution has a right Pareto tail with Pareto inequality  $\theta = \max(\theta_E, \frac{r - \rho}{\eta})$ . *Q.E.D.*

**PROOF FOR PROPOSITION 3:** We prove the proposition in three steps. We first prove two distinct lemmas, which are combined in the third step.

*Step 1* Denote  $\mathbf{u}(\theta, r)$  and  $\mathbf{v}(\theta, r)$  the left and right eigenvectors associated with the dominant eigenvalue of the matrix  $\frac{1}{\theta} \mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}$ , normalized so that  $\mathbf{u}' \mathbf{1} = \mathbf{u}' \mathbf{v} = 1$ . As proven in Proposition 2, Pareto inequality  $\theta$  is implicitly characterized by the following equation

$$\left( \frac{1}{\theta} \mathcal{D}(\boldsymbol{\mu}) + \mathcal{T} \right) \mathbf{v}(\theta, r) = (\eta + \tau) \mathbf{v}(\theta, r).$$

Differentiating with respect to  $r$  gives

$$\left( \frac{1}{\theta} \mathcal{D}(\partial_r \boldsymbol{\mu}) - \frac{1}{\theta^2} \mathcal{D}(\boldsymbol{\mu}) \partial_r \theta \right) \mathbf{v} + \left( \frac{1}{\theta} \mathcal{D}(\boldsymbol{\mu}) + \mathcal{T} \right) (\partial_r \mathbf{v} + \partial_\theta \mathbf{v} \partial_r \theta) = (\eta + \tau) (\partial_r \mathbf{v} + \partial_\theta \mathbf{v} \partial_r \theta).$$

Left-multiplying by  $\mathbf{u}$  gives:

$$\mathbf{u}' \left( \frac{1}{\theta} \mathcal{D}(\partial_r \boldsymbol{\mu}) - \frac{1}{\theta^2} \mathcal{D}(\boldsymbol{\mu}) \partial_r \theta \right) \mathbf{v} = 0,$$

where we used the fact that, by definition of  $\mathbf{u}$ ,  $\mathbf{u}'(\frac{1}{\theta}\mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}) = \mathbf{u}'(\eta + \tau)$ .

A similar derivation gives that  $\mathbf{u}'\mathcal{D}(\boldsymbol{\mu})\mathbf{v}$  can be seen as the derivative of  $\xi \rightarrow \varrho(\xi\mathcal{D}(\boldsymbol{\mu}) + \mathcal{T})$  at  $\xi = 1/\theta$ , which is a strictly convex function (see [Beare et al., 2021](#)). Hence, we can divide the previous equation by  $\mathbf{u}'\mathcal{D}(\boldsymbol{\mu})\mathbf{v}$  to obtain:

$$\partial_r \log \theta = \frac{\mathbf{u}'\mathcal{D}(\partial_r \boldsymbol{\mu})\mathbf{v}}{\mathbf{u}'\mathcal{D}(\boldsymbol{\mu})\mathbf{v}} = \frac{(\mathbf{u} \circ \mathbf{v})'\partial_r \boldsymbol{\mu}}{(\mathbf{u} \circ \mathbf{v})'\boldsymbol{\mu}}, \quad (42)$$

where  $\mathbf{u} \circ \mathbf{v}$  denotes the multiplication element-wise. Equation (42) says that the semi-elasticity of Pareto inequality with respect to  $r$  can be written as the ratio between the average derivative of the growth rate of wealth  $\partial_r \mu$  and the average growth rate of wealth  $\mu$ , where the average is taken with respect to some density across productivity states  $\mathbf{u} \circ \mathbf{v}$ .<sup>53</sup> The rest of the proof shows that this density can be interpreted as the density of past states for individuals in the right tail of the wealth distribution.

*Step 2.* Consider a function  $f$  defined on the set of states  $\{1, \dots, S\}$ . For an individual  $i$  in the wealth distribution, denote  $F_i = \int_0^{a_i} f(s_{ia}) da$  the cumulative sum of  $f(s_{ia})$  since birth. Denote  $\mathbf{p}_E(\omega, F)$  the cross-sectional density of productivity state  $s$ , log wealth  $\omega$ , and  $F$  for entrepreneurs. Denote  $\mathbf{m}_E(\omega, \beta) \equiv \int_{\mathbb{R}} e^{\beta F} \mathbf{p}_E(\omega, F) dF$  the moment generating function of  $F$  and  $\tilde{\mathbf{m}}_E(\xi, \beta) \equiv \int_{\mathbb{R}^2} e^{\xi\omega + \beta F} \mathbf{p}_E(\omega, F) d\omega dF$  the joint moment generating function of  $F$  and  $\omega$ .

Applying the Laplace transform on the Kolmogorov-Forward equation for  $\mathbf{p}_E(\omega, F)$  gives a closed form solution for  $\tilde{\mathbf{m}}_E$ :

$$0 = \mathcal{D}(\mathbf{q})^\xi (\beta\mathcal{D}(\mathbf{f}) + \xi\mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}' - (\eta + \tau)\mathcal{I}) \mathcal{D}(\mathbf{q})^{-\xi} \tilde{\mathbf{m}}_E(\xi, \beta) + \eta\mathcal{D}(\mathbf{q})^\xi \boldsymbol{\psi}, \quad (43)$$

where  $\mathbf{f} \equiv (f(s_1), \dots, f(s_S))$ .

We know that  $\varrho(\xi\mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}) = \eta + \tau$  has a unique positive solution, given by  $\xi = 1/\theta$ . Hence, for  $\beta$  close enough to zero,  $\varrho(\beta\mathcal{D}(\mathbf{f}) + \xi\mathcal{D}(\boldsymbol{\mu}) + \mathcal{T}) = \eta + \tau$  also has a unique positive solution, which we denote  $\xi^*(\beta)$ . Given (43), this implies that  $\xi \rightarrow \tilde{\mathbf{m}}_E$  has a pole in  $\xi^*(\beta)$ . As shown in [Beare et al. \(2021\)](#), this implies

$$\log \mathbf{m}_E(\omega, \beta) \sim -\xi^*(\beta)\omega \mathbf{1} \text{ as } \omega \rightarrow +\infty. \quad (44)$$

Finally, note that the expectation of  $F$  is related to the derivative of  $\mathbf{m}_E$  at zero:

$$\mathbb{E}[F_i | \omega_i = \omega, s_i = s] = \frac{\int_{\mathbb{R}} F p_{Es}(\omega, F) dF}{\int_{\mathbb{R}} p_{Es}(\omega, F) dF} = \partial_{\beta=0} \log m_{Es}(\omega, \beta),$$

where  $p_{Es}$  and  $m_{Es}$  denote the  $s^{th}$  element of the vectors  $\mathbf{p}_E$  and  $\mathbf{m}_E$ . Combining with (44) gives:

$$\mathbb{E}[F_i | \omega_i = \omega] \sim -\xi^{*'}(0)\omega \text{ as } \omega \rightarrow +\infty.$$

A similar derivation as in Step 1 gives  $\xi^{*'}(0) = -((\mathbf{u} \circ \mathbf{v})'\mathbf{f})/((\mathbf{u} \circ \mathbf{v})'\boldsymbol{\mu})$ , which implies:

$$\mathbb{E}[F_i | \omega_i = \omega] \sim \frac{(\mathbf{u} \circ \mathbf{v})'\mathbf{f}}{(\mathbf{u} \circ \mathbf{v})'\boldsymbol{\mu}} \omega \text{ as } \omega \rightarrow +\infty.$$

<sup>53</sup>Note that  $\mathbf{u} \circ \mathbf{v}$  corresponds to a density as  $\mathbf{u}$  and  $\mathbf{v}$  are positive element-wise (they correspond to the eigenvectors associated with the dominant eigenvalue) and as we used the normalization  $\mathbf{u}'\mathbf{v} = 1$ .



