

A Q-Theory of Inequality*

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Abstract

We study the effect of interest rates on top wealth inequality. While lower rates decrease the average growth rate of existing fortunes, they *increase* the growth rate of new fortunes by making it cheaper to raise capital. Using a sufficient statistic approach, we show that the second effect dominates whenever individuals in the right tail of the wealth distribution are on average net equity issuers. Quantitatively, we find that the secular decline in real interest rates has been a major contributor to the rise of top wealth inequality in the U.S.

1 Introduction

Since the seminal work [Wold and Whittle \(1957\)](#), a widely held view is that wealth inequality goes hand-in-hand with high average returns on wealth (see, in particular, [Piketty, 2015](#)). However, this view seems at odds with the recent U.S. data: wealth inequality increased substantially in the past forty years, in a period marked by declining real interest rates. Relatedly, a growing body of evidence suggests that the recent rise in top wealth inequality is driven by the creation of new fortunes, rather than the growth of existing ones ([Bach et al., 2017](#); [Campbell et al., 2019](#); [Gârleanu and Panageas, 2017](#); [Gomez, 2019](#); [Zheng, 2019](#)).

In this paper, we argue that lower discount rates (i.e., lower required return on wealth) can actually *increase* wealth inequality. This is because low discount rates imply high valuations, which allow successful entrepreneurs to raise capital more cheaply. Put differently, while lower rates decrease average returns, they can increase the realized returns of those making it to the top.

We formalize this intuition in a parsimonious model of wealth accumulation. We show that the long-run effect of discount rates on top wealth inequality depends on the equity issuance rate as well as the leverage of the firms owned by entrepreneurs *making it to the top*. In the U.S., entrepreneurs in the right tail of the distribution tend to be net net equity issuers, and be moderately levered, which implies that a decrease in discount rates increases wealth inequality.

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Using a sufficient statistic approach, we find that the effect of discount rates on top wealth inequality is quantitatively large. According to our preferred measures, the long-run decline of discount rates since 1985 can explain roughly two-thirds of the rise in top wealth inequality.

Our paper has three main parts. We first present our idea in the simplest model possible. Entrepreneurs are born with trees that require constant investment to grow. In order to fund the growth of their trees, entrepreneurs sell shares to outside investors (i.e., rentiers). Trees mature with a constant hazard rate. When trees mature, they generate a one-time dividend proportional to their size, at which point entrepreneurs themselves become rentiers, investing their wealth in a diversified portfolio of trees.

In this stylized economy, a decline in discount rates *increases* top wealth inequality (i.e., decrease the Pareto exponent of the wealth distribution). Even though lower rates decrease the average return of rentiers, they increase the growth rate of wealth for successful entrepreneurs (i.e., the ones with trees that keep growing). When rates are low, valuations are high, which decreases the rate at which these entrepreneurs get diluted over time.

Second, to quantify this mechanism, we build a more realistic model of wealth accumulation. Entrepreneurs are born with firms. Each firm's TFP follows an idiosyncratic Markov chain with an arbitrary transition matrix. Firms optimally invest to maximize their values, trading off the marginal productivity of capital with adjustment costs (q-theory of investment).

Despite the large degree of heterogeneity in the model, we are able to derive an analytical expression for the effect of the interest rate on the Pareto exponent of the wealth distribution. What matters is the effect of the interest rate on the growth rate of individuals *making it to the top*, rather than its effect on the average growth rate of individuals *already at the top*. Put differently, the only thing that matters for tail inequality is the effect of the interest rate on the *past* growth rate of individuals currently in the right tail of the wealth distribution, not on their *current* growth rate.

Using this result, we develop a sufficient statistic approach to measure the effect of discount rates on tail inequality. It depends on two key statistics: the lifetime average payout yield of the firms owned by entrepreneurs in the right tail of the wealth distribution and the sensitivity of their values to a change in interest rates (i.e., their duration). The more negative the payout yield is, or the higher the duration, the bigger the effect of a decline in discount rates on top wealth inequality.

We extend the model to allow entrepreneurs to be over-levered with respect to their firms. In this case, the effect of discount rates on tail inequality also depends on the average leverage of the firms owned by entrepreneurs making it to the top. The higher the leverage, the bigger the effect of discount rates on top wealth inequality. Quantitatively, the "leverage" effect is important for firms that do not raise equity over their lifetime, such as family firms. However, it is relatively small for firms backed by VC funding, which tend to be all-equity firms.