We refer the reader to [Lecomte \(2007\)](#) and [Chetrite and Touchette \(2015\)](#) for similar derivations in the context of large deviation theory. *Step 3.* Combining (42) with the previous formula in the special case  $f(s) = \partial_r \mu_s$  gives

$$\mathbb{E} \left[ \int_0^{a_i} \partial_r \mu_{s_{ia}} da \mid \omega_i = \omega \right] \sim (\partial_r \log \theta) \omega \text{ as } \omega \rightarrow +\infty,$$

which implies

$$\begin{aligned} \partial_r \log \theta &= \lim_{\omega \rightarrow \infty} \frac{1}{\omega} \mathbb{E} \left[ \int_0^{a_i} \partial_r \mu_{s_{ia}} da \mid \omega_i = \omega \right] \\ &= \lim_{W \rightarrow \infty} \mathbb{E} \left[ \frac{\frac{1}{a_i} \int_0^{a_i} \partial_r \mu_{s_{ia}} da}{\frac{1}{a_i} \int_0^{a_i} \mu_{s_{ia}} da} \mid W_i = W \right]. \end{aligned}$$

*Q.E.D.*

#### B.4. Extension: debt issuance

We now provide a derivation of Equation 15. Similarly to the stylized model, we have, in the entrepreneur regime

$$\theta = \frac{-\frac{i_\lambda}{q_\lambda} + g - \rho}{\eta + \tau}.$$

Hence, the effect of a small change in the required return on debt  $dr_f$  and in the required return on unlevered equity  $dr$  on  $\theta$  is:

$$d \log \theta = d \log \left( -\frac{i_\lambda}{q_\lambda} + g - \rho \right) = \frac{d \left( -\frac{i_\lambda}{q_\lambda} \right)}{-\frac{i_\lambda}{q_\lambda} + g - \rho}, \quad (45)$$

where the second equality uses the fact that  $g$  and  $\rho$  are exogenous parameters. Differentiating (13) and (14) gives

$$\begin{aligned} di_\lambda &= (\lambda - 1) dr_f, \\ d \log q_\lambda &= \lambda_M d \log q, \end{aligned}$$

where  $\lambda_M \equiv \lambda q / q_\lambda$  denotes market leverage. Finally, Equation (2) gives  $d \log q = \partial_r \log q dr$ ; that is, the value of  $q$  only depends on  $r$ , not  $r_f$ . Combining these equations gives

$$\begin{aligned} d \left( -\frac{i_\lambda}{q_\lambda} \right) &= -\frac{di_\lambda}{q_\lambda} + \frac{i_\lambda}{q_\lambda} d \log q_\lambda \\ &= -\frac{\lambda - 1}{q_\lambda} dr_f - \frac{i_\lambda}{q_\lambda} \lambda_M (-\partial_r \log q) dr \\ &= -(\lambda_M - 1) dr_f - \frac{i_\lambda}{q_\lambda} \lambda_M (-\partial_r \log q) dr. \end{aligned}$$

Combining with (45) and rearranging gives (15).

### B.5. Constraints on external financing

We now consider an extension of the investment model described in [Endogenous investment](#) with constraints on the amount of external financing. Formally, we assume that the book leverage of the firm must be  $\lambda$  (a constraint on debt issuance) and that the equity payout yield must be higher than a certain threshold  $-B$  (a constraint on equity issuance). Formally, the investment problem faced by entrepreneur is now

$$\begin{aligned} r q &= \max_g \left\{ a - \iota(g) + g q + \tau(1 - q) \right\} \\ \text{s.t.} \quad i_\lambda &\leq B q_\lambda, \end{aligned} \tag{46}$$

where, similarly to the leverage extension,  $\lambda$  denotes book leverage,  $i_\lambda = g - r_f + \lambda(\iota(g) - a - (g - r_f))$  denotes the flow of equity financing as a share of book equity, and  $q_\lambda = 1 + \lambda(q - 1)$  denotes the market value of equity divided by its book value.

The baseline model can be seen as a special case where  $B = +\infty$  (i.e., no constraint on equity issuance). Another special case often considered in the literature on entrepreneurship is  $B = 0$  (i.e., no equity issuance). More generally, this model allows us to consider the intermediate case  $0 < B < +\infty$ .

Denoting by  $v/\lambda \geq 0$  the Lagrange multiplier on the financial constraint, the first-order condition for  $g$  in (46) gives:

$$(1 + v)(1 + \lambda(\iota'(g) - 1)) = q_\lambda. \tag{47}$$

When the constraint does not bind (i.e.,  $v = 0$ ), we obtain  $\iota'(g) = q$ , as in the model without constraints on external financing. In contrast, when the constraint binds (i.e.,  $v > 0$ ), investment is inefficiently low.

We now assume that the constraint binds (otherwise, this reverts to the model without constraints on external financing). As in the stylized model (see Equation 7), the effect of a change in the required return on debt  $r_f$  and in the required return on (unlevered) equity  $r$  on Pareto inequality is given by the relative change in the growth rate of entrepreneurs:

$$\begin{aligned} d \log \theta &= d \log \left( -\frac{i_\lambda}{q_\lambda} + g - \rho \right) \\ &= \frac{dg}{-\frac{i_\lambda}{q_\lambda} + g - \rho}, \end{aligned}$$

where the second equality uses the fact that the constraint binds. To compute the change  $dg$ , we differentiate the constraint on external financing

$$\begin{aligned} di_\lambda &= B q_\lambda d \log q_\lambda \\ \implies (\lambda - 1) dr_f + (1 + \lambda(\iota'(g) - 1)) dg &= B q_\lambda \lambda_M d \log q \\ \implies (\lambda - 1) dr_f + \frac{q_\lambda}{1 + v} dg &= i_\lambda \lambda_M d \log q \\ \implies dg &= (1 + v) \left( dr_f + \lambda_M \left( \frac{i_\lambda}{q_\lambda} \partial_r \log q dr - dr_f \right) \right). \end{aligned}$$

Plugging this into the expression for  $d \log \theta$  gives:

$$d \log \theta = (1 + v) \frac{dr_f + \lambda_M \left( \frac{i_\lambda}{q_\lambda} \partial_r \log q dr - dr_f \right)}{-\frac{i_\lambda}{q_\lambda} + g - \rho}.$$

This is the same as the sufficient statistic with leverage (Appendix B.4) with one key difference: the formula for the effect of  $r$  on  $\log \theta$  is multiplied by  $(1 + v)$ .

How important is this multiplier? To answer this question, note that the multiplier can be rewritten as  $1 + v = (1 + \lambda(q - 1))/(1 + \lambda(l'(g) - 1))$  using the first-order condition for investment (47). Since  $1 < l'(g) \leq q$ , we obtain that the multiplier is bounded below by 1 and bounded above by  $q_\lambda$ ; that is,  $1 < 1 + v \leq q_\lambda$ . The lower bound is attained in the limit  $B \rightarrow \infty$  (i.e., there are no financial frictions) while the upper bound is attained in the limit  $l'(g) \rightarrow 1$  (i.e., there are no adjustment costs). In the latter case, the constraint on external financing is the only force that keeps the growth rate of the firm from being infinite, as in [Cagetti and De Nardi \(2006\)](#) and [Moll \(2014\)](#).

## APPENDIX C: APPENDIX FOR SECTION 4

### C.1. Estimating the sufficient statistic

#### C.1.1. Methodology

We use annual data from Compustat ([SP Global Market Intelligence, 2023](#)) to estimate the equity payout yield and market leverage of the firms owned by top individuals in the U.S.

*Equity payout yield.* We start by showing that the equity payout yield can be written as the sum of a dividend yield and a buyback yield. Denote by  $CF_t dt$  the amount of cash distributed to equity holders during a time period  $dt$ . This cash can be distributed through dividends or through share repurchases. Denoting  $D_t$  the dividend per share,  $P_t$  the price per share, and  $N_t$  the number of outstanding shares, the following accounting identity holds:

$$CF_t dt = N_t D_t dt - P_t dN_t.$$

Dividing by the market value of the firm equity  $N_t P_t$ , we obtain

$$\underbrace{\frac{CF_t}{N_t P_t} dt}_{\text{Equity payout yield}} = \underbrace{\frac{D_t}{P_t} dt}_{\text{Dividend yield}} - \underbrace{\frac{dN_t}{N_t}}_{\text{Buyback yield}}.$$

This says that the equity payout yield is the sum of the dividend yield and the buyback yield, where the buyback yield is defined as the opposite of the growth of the number of shares. Note that the buyback yield is positive when firms repurchase shares and negative when firms issue shares.

We first describe the construction of the dividend yield. In years in which a company is public, we observe in Compustat the amount of dividends during the year,  $dv_t$ , and the market value of equity at the end of the year  $mkvalt_t$  (or the number of common shares outstanding  $csho_t$  times the price per share  $prcc\_f_t$  if it is missing) in Compustat. We then construct the dividend yield during the year as:

$$\text{Dividend yield}_t = \frac{dv_t}{(mkvalt_{t-1} + mkvalt_t)/2}$$

We winsorize this variable at the 1st and 99th percentile to decrease the effect of measurement errors. Finally, we set the dividend yield to zero in years in which a company is private.

We now describe the construction of the buyback yield in years in which a company is public. We observe in Compustat the number of common shares outstanding  $chso_t \times adj\_f_t$ , where the adjustment accounts for the effect of stock splits.<sup>54</sup> We construct the buyback issuance yield in years in which a company is public as the opposite of the logarithmic growth in its number of common shares outstanding.

Our baseline model assumes that firms redistribute to equity holders the component of earnings that they do not invest in their production technology. In reality, firms may also use this cash to acquire other firms. To account for these additional financial transactions, we adjust our measure of buyback yield by the net cash used for acquisition,  $acq_t - sppe_t$ , divided by firm market equity. Intuitively, this means that we treat similarly a firm repurchasing its own shares and a firm purchasing the shares of another firm's share. Overall, our final measure for the buyback yield in years post-IPO is:

$$\text{Buyback yield}_t = \log \left( \frac{chso_{t-1} \times adj\_f_{t-1}}{chso_t \times adj\_f_t} \right) + \frac{acq - sppe}{(mkvalt_{t-1} + mkvalt_t)/2}$$

We winsorize this variable at the 1st and 99th percentile to decrease the effect of measurement errors.

We estimate the buyback yield in years leading (and including) the IPO using hand-collected data. We rely on the ownership share of founders (immediately post-IPO) as reported on their S-1 forms, denoted  $\Omega$ . Assuming that founders neither sold or purchased shares or received shares as part of their compensation, this ownership share corresponds to the inverse of the cumulative growth in the number of shares since founding date.<sup>55</sup> Overall, our measure for the buyback yield in years leading (and including) the IPO is:

$$\text{Buyback yield}_t = \frac{\log(\Omega)}{t_{\text{IPO}} - t_0}$$

where  $t_{\text{IPO}}$  denotes the year of the IPO and  $t_0$  denotes the year in which the firm was incorporated. Finally, we set the buyback yield of firms that are private in 2015 to zero.

Figure C.1 plots the average annual equity payout yield of firms that are public in 2015 as a function of their age. One can see that the equity payout yield gradually increases, from -10% in early years to 5% in later years. This reflects the fact that, similarly to the trees in the stylized model, firms initially raise cash from equity holders (through equity issuance) and then start paying positive cash flows as they age (through dividend payments and/or equity repurchases).

*Market leverage.* In years in which a company is public, we compute market leverage as the ratio between the market value of assets and the market value of equity. The market value of assets is computed as the market value of equity plus the value of debt, where the value of debt is constructed using Compustat asset at minus cash che minus shareholder equity seq. Overall, our measure of market leverage in years in which a firm is public is:

$$\text{Market leverage} = \frac{mkvalt + (at - che - seq)}{mkvalt}$$

<sup>54</sup>See Fama and French (2001) or Boudoukh et al. (2007) for similar measurements of the buyback yield post-IPO.

<sup>55</sup>Note that this assumption leads us to underestimate (in magnitude) the buyback yield in case founders purchased or received shares before IPO.

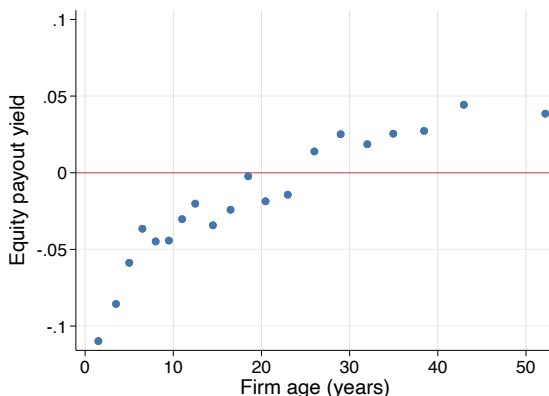


FIGURE C.1.—Annual equity payout yield as a function of firm age

Notes. This figure plots the average annual equity payout yield in 20 bins of firm age for the set of firms that are public in 2015.

We winsorize this variable at the 1st and 99th percentile to decrease the effect of measurement errors. We then construct the market leverage of firms before IPO as their market leverage in the year following the IPO. Finally, we set the market leverage of firms that are private in 2015 as the average market leverage of firms that are public in 2015.

*Growth rate of wealth.* We measure normalized wealth in 2015,  $W_{2015}$ , by dividing individual wealth reported in the 2015 Forbes list by the aggregate household net worth of households from the Financial Accounts of the United States in 2015 (Board of Governors of the Federal Reserve System (US), 2023b), divided by the number of households from the Census (U.S. Census Bureau, 2023). We then compute the lifetime average growth rate of each entrepreneur using Equation 20.

### C.1.2. Sensitivity analysis and sampling uncertainty

We now assess the robustness of our estimated sufficient statistic along two dimensions.

*Sensitivity analysis.* Our individual statistics (i.e., lifetime average equity payout yield, market leverage, and growth rate of wealth) might be measured with a bias. To assess the sensitivity of our estimated sufficient statistic to these potential biases, we re-estimate the sufficient statistic after shifting uniformly all individual statistics by a given number. We report the results in Table C.I, with bootstrapped 95% confidence intervals.

The first two rows report the sufficient statistic after shifting the equity payout yield of all individuals in our sample by  $\pm 0.5$  pp. The third and fourth rows report the sufficient statistic after shifting the market leverage by  $\pm 0.1$ . Unsurprisingly, decreasing the equity payout yield or increasing the market leverage both tend to decrease the sufficient statistic  $\partial_r \log \theta$ , as they correspond to an increased reliance on external sources of financing.

Finally, the fifth and sixth rows report the sufficient statistic after shifting the growth rate of wealth by  $\pm 5$  pp. Alternatively, we explore the sensitivity of our estimated sufficient statistic to the lifetime average growth rate of wealth by setting the entrepreneur's wealth the year of incorporation,  $W_0$ , to ten times more or less than the average wealth in the economy in (20) (as opposed to one in the baseline). We find that this does not change the sufficient statistic much. The reason is that the terminal wealth of the individuals in our sample is so high that small changes in their initial wealth do not matter much for their lifetime average growth rate.

TABLE C.I  
SENSITIVITY ANALYSIS FOR  $\widehat{\partial_r \log \theta}$

	Estimate	95% Confidence interval	
		Lower bound	Upper bound
Equity payout yield -0.5pp	-5.0	-6.3	-4.0
Equity payout yield +0.5pp	-3.4	-4.6	-2.5
Market leverage -0.1	-3.7	-4.8	-2.8
Market leverage +0.1	-4.8	-6.0	-3.8
Growth rate of wealth -5pp	-5.0	-6.4	-3.9
Growth rate of wealth +5pp	-3.7	-4.7	-2.8
Growth rate of wealth assuming initial wealth $W_{t_0} = 10$	-5.1	-6.5	-3.9
Growth rate of wealth assuming initial wealth $W_{t_0} = 0.1$	-3.6	-4.7	-2.8

Note that, in all specifications, the lower bound of the confidence interval remains well below zero, which suggests that the sign of the sufficient statistic (if not its magnitude) are robust to potential biases and sampling uncertainty.

*Alternative estimator.* For the sake of simplicity, we estimated the sufficient statistic in the main text as a ratio of two averages: the average effect of required returns on the growth rate of wealth, in the numerator, and the average growth rate of wealth, in the denominator (see Equation 19). In the presence of firm heterogeneity, however, theory instructs us to compute the sufficient statistic as the average of a ratio computed at the individual level (see Equation 12). To examine the difference between these two methods, we now consider an alternative estimator for our sufficient statistic:

$$\widehat{\partial_r \log \theta}_{\text{alt}} = \frac{1}{N} \sum_{i=1}^N \frac{1 + \text{Market leverage}_{i,T} \times (\text{Equity payout yield}_{i,T} \times \text{Duration} - 1)}{\text{Growth rate}_{i,T}}; \quad (48)$$

that is, an average of ratios rather than a ratio of averages. We report this estimate, as well as the bootstrapped 95% confidence interval, in Table C.II. We find that this alternative estimate is very close to the original one.

TABLE C.II  
ESTIMATES FOR  $\widehat{\partial_r \log \theta}_{\text{ALT}}$

	Estimate	95% Confidence interval	
		Lower bound	Upper bound
Duration = 35 years (baseline)	-4.1	-5.4	-3.1
Duration = 20 years	-4.0	-5.2	-3.1
Duration = 50 years	-4.2	-5.6	-3.1

*Notes.* The alternative sufficient statistic is constructed using Equation (48). The 95% confidence-interval is constructed as a percentile bootstrap confidence interval using 1000 replications. Data are from Forbes, Compustat, and S-1 filings.

## C.2. Estimating required returns

### C.2.1. The required return on business liabilities

We now describe our methodology to estimate the required return on business liabilities (i.e., corporate equities and debts). We use publicly available annual data from the *Integrated Macroeconomic Accounts* (Bureau of Economic Activity, 2023), which combines sectoral data on income and expenditure from the National Accounts with data on financial transactions and holdings from the Financial Accounts. We focus on the corporate nonfinancial sector (i.e., Table S5) and deflate all variables using the Consumer Price Index for All Urban Consumers (U.S. Bureau of Labor Statistics, 2023).