Finally, we use our sufficient statistic approach to quantify the effect of discount rates on tail inequality. We estimate the lifetime average payout yield, duration, and leverage for the top 100 wealthiest individuals in the U.S. in 2015. We find that the average lifetime payout yield of firms owned by top individuals is -3%. It is negative, which means that they tend to have been net equity issuers. The distribution of payout yields is negatively skewed: while

some older individuals have a positive payout yield, the payout yields of firms funded by VCs can be as low as -15% (e.g., Uber or Tesla). We estimate an average duration of 30 years. Finally, we estimate an average leverage of 1.5 (i.e., individuals at the top tend to be over-levered).

Combining these estimates, we find that the effect of discount rates on wealth inequality is large. According to our preferred measures, a decline of 1 percentage point in discount rates increases tail inequality by 7%. Since tail inequality has increased by 30% since 1985 (i.e., the Pareto exponent has decreased from 1.65 to 1.3), this suggests that a decline of discount rates by 3 percentage points since 1985 can explain two-thirds of the rise in top wealth inequality.

Related Literature. This paper contributes to a growing literature that seeks to understand the factors that caused wealth inequality to increase in the U.S. over the post-1980 period, as documented in [Saez and Zucman \(2016\)](#) and [Batty et al. \(2019\)](#).

Most of the existing literature argues that the recent rise in wealth inequality comes from an increase in the (average) return on wealth ([Piketty, 2015](#); [Kuhn et al., 2017](#); [Pastor and Veronesi, 2018](#); [Moll et al., 2019](#); [Hubmer et al., 2019](#)).¹ In contrast, we argue that it may come from a *decrease* in the average return of wealth. In our model, a lower (ex-ante) rate of return reduces the growth rate of the average entrepreneur, but it increases the (ex-post) growth rate of the most successful entrepreneurs. As discussed earlier, the reason is that low rates imply high valuations, which allow successful entrepreneurs to fund the growth of their firm cheaply by issuing equity. By stressing the importance of the distribution of ex-post (cumulated) returns, our paper is related to a growing literature that focuses on the role of return heterogeneity for top wealth inequality ([Benhabib et al., 2011](#); [Toda, 2014](#); [Bach et al., 2015](#); [Fagereng et al., 2016](#); [Bach et al., 2017](#); [Gomez, 2019](#); [Campbell et al., 2019](#); [Zheng, 2019](#); [Atkeson and Irie, 2020](#)).

We build on the existing literature by modeling top inequality through random growth ([Wold and Whittle, 1957](#); [Acemoglu and Robinson, 2015](#); [Jones, 2015](#)). Recently, this literature has moved towards models with *persistent* growth rate heterogeneity ([Luttmer, 2011](#); [Jones and Kim, 2016](#); [Benhabib et al., 2015](#); [Gabaix et al., 2016](#)). In this case, the Pareto exponent can be obtained as the principal eigenvalue of an operator related to the transition matrix between states (see [de Saporta, 2005](#); [Beare et al., 2019](#)). Relative to that literature, a contribution of our paper is to obtain a closed-form expression for the derivative of the Pareto exponent with respect to a parameter (here, the interest rate). Remarkably, it can be expressed in terms of moments that we can measure in the data, namely the lifetime average payout yield and leverage of firms owned by entrepreneurs *reaching the top of the wealth distribution*. This “sufficient statistic” approach allows us to directly quantify the effect of discount rates on inequality.

Our model also relates to the literature on entrepreneurial wealth accumulation ([Quadrini, 2000](#), [Cagetti and De Nardi, 2006](#); [Moll, 2014](#); [Guvenen et al., 2019](#); [Peter, 2019](#); [Tsiaras, 2019](#)). As in these papers, we assume that entrepreneurs remain exposed to their firms, which plays a key role in shaping the wealth distribution. One key difference is that we consider a model where firms can freely issue equity. In our model, as in the data, the most successful firms continuously raise equity and, therefore, vastly outgrow their founder. This allows us to dis-

¹[Hubmer et al., 2019](#) argue that the decline in tax progressivity has played a key role in increasing the average *after-tax* return on wealth. [Kaymak and Poschke, 2016](#) also emphasize the importance of the decline in tax progressivity.

tinguish two channels by which lower rates can increase top wealth inequality: the “leverage” effect (i.e., entrepreneurs are over-levered) and the “dilution” effect (i.e. entrepreneurs issue equity). The “dilution” effect turns out to be more important quantitatively, consistent with the fact that firms backed by VC funding tend to be all-equity firms.