*Return definition.* Consider the return associated with a trading strategy that consists of holding all liabilities issued by the corporate sectors and purchasing all new issuances in every year. The realized return of owning the corporate sector between year  $t$  and  $t + 1$  is given by

$$r_{\text{corp},t+1} = \frac{\text{net operating surplus}_{t+1} - \text{net capital formation}_{t+1}}{\text{net liabilities}_t} + \frac{\text{net liabilities}_{t+1} - \text{net liabilities}_t}{\text{net liabilities}_t}. \quad (49)$$

Net operating surplus (line item 8) is a measure of net corporate profit (i.e., value-added minus worker compensation and capital depreciation). Net capital formation (line 28) measures capital formation (which includes investments in real estate, equipment, and intellectual property products) net of depreciation. Net operating surplus minus net capital formation thus accounts for all of the cash flows generated by the corporate sector. Finally, net liabilities measures the market value of debts and equities issued by the corporate sector minus the financial assets held by the corporate sector.

The first term in (49) corresponds to the payout yield. Corporate cash flows can be used to pay interests, dividends, stock buybacks or debt repurchases. Given the trading strategy that we consider, all of these uses of corporate cash flows have the same economic implication: they represent flows of cash from corporations to households (see Abel et al., 1989 for an early discussion of this idea). The second term accounts for the contribution of the growth in the market value of liabilities.

To map (49) more closely to the model, we define the following variables:

$$\begin{aligned} \text{rok}_{t+1} &\equiv \frac{\text{net operating surplus}_{t+1}}{\text{capital}_t}, && \text{(Return on capital)} \\ g_{t+1} &\equiv \frac{\text{net capital formation}_{t+1}}{\text{capital}_t}, && \text{(Net capital formation rate)} \\ Q_t &\equiv \frac{\text{net liabilities}_t}{\text{capital}_t}. && \text{(Tobin's Q)} \end{aligned}$$

Given these definitions, the realized return defined in (49) can thus be rewritten as

$$\begin{aligned} r_{\text{corp},t+1} &= \frac{\text{rok}_{t+1} - g_{t+1}}{Q_t} + \frac{g_{t+1} \times \text{capital}_{t+1} - Q_t \times \text{capital}_t}{Q_t \times \text{capital}_t}, \\ &= \frac{\text{rok}_{t+1} - g_{t+1}}{Q_t} + g_{t+1} + \underbrace{\frac{\text{capital}_{t+1} - (1 + g_{t+1})\text{capital}_t}{\text{capital}_t}}_{\text{revaluation gain}} + \frac{\text{capital}_{t+1}}{\text{capital}_t} \times \frac{Q_{t+1} - Q_t}{Q_t}. \end{aligned}$$

We call the sum of the last two terms the “revaluation gains”. This term combines the growth in the replacement value of capital and the growth in Tobin’s Q (revaluation of net financial assets liabilities relative to the replacement value of capital). In practice, the first term in this sum is mainly driven by the revaluation of real estate prices (real estate capital is reported using market values), and it averages to roughly zero in our sample.

*Required returns.* To obtain a measure of expected returns, we make two assumptions. First, the investment rate and the return on capital are known one period in advance (i.e.,  $\mathbb{E}_t [g_{t+1}] = g_{t+1}$  and  $\mathbb{E}_t [\text{rok}_{t+1}] = \text{rok}_{t+1}$ ). Second, expected revaluation gains are zero. See [Campbell \(2017\)](#) chapter 5.5.2, for an analogous set of assumptions in the context of stock market returns. Combining the definition of realized returns (49) with these two assumptions, we obtain that expected returns can be written as:

$$\mathbb{E}_t [r_{\text{corp},t+1}] = \frac{\text{rok}_{t+1} - g_{t+1}}{Q_t} + g_{t+1}, \quad (50)$$

which is directly observable. From now on, we refer to  $\mathbb{E}_t [r_{\text{corp},t+1}]$  as the *required* return on wealth. The idea is that, the value of net liabilities  $Q_t$  is such that the expected return on investor’s wealth is equal to their required return.

*Aggregate per-capita growth.* What matters in our sufficient statistic approach is the decline in *r net of aggregate growth per capita*. One simple method is to deflate our measure of required returns by the growth rate of capital per capita. This deflated measure of required returns simply corresponds to the payout yield,  $(\text{rok}_{t+1} - g_{t+1})/Q_t$ , plus the rate of population growth ([Board of Governors of the Federal Reserve System \(US\), 2023c](#)). A second method is to deflate our measure of returns by TFP growth, as constructed in [Feenstra, Inklaar, and Timmer \(2015\)](#) ([University of Groningen and University of California, Davis, 2023](#)).

*Results.* We plot the our required returns on wealth series in [Figure C.2](#). The key observation is that, for both deflators, there is a substantial decline in the required return net of per-capita growth.

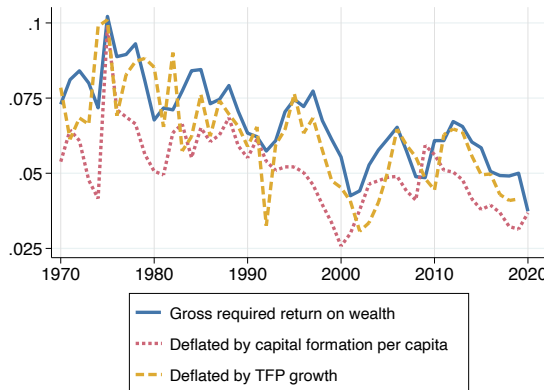


FIGURE C.2.—Required return on business liabilities net of aggregate growth

*Notes.* Data are from [Bureau of Economic Activity \(2023\)](#), [Board of Governors of the Federal Reserve System \(US\) \(2023c\)](#), [U.S. Bureau of Labor Statistics \(2023\)](#), and [University of Groningen and University of California, Davis \(2023\)](#).



Table C.III contains summary statistics on the return on capital, the required return on wealth (deflated or not), and Tobin's  $Q$  from 1985 through 2015. Computing the decline as the change in the average in 2015–2020 compared to the average in 1980–1985, we obtain that the change in the required return on corporate sector liabilities is  $-2.7$  pp. Deflating by the growth rate of capital per capita gives a change of  $-2.3$  pp. To be conservative we use a change in required returns of  $-2$  pp. in the main text.

TABLE C.III  
REQUIRED RETURNS AND VALUATIONS OF THE U.S. CORPORATE SECTOR

Moment	1980-1985	1980-2020	2015-2020
Return on capital (%)	6.5	7.6	7.6
Required return on wealth (%)	7.6	6.3	4.9
Required return on wealth net of capital growth p.c. (%)	5.8	4.8	3.6
Required return on wealth net of TFP growth (%)	7.3	5.7	4.5
Tobin's $q$	78.9	148.2	198.4

*Notes.* This table reports moments for the U.S. non financial corporate sector. The construction of each variable is detailed in Appendix C.2. Data are from the [Bureau of Economic Activity \(2023\)](#), FRED, and [Feenstra, Inklaar, and Timmer \(2015\)](#).

### C.2.2. Required returns on corporate liabilities versus required returns on corporate debt

To implement our sufficient statistic, we have assumed that the change in the required return on corporate liabilities  $dr$  was equal to the change in the required return on corporate debt  $dr_f$ . To test this assumption, we now separately estimate the change in the required return on corporate debt. We then discuss the effect of this estimate on the change in Pareto inequality due to the change in required returns.

Debt issued by the corporate sector can take the form of bonds or bank loans. Assuming away the probability of default does not change over time, we can directly estimate this required return from the interest rate paid by the corporate sector.<sup>56</sup> Figure C.3 plots the required returns on debt, using Moody's data on corporate bond yields for firms rated AAA and BAA ([Moody's, 2023a](#) and [Moody's, 2023b](#)) and the bank lending rate ([Board of Governors of the Federal Reserve System \(US\), 2023a](#)). We deflate these required returns using a lagged three-years average inflation ([U.S. Bureau of Labor Statistics, 2023](#)) in order to obtain real returns. We find that both rates have declined substantially over time.

Following [Barkai \(2020\)](#), we construct the required return on corporate debt by averaging the two series, with weights given by the relative quantity of each type of debt according to the *Integrated Macroeconomic Accounts*. Similarly to the case of the required return on all corporate liabilities, what matters is the interest rate on debt relative to the growth rate of the economy. Figure C.4 plots the resulting interest rate using the same deflators as in Figure C.2, while Table C.IV reports the average of interest rates in different periods.

We obtain that the change in the real interest rate paid by firms  $-2.7$  pp. Deflating by the growth rate of capital per capita gives a change of  $-2.3$  pp., which is the same as the change in the required return on all corporate liabilities. This justifies our approach of considering a homogeneous declines in the rate of return across all securities.

<sup>56</sup>More precisely, since we only need to estimate the declined in required returns, we only need to assume that the probability of default does not change over time.

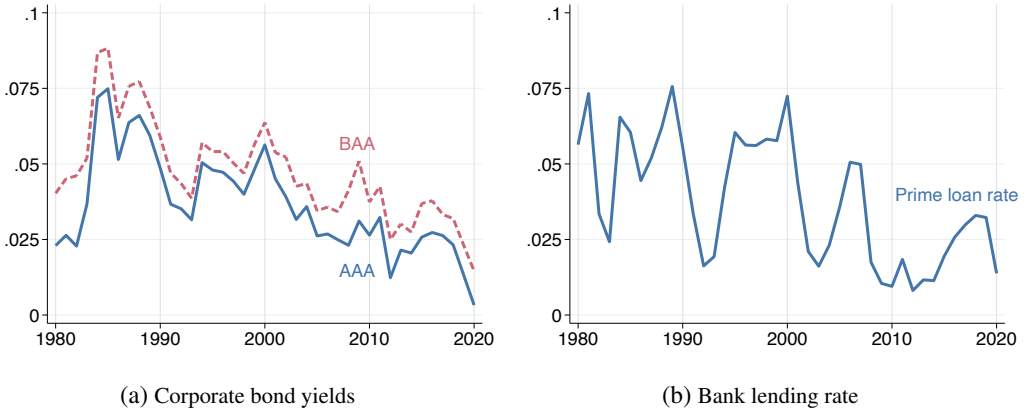


FIGURE C.3.—Required returns on corporate debt

Notes. Panel (a) plots the evolution of U.S. corporate bond yields by Moody's ratings. Panel (b) plots the evolution of the bank lending rate. Both are in real terms. Data are from [Moody's \(2023a\)](#), [Moody's \(2023b\)](#), and [Board of Governors of the Federal Reserve System \(US\) \(2023a\)](#).

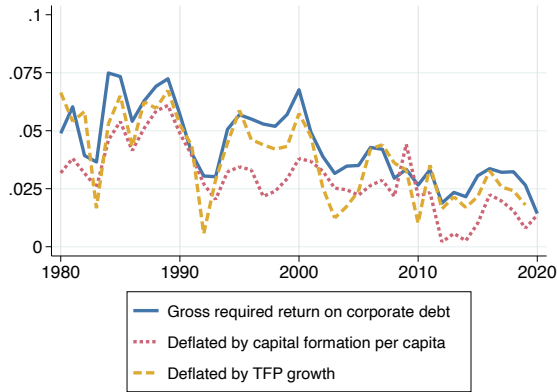


FIGURE C.4.—Required returns on corporate debt net of aggregate growth

TABLE C.IV  
REQUIRED RETURNS ON CORPORATE DEBT

Moment	1980-1985	1980-2020	2015-2020
Interest on corporate debt (%)	5.6	4.3	2.8
Interest on corporate debt net of capital growth p.c. (%)	3.8	2.9	1.5
Interest on corporate debt net of TFP growth (%)	5.2	3.8	2.4

### C.3. Estimating Pareto inequality

We now describe how we estimate Pareto inequality in the data. First, we use data from [Smith et al. \(2023\)](#), who use an improved version of the capitalization method developed in [Saez and Zucman \(2016\)](#) to construct wealth share estimates in the US. Relative to [Saez and Zucman](#)

(2016), the methodology allows for more granular return heterogeneity.<sup>57</sup> Using this data, we construct three alternative estimates of Pareto inequality using the “top share estimator” defined in Equation 22, with  $p = 0.001\%$ ,  $p = 0.01\%$ , and  $p = 0.1\%$ .

As a robustness check, we also construct two alternative sets of estimates for Pareto inequality using Forbes data on the wealthiest 400 individuals.<sup>58</sup> We only use data on their rank in the list and on their stated wealth. First, we use the log-rank estimator proposed by Gabaix and Ibragimov (2011). The idea is to estimate a cross-sectional regression of log wealth on the log rank minus 1/2 and use the slope of this regression as an estimate of the Pareto exponent of the wealth distribution. To get an estimate of Pareto inequality, we simply take the inverse of this coefficient. Second, following Saez (2001), we use the mean-min estimator  $\theta = 1 - \mathbb{E}[W|W > \underline{W}]/\underline{W}$ , where  $\mathbb{E}[W|W > \underline{W}]$  is the average wealth of households in the Forbes 400 list and  $\underline{W}$  is the wealth of the last household in list.

Table C.V contains the beginning, average, and end value of the Pareto inequality estimates over our time period of interest (i.e., 1980 to 2020). Taking the log difference between the average value in the last five years of our sample minus the average value in the first five years of our sample, and averaging across the different measures of Pareto inequality, gives an estimate for the rise in Pareto inequality of 22 log points.

TABLE C.V  
ESTIMATES OF PARETO INEQUALITY

Estimate	1980-1985	1980-2020	2015-2020
Ratio top shares (1%-0.1%)	0.59	0.64	0.67
Ratio top shares (0.1%-0.01%)	0.53	0.62	0.66
Ratio top shares (0.01%-0.001%)	0.47	0.59	0.65
Mean-min (Forbes 400)	0.58	0.67	0.71
Log-rank (Forbes 400)	0.57	0.69	0.73

*Notes.* The table reports estimates of Pareto inequality using, successively, the log ratio between the top 0.1% and the top 1%, the log ratio between the top 0.1% and the top 0.01%, the log ratio between the top 0.001% and the top 0.1%, the log ratio between the average wealth in Forbes 400 and the wealth of the last person in Forbes 400, and the slope estimate in a regression of log rank minus 1/2 on log wealth. Data from Smith Matthew (2022) and Forbes.

#### C.4. Evidence beyond the top 100

*Private businesses.* We now use data from the 2016 wave of the *Survey of Consumer Finances* (SCF) from Federal Reserve Board (2016) to quantify the prevalence of entrepreneurs at the top of the wealth distributions (i.e., individuals who founded or acquired a business that they actively manage). Table C.VI presents summary statistics. First, notice that entrepreneurs

<sup>57</sup>The authors summarize the influence of return heterogeneity on estimated top wealth shares as follow: “*In terms of top portfolios, we find that accounting for estimated return heterogeneity makes a difference. First, relative to an equal returns approach, we find a larger role for pass-through business wealth and a smaller role for fixed income wealth. Second, the fixed income portfolio share falls and the equity share rises with wealth at the top. Pass-through business and C-corporation equity wealth are the primary sources of wealth at the top. At the very top, C-corporation equity is the largest component, accounting for 53% of top 0.001% wealth, and pass-through business accounts for 22%. In contrast, pensions and housing account for almost all wealth of the bottom 90%. Third, we find that the fixed income portfolio share at the very top remained relatively stable, whereas under equal returns, the fixed income portfolio share increased substantially since 2000.*”

<sup>58</sup>We use data from Gomez (2023b).

are over-represented at the top. As in [Cagetti and De Nardi \(2006\)](#), we find that wealthier individuals are much more likely to be entrepreneurs. In the full population, 11% of individuals are entrepreneurs while in the top 0.01%, the fraction increases to 66%. Second, the businesses founded by wealthy individuals tend to be pass-through entities, which is consistent with the evidence in [Cooper et al. \(2016\)](#). For instance, 93% of businesses owned by households in the top 0.01% are partnerships or S corporations. This is in a sharp contrast with the fact that roughly two-thirds of entrepreneurs in the top 100 own public firms (i.e., C corporations).

TABLE C.VI  
ENTREPRENEURS IN THE TOP PERCENTILES (SCF, 2016)

Top percentile groups	Total	Top 1%	Top 0.1%	Top 0.01%
Entrepreneurs	0.11	0.43	0.59	0.66
Sole proprietorship	0.48	0.09	0.06	0.02
Partnership	0.35	0.60	0.64	0.63
S corporation	0.11	0.21	0.24	0.30
Other corporations	0.06	0.11	0.06	0.05

The effect of required returns on wealth inequality in our model depends on the extent to which these businesses rely on external financing (through either equity or debt financing). Due to data limitations, we are unable to produce estimates of the equity issuance and leverage of the firms owned by entrepreneurs in the top 1% in the US. However, we now present evidence from [Kochen \(2022\)](#) that firms in high-income countries frequently use external financing.

[Kochen \(2022\)](#) harmonizes data for 11 high-income countries (i.e., Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, and the United Kingdom) over the 1996–2018 period using the Orbis database. The dataset contains firm-level data on millions of companies, most of which are private. [Table C.VII](#) summarizes the importance of debt and equity financing. First, notice that firms use a substantial amount of leverage, which amounts (on average) to 1.5 using book values. This is somewhat higher than what we find among public firms in our sample (see [Table II](#)). Regarding equity issuances, on average 8% of firms issue equity in a given year and, conditional on doing an equity issuance, it amounts to roughly 18% of book equity. Putting together, this represents a roughly  $8\% \times 18\% \approx 1.5\%$  annual net equity issuance yield (or a  $-1.5\%$  buyback yield), which is roughly half as much as the firms in our sample (see [Table II](#)).

How would the sufficient statistic change after incorporating these entrepreneurs? On the one hand, the fact that they use less equity financing will tend to decrease the numerator in the sufficient statistic (19). On the other hand, the fact that their lifetime average growth rate is (most likely) smaller than the entrepreneurs in *Forbes* (i.e., most of them did not become billionaires) will tend to decrease the denominator in the sufficient statistic. As a result, the overall effect of incorporating these less successful entrepreneurs is ambiguous (see [Appendix C.1.2](#) for the sensitivity of the sufficient statistic to the equity issuance and growth rate of entrepreneurs).

*Venture capital backed firms.* Firms backed by venture capitalists (henceforth VCs) are an important part of the US economy. According to Capshare, 10,400 companies received venture funding in 2018.<sup>59</sup> On average, the ownership share of the founders decreases by roughly 25%

<sup>59</sup>Capshare is a “web application that helps businesses manage their stock and assets on one organized platform”. Our statistics are taken from their “2018 Private Company Equity Statistics Report”.

TABLE C.VII  
DEBT AND EQUITY FINANCING (ORBIS)

Variable	Mean
Leverage	1.5
Frequency of equity issuance	0.08
Size of equity issuance	0.18

*Notes.* “Leverage” represents the ratio of total asset (i.e., debt plus book equity) divided by the book value of equity, “Frequency of equity issuance” represents the share of firms that have issued equity in the current year; “Size of equity issuance” represents the ratio of equity issuance to capital, conditional on equity issuance being positive. All averages are weighted by capital and obtained in Appendix Table A.3. in [Kochen \(2022\)](#). While the paper reports the average debt-to-capital ratio, we transform this into an estimate of the average book leverage (capital-to-book-equity ratio) as  $1/(1 - \text{debt-to-capital ratio})$ . Similarly, we report the size of equity issuances as a share of book equity while the paper reports it as a share of capital.

every funding round. Since funding rounds tend to happen every 18 month, this corresponds to an annual dilution rate of 16% (i.e., an equity payout yield of  $-16\%$ , assuming that no dividends are paid out).

Another way to obtain a measure of the average dilution rate is to divide the total equity raised by firms funded by VC to their total market capitalization. Pitchbook estimates that VC-backed companies have a combined market capitalization of around \$3 trillion in 2022 and that they collectively raised \$130 billion that year.<sup>60</sup> Combining these two figures gives a (market-capitalization weighted) dilution rate of 4.5% (i.e., an equity payout yield of  $-4.5\%$ , assuming that no dividends are paid out). The fact that this estimate is lower than the previous figure reflects the fact that the largest dilution happens in early rounds, that is, in companies with smaller market capitalizations.

While the number of VC firms is small relative to the number of households, it is worth noting that many key employees of these firms receive a substantial proportion of their income in the form equity (i.e., equity grants, stock options, etc.). Equity compensation typically leads to concentrated portfolios due to a mix of vesting time, other restrictions on stock sales, and illiquidity (especially pre-IPO). Our notion of “entrepreneurs” in the model can be interpreted as including not only the founder of a firm, but also any individual who invests the majority of their wealth in the firm. In particular, it also includes employees that receive a substantial proportion of their income as equity. [Eisfeldt et al. \(2019\)](#) reports that equity compensation represents almost 45% of total compensation to high-skilled labor in recent years and that employees working in VC-backed firms account for approximately 2% of the workforce. Despite the lack of data on the portfolio of such “human capitalists”, we think that many wealthy, high-skilled employees have portfolios with concentrated holdings. We expect these concentrated holdings to be particularly important for firms that are net equity issuers.

## APPENDIX D: APPENDIX FOR SECTION 5

### D.1. *Characterization of equilibrium*

*Firm policy functions.* The optimal labor and investment satisfies

$$w_t = (1 - \alpha)(K_t/L_t)^\alpha,$$

$$q_{s,t} = 1 + \chi(g_{s,t} - g_s),$$

<sup>60</sup>These two statistics are taken from their “2022 Quantitative Perspectives: US Market Insights”.

where  $q_s \equiv V_s(K)/K$ . Notice that while firms  $s = 1, 2$  choose different growth rates, they choose the same capital to output ratio. This is because their production technology is identical. The solution is

$$g_{s,t} = \underline{g}_s + \frac{1}{\chi}(q_{s,t} - 1),$$

$$L_{s,t} = (1 - \alpha)^{-\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}} K_{s,t}.$$

*Firm valuations.* Using the optimal policy functions and the definition  $\text{MPK} \equiv F_K(K, L)$ , we have:

$$0 = \left( \text{MPK}_t - r_t + \tau(\psi q_{1,t} - 1) - (r_t + \tau - \underline{g}_0)(q_{0,t} - 1) + \frac{1}{2\chi}(q_{0,t} - 1)^2 \right) dt + \mathbb{E}_t [dq_{0,t}], \quad (51)$$

$$0 = \left( \text{MPK}_t - r_t - (r_t - \underline{g}_1)(q_{1,t} - 1) + \frac{1}{2\chi}(q_{1,t} - 1)^2 \right) dt + \mathbb{E}_t [dq_{1,t}], \quad (52)$$

Along a balanced growth path (i.e.,  $\text{MPK}_t = \text{MPK}$ ,  $r_t = r$ ), we have

$$0 = \text{MPK} - r + \tau(\psi q_1 - 1) - (r + \tau - \underline{g}_0)(q_0 - 1) + \frac{1}{2\chi}(q_0 - 1)^2,$$

$$0 = \text{MPK} - r - (r - \underline{g}_1)(q_1 - 1) + \frac{1}{2\chi}(q_1 - 1)^2.$$

*Implications of labor market clearing.* From the first-order condition for labor, we have that the capital to labor ratio is the same at both types of firms. Using the labor market clearing condition (i.e.,  $L_{0,t} + L_{1,t} = 1 - \pi$ ), the equilibrium wage and MPK must be

$$w_t = (1 - \alpha) \left( \frac{K_t}{1 - \pi} \right)^{1-\alpha}, \quad \text{MPK}_t = \alpha \left( \frac{K_t}{1 - \pi} \right)^{\alpha-1}.$$

*Law of motion for capital.* The law of motion for detrended capital by firm type is

$$dK_{0,t} = (g_{0,t} - \tau - \eta)K_{0,t} dt + \eta\pi\bar{K} dt, \quad dK_{1,t} = (g_{1,t} - \eta)K_{1,t} dt + \tau\psi K_{0,t} dt.$$

In steady-state, we have

$$K_0 = \frac{\eta}{\eta + \tau - g_0} \pi \bar{K}, \quad K_1 = \frac{\tau\psi}{\eta - g_1} K_0, \quad K = \frac{\eta - g_1 + \tau\psi}{\eta - g_1} \frac{\eta}{\eta + \tau - g_0} \pi \bar{K},$$

where  $K$  is aggregate capital and  $g_s$  is the steady-state growth rate of the firm of each type, i.e.  $g_s = \underline{g}_s + \frac{1}{\chi}(q_s - 1)$ .

*Duration of aggregate wealth.* In steady state, the (aggregate) Tobin's Q is defined as the capital-weighted average of individuals  $q_s$ :

$$Q = \frac{\eta - g_1}{\eta - g_1 + \tau\psi} q_0 + \left( 1 - \frac{\eta - g_1}{\eta - g_1 + \tau\psi} \right) q_1.$$

In particular, the duration of aggregate wealth is given by

$$D \equiv -\frac{\partial_r Q}{Q} = -\frac{\partial_r (q_0 K_0 + q_1 K_1)}{QK} = \frac{q_0 K_0}{QK} D_0 + \frac{q_1 K_1}{QK} D_1,$$

where  $D_s \equiv -\partial_r q_s / q_s$  denotes the duration of a firm in state  $s$ .

*Entrepreneur wealth.* Let  $\text{epy}_{s,t}$  be the equity payout yield of firm in state  $s$  at time  $t$ :

$$\text{epy}_{s,t} \equiv \frac{r_t - g_{s,t} + \lambda (\text{MPK}_t - \iota_s(g_{s,t}) - (r_t - g_{s,t}))}{q_{s,\lambda,t}},$$

where  $q_{s,\lambda,t} \equiv 1 + \lambda(q_{s,t} - 1)$ . Denoting by  $T$  the (random) time at which the firm matures. Assuming that  $1 + \lambda(\psi q_{1,t} - 1) > 0$  (this ensures that the entrepreneur does not default when transitioning from a growth to a mature firm, and it will be satisfied in our calibration), the wealth of an entrepreneur evolves according to:

$$\frac{dW_t}{W_t} = \begin{cases} (\text{epy}_{0,t} + g_{0,t} - \rho) dt + \frac{dq_{0,\lambda,t}}{q_{0,\lambda,t}} & \text{if } t < T \\ \frac{1 + \lambda(\psi q_{1,t} - 1)}{1 + \lambda(q_{0,t} - 1)} - 1 & \text{if } t = T \\ (\text{epy}_{1,t} + g_{1,t} - \rho) dt + \frac{dq_{1,\lambda,t}}{q_{1,\lambda,t}} & \text{if } t > T. \end{cases} \quad (53)$$

Denote by  $W_{E,s,t}$  the detrended total wealth of entrepreneurs owning firms in state  $s \in \{0, 1\}$ . Its law of motion is given by:

$$\begin{aligned} dW_{E,0,t} &= \left( (\text{epy}_{0,t} + g_{0,t} - \rho - \tau - \eta) dt + \frac{dq_{0,\lambda,t}}{q_{0,\lambda,t}} \right) W_{E,0,t} + \eta \pi \frac{\bar{K} q_{0,\lambda,t}}{\lambda} dt, \\ dW_{E,1,t} &= \left( (\text{epy}_{1,t} + g_{1,t} - \rho - \eta) dt + \frac{dq_{1,\lambda,t}}{q_{1,\lambda,t}} \right) W_{E,1,t} + \tau \frac{1 + \lambda(\psi q_{1,t} - 1)}{1 + \lambda(q_{0,t} - 1)} W_{E,0,t} dt. \end{aligned}$$

*Mutual fund wealth.* By Walras' law, labor and product market clearing implies financial market clearing. The mutual fund must therefore hold all wealth not held by entrepreneurs:

$$W_{M,t} = Q_t K_t - W_{E,0,t} - W_{E,1,t}.$$

Since the entrepreneurs own levered claims on firms (i.e., levered equity shares), it means that the mutual fund must hold debt. In a steady-state, this is inconsequential, since all assets have the same return. But over a transition path, it means that the revaluation gains of the mutual fund will differ from those of entrepreneurs. We obtain the mutual fund's revaluation gains as a residual

$$\frac{dq_{M,t}}{q_{M,t}} = \frac{Q_t K_t \frac{dQ_t}{Q_t} - W_{E,0,t} \frac{dq_{0,\lambda,t}}{q_{0,\lambda,t}} - W_{E,1,t} \frac{dq_{1,\lambda,t}}{q_{1,\lambda,t}}}{Q_t K_t - W_{E,0,t} - W_{E,1,t}}.$$

*Worker wealth.* Denote  $W_{L,t}$  to be detrended worker wealth. Its law of motion is:

$$dW_{L,t} = \left( (r_t - \rho_L - \eta) dt + \frac{dq_{M,t}}{q_{M,t}} - \mathbb{E}_t \left[ \frac{dq_{M,t}}{q_{M,t}} \right] \right) W_{L,t} + (1 - \pi) w_t dt - \rho_L H_t dt, \quad (54)$$

where  $H_t \equiv \mathbb{E}_t \left[ \int_0^\infty e^{-\int_0^s r_{t+h} dh} w_{t+s} ds \right]$  denotes the human wealth of a worker at time  $t$ .

*Foreigner wealth.* The law of motion for detrended foreigner wealth is

$$dW_{F,t} = S_{F,t} dt + \left( \frac{dq_{M,t}}{q_{M,t}} - \eta dt \right) W_{F,t},$$

where  $S_{F,t}$  is the flow of savings by foreigners.

*Pareto inequality.* The formula for steady-state Pareto inequality is almost exactly as in the stylized model (see Section 2):

$$\theta = \max \left( \frac{\text{epy}_0 + g_0 - \rho}{\eta + \tau}, \frac{r - \rho}{\eta} \right). \quad (55)$$

The key difference is that, in the stylized model,  $r - \rho$  corresponds to the return of rentiers (i.e., return on a diversified portfolio). Now, it corresponds to the return of holding a mature firm. Since the return of mature firms is deterministic, it must be  $r$  (both in expectation and ex-post).

## D.2. Neoclassical growth model as a limiting case

We now show that our model nests the neoclassical growth model as a special case where:

- (1) Capital is fully elastic ( $\chi = 0$ ) and there is no firm heterogeneity ( $\psi = 0$ );
- (2) All agents are workers ( $\pi = 1$ ) and there is no population renewal ( $\eta = 0$ ).

For simplicity, we focus on a closed-economy steady-state equilibrium. Using the parameter restriction (1) and the firm valuation equations (51), we obtain

$$q_0 = 1, \quad \text{MPK} - \tau = r.$$

In words, this says that there are no rents in equilibrium (i.e., the cost of capital  $r$  equals the net return on capital), which implies that Tobin's  $Q$  is one. Notice that the parameter  $\tau$  now has the interpretation of a depreciation rate.

Using the parameter restriction (2), we have that existing agents own all of future wages and payments to capital, which means that their total wealth is  $W = Y/r$ , where  $Y = K^\alpha L^{1-\alpha}$ . Given the log utility assumption, their optimal consumption is  $C = \rho_L Y/r$ . Using the product market clearing condition (i.e.,  $C = Y$ ), we have that

$$r = \rho_L.$$

In words, this means that the required return is equal to the subjective discount factor.

Putting together, we obtain the steady-state allocation in the neoclassical growth model, where the net marginal product of capital is equal to the subjective discount factor (i.e.,  $\text{MPK} - \tau = \rho_L$ ). In the calibrated model, we relax (1) in order to generate a wedge between the return on capital and the cost of capital and relax (2) in order to have wealth inequality due to concentrated portfolios.



### D.3. Domestic savings glut

In the baseline model experiment, we generate an equilibrium decline in  $r$  by feeding in an exogenous rise in savings by foreigners. We now consider an asset-demand shock that originates domestically (i.e., a domestic savings glut). We do so by changing the subjective discount factor  $\rho$  of domestic agents. This captures, in a reduced-form way, a number of forces, such as rising longevity, that pushes up the desire to save. The key difference with the baseline model experiment is that a decline in  $\rho$  has a direct on top wealth inequality, a force that we now quantify.

To implement the model experiment, we consider a model extension where workers and entrepreneurs have time-varying subjective discount factors and where the flow of savings by foreigners is constant over time.

The model experiment consists of a steady-state comparative static where we shock the subjective discount factors of both workers and entrepreneurs by a common shifter  $\varepsilon$ . Other than that, all other model parameters are exactly as in the baseline calibration. The path of foreign savings is constant at some value  $S_F$ . We choose the value  $S_F$  as being equal to its value in the  $r = 6\%$  (i.e., 1985-2015 average) steady-state of the baseline model. That way, we match the fact that the NFA to domestic wealth ratio is 5% (i.e., a targeted moment in the baseline model calibration).

TABLE D.I  
MODEL EXPERIMENT WITH A DOMESTIC SAVINGS GLUT (LONG-RUN, PERCENTAGE POINTS)

Model	$\Delta r$	$\Delta \rho$	$\Delta \log \theta$
Baseline	-2.0	0.0	11
Domestic savings glut	-2.0	-1.5	16

Table D.I reports the long-run change in the required return, the subjective discount factor, as well as the change in (log) Pareto inequality. Overall, we find that the rise in Pareto inequality is roughly 1.5 times larger than in the baseline model. To understand the forces at play, it is instructive to use an comparative statics formula for the change in Pareto inequality in response to an infinitesimal change in the subjective discount factor  $d\rho$  and the required return  $dr$ : Totally differentiating the expression for Pareto inequality (55) in steady-state, and assuming that we are in the entrepreneur regime, we have that

$$d \log \theta = \partial_\rho \log \theta d\rho + \partial_r \log \theta dr$$

with

$$\begin{aligned} \partial_\rho \log \theta &= -\frac{1}{\text{epy}_0 + g_0 - \rho} \\ \partial_r \log \theta &= \lambda \frac{q_0}{q_{0,\lambda}} \times \frac{(\text{epy}_0 \times (-\partial_r \log q_0) - 1)}{\text{epy}_0 + g_0 - \rho}. \end{aligned}$$

Or, in words,

$$\partial_\rho \log \theta = \frac{-1}{\text{Growth rate of wealth}}$$

$$\partial_r \log \theta = \frac{1 + \text{Market leverage} \times (\text{Equity payout yield} \times \text{Duration} - 1)}{\text{Growth rate of wealth}}.$$

The formula expresses the change in Pareto inequality as a linear function of the change in the subjective discount factor  $\rho$  and in the required return  $r$ . In the model experiment, the required return  $r$  declines by 2 pp. while the subjective discount factors decline by roughly 1.5 pp.<sup>61</sup> However, the sensitivity of Pareto inequality to required returns is higher than its sensitivity to the subjective discount factor. The reason is that a change in  $\rho$  moves the growth rate of wealth of wealth one-for-one for all entrepreneurs, while a change in  $r$  affects the growth rate of successful entrepreneurs more than one-for-one, due to the fact that these entrepreneurs use a lot of external financing and own high-duration firms (i.e.,  $1 + \text{Market leverage} \times (\text{Equity payout yield} \times \text{Duration} - 1) > 1$ , see Table IV).

#### D.4. *Quantifying the intensive and extensive margins of top wealth share growth*

We now decompose the rise in top wealth shares in our model into an intensive and extensive margin. This allows us to measure the relative contribution of the growth rate of existing fortunes, as opposed to the inflow of new fortunes, in the rise in top wealth inequality.

Following Gomez (2023a), we now decompose the growth rate of the share of aggregate wealth owned by a top percentile, at each time period, into an intensive and extensive term. The intensive term holds constant the composition of individuals in the top percentile over the period of time: it is defined as the wealth growth of individuals who are initially in the top percentile relative to the growth of the average wealth in the economy. In contrast, the extensive term, which is defined as a residual, accounts for all composition changes in the top percentile. More precisely, in our model, this extensive term is the sum of a positive force—i.e., the flow of successful entrepreneurs in the top percentile (of type 0) who displace the less successful ones (of type 1)—as well as a negative force, population growth.<sup>62</sup>

Figure D.1 plots the (annualized) growth of the top 0.1% wealth share in the baseline model experiment, as well as its decomposition into an intensive and an extensive margin, as discussed above. For the first 5 years, the rise in the intensive term explains most of the rise in the top wealth share. This is because the realized returns of individuals at the top (26) are high relative to the rest of the distribution. This comes from the fact that revaluation gains are particularly high for individuals at the top of the wealth distribution, who tend to own levered positions in high-duration firms. However, as realized returns start declining, the contribution of the intensive term declines. In fact, the intensive term is ultimately lower in the new steady state compared to the initial steady state, as the average return on wealth in this economy is now lower.

The rise in the top 0.1% wealth share is ultimately driven by a rise of the extensive term. This increase in the extensive term reflects the fact that, in a low-rate environment, the most successful entrepreneurs accumulate capital more quickly as they face a lower cost of capital. As shown in Figure D.1, this higher inflow of new fortunes in the top 0.1% more than compensates for the lower growth rate of existing fortunes. Overall, these results are consistent with evidence from Gomez (2023a), Zheng (2019) and Atkeson and Irie (2022), who argue that an

<sup>61</sup>The reason why a 1.5 pp. decline in the subjective discount factors of both workers and entrepreneurs leads to a 2 pp. decline in the required return is that there is a reallocation of wealth towards entrepreneurs who have a lower subjective discount factor than workers (see Table IV).

<sup>62</sup>Gomez (2023a) further decomposes the extensive margin as the sum of a positive “between” term, which account for the dispersion in wealth growth within top individuals, and a negative “demography” term, which accounts for demographic changes such as a death and population growth.

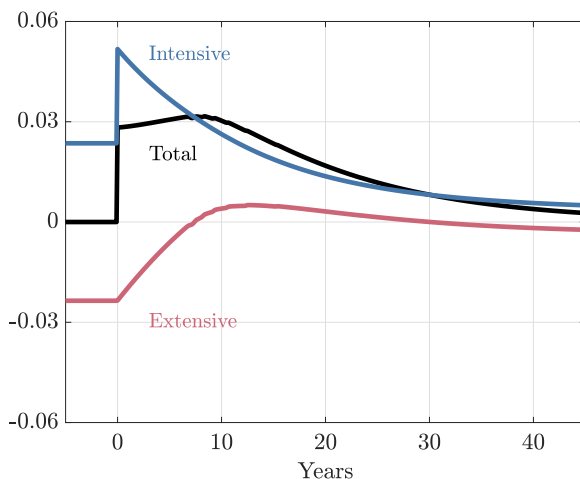


FIGURE D.1.—Decomposing the growth of the top 0.1% into an intensive and extensive term

increase in the flow of new fortunes in top percentiles has played a substantial role in the recent rise in U.S. top wealth inequality.

Finally, this decomposition is useful to relate our theory to the central idea in [Piketty and Zucman \(2015\)](#), which is that Pareto inequality increases with “ $r - g$ ” (i.e., the required return net of per-capita growth). On the one hand, it is true that a decline in “ $r - g$ ” leads to a decrease in the growth rate of existing fortunes relative to the economy, which tends to push down top wealth inequality. This is captured by the long-run decline in the intensive term in [Figure D.1](#). However, what our decomposition shows is that the lower growth rate of existing fortunes in a low-rate environment is more than compensated by the larger inflow of new fortunes in the top percentiles (as the decline in the intensive term is more than compensated by the increase in the extensive term).

#### D.5. Elastic capital calibrations

*Calibrations.* [Table D.II](#) reports the model fit for the three elastic capital extensions (i.e., low-elasticity, medium-elasticity, and high-elasticity).

*Evidence from investment regressions.* In [Section 5.5](#), we consider three alternative calibrations where we use the parameter  $\chi$ —which governs the degree of investment adjustment costs—to match, respectively, a 0.5, 1, and 1.5 percentage point decline of the return on capital in the model experiment. [Table D.III](#) reports model objects for four calibrations of the model: the baseline calibration, the three elastic capital calibrations, as well as a “very high elasticity” calibration where we target a 4 percentage points decline of the return on capital.

The first column reports the targeted long-run decline of the return on capital (i.e.,  $\Delta r_{ok}$ ). As discussed earlier, we target values from 0 pp. (in the baseline model) to  $-4$  pp. (in the very high elasticity calibration). The second column reports the long-run increase in (log) Pareto inequality (i.e.,  $\Delta \log \theta$ ). Notice that the rise in Pareto inequality is monotonically decreasing in the degree of capital elasticity. In the most aggressive calibration (i.e., the very high elasticity calibration), the rise is 4 log points, which is about a third of the rise in the baseline model.

The last column reports the inverse of the parameter  $1/\chi$ , which we will use to assess whether our calibrations are consistent with the existing empirical evidence on the sensitivity of firm-

TABLE D.II  
TARGETED MOMENTS (ELASTIC CAPITAL CALIBRATIONS)

Moment	Period	Data	Low	Medium	High
Conditional micro moments					
Equity payout yield	1985-2015	-0.022	-0.022	-0.022	-0.022
Growth rate of wealth	1985-2015	0.32	0.32	0.32	0.32
Market leverage	1985-2015	1.4	1.4	1.4	1.4
Duration	1985-2015	35	34	34	34
Macro moments					
Return on capital	1985	0.07	0.071	0.072	0.072
Depreciation rate	1985-2015	0.08	0.08	0.081	0.081
Pareto inequality	1985-2015	0.6	0.6	0.6	0.6
Aggregate duration	1985-2015	20	21	21	21
NFA to domestic wealth	1985-2015	-0.05	-0.05	-0.05	-0.05
Change in return on capital	1985-2015	0	-0.005	-0.01	-0.015

TABLE D.III  
MODEL-IMPLIED REGRESSION COEFFICIENTS (PERCENTAGE POINTS)

Calibration	$\Delta \text{rok}$	$\Delta \log \theta$	$1/\chi$
Baseline	0.0	10.9	0
Low elasticity	-0.5	9.3	0.3
Medium elasticity	-1.0	7.9	0.6
High elasticity	-1.5	6.7	1
Very high elasticity	-4.0	3.5	3.6

level investment to the cost of capital. Recall that, in the model, the following structural relationship holds

$$g_{s,t} = \underline{g}_s + \frac{1}{\chi}(q_{s,t} - 1). \quad (56)$$

If the state  $s$  was observed, we could therefore consistently estimate  $1/\chi$  by running a regression of the firm-level investment rate  $g$  on  $q$  with a state fixed-effect. Alternatively, if the state  $s$  is sufficiently persistent, the state fixed-effect could be proxied by a firm fixed-effect. For instance, Table 2 of [Peters and Taylor \(2017\)](#) reports comparable regression coefficients of investment rate on  $q$  (with year and firm fixed effects) using Compustat data on public firms from 1975 to 2011. They report the values for different types of investment: physical, intangibles, and R&D. In Panel B of [Peters and Taylor \(2017\)](#), the authors use the usual definition of  $q$  (i.e., enterprise value over physical capital) and report regression coefficient values ranging from 0.3 pp. (for R&D investment) to 0.6 pp. (for physical investment). Taking these numbers at face value, our baseline calibration (with an implied value of  $1/\chi = 0$ ) is not too far off and the “medium elasticity” calibration compares favorably to the data. The authors also propose an improved measure of  $q$ , which accounts for the presence of intangible capital, and obtain larger regression coefficient values, ranging from 1.3 pp. (for R&D investment) to 2.9 pp. (for physical capital). Those values are almost an order of magnitude larger, and are closer to our high elasticity calibrations.