Finally, a growing literature in macro-finance examines the role of equity issuance, or “displacement”, for asset prices (Gârleanu et al., 2012; Gârleanu and Panageas, 2017; Gârleanu and Panageas, 2019; Kogan et al., 2020; Gârleanu and Panageas, 2020). A central idea in this literature is that lower discount rates, i.e. higher valuations, benefit people that sell firms at the expense of people that buy firms. Our key contribution is to assess the role of this mechanism for top wealth inequality, both in theory and in the data.² Finally, our focus on equity issuance relates this paper to a large literature studying how firms raise capital: this includes VC funding (Cochrane, 2005; Opp, 2019; Gornall and Strebulaev, 2015; Gornall and Strebulaev, 2020), equity-based compensation (Ofek and Yermack, 2000; Frydman and Jenter, 2010; Ai et al., 2018; Eisfeldt et al., 2019), IPOs (Ritter and Welch, 2002; Pastor and Veronesi, 2005), as well as seasoned equity offering (Fama and French, 2004; Baker and Wurgler, 2006; Boudoukh et al., 2007).

2 Stylized model

In this section, we describe our mechanism in a stylized model. We obtain a closed-form formula for tail inequality, which allows us to show that tail inequality is a “u-shaped” function of the interest rate.

2.1 Environment

The economy is populated by infinitely-lived agents and population grows at rate η . Agents are born as “entrepreneurs” and are endowed with a tree. Trees require investment by outside investors (from now on “rentiers”) to grow until they mature. When the tree matures, entrepreneurs cash out and become rentiers themselves and they invest in a diversified portfolio of trees.

Trees. Each tree starts with a size of one and grows at rate g . With hazard rate δ , the tree matures and returns a one-time dividend equal to its size. To grow, trees require a continuous flow of investment proportional to their size. Formally, the instantaneous cash-flow dD_t of a tree is given by

$$dD_t = \begin{cases} -ie^{\delta t} dt & \text{if } t < T \\ e^{\delta t} & \text{if } t = T, \end{cases}$$

²In these models, lower discount rates typically decrease top wealth inequality. This is because equity issuance is modeled as a one-off event, i.e. something that happens at birth or with a constant hazard rate. In contrast, in our model, some successful entrepreneurs continuously raise equity over their lifetime. This turns out to be key for lower discount rates to actually increase top wealth inequality.

where T is the stochastic time at which the tree matures. We assume that $g - \delta < \eta$ so that trees do not grow faster than the population. We also assume that $i < \delta$ so that trees return a positive amount of dividend in expectation.

Pricing. Because dividends are proportional to the size of a tree, the value of a tree is proportional to its size. Denote q the value of a tree of size one. The instantaneous return of holding a tree is given by

$$\frac{dR_t}{R_t} = \begin{cases} \left(-\frac{i}{q} + g\right) dt & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T. \end{cases} \quad (2.1)$$

While the tree is still growing, i.e. $t < T$, the instantaneous return dR_{jt}/R_{jt} is the sum of the payout yield, $-i/q dt$, and a capital gain equal to the growth of the tree $g dt$. When the tree matures, i.e. $t = T$, the instantaneous return is $1/q - 1$, since the tree returns a one-time dividend equal to its size.

The return of the tree *conditionally on not maturing*, $-i/q + g$, can be seen as the growth rate of the tree g , minus an adjustment that corresponds to the fraction of new shares i/q that must be sold to outside investor to raise i . This adjustment reflects the fact that existing shareholders are diluted due to the issuance of these new shares.

Denote r the discount rate, i.e. the required rate of return in the economy. Assume that $r > g - \delta$ to ensure that the price of the tree is finite. The price q is pinned down by the fact that the expected return of a tree must equal the discount rate r :

$$r = -\frac{i}{q} + g + \delta \left(\frac{1}{q} - 1\right) \quad (2.2)$$

Note that there is no risk adjustment for the risk of maturity, since it is purely idiosyncratic. Because we assumed that $\delta - i > 0$, the price q decreases as a function of the discount rate r , as usual.

Wealth accumulation. Agents have log utility and discount the future at rate ρ which implies that they optimally consume a constant fraction ρ of their wealth. Let W_t the wealth of an individual.

We make the assumption that entrepreneurs must remain fully exposed to their tree while it is growing. The instantaneous wealth growth of an entrepreneur is $dW_t/W_t = dR_t/R_t - \rho dt$, i.e.

$$\frac{dW_t}{W_t} = \begin{cases} \left(-\frac{i}{q} + g - \rho\right) dt & \text{if } t < T \\ \frac{1}{q} - 1 & \text{if } t = T. \end{cases} \quad (2.3)$$

While the tree still grows, the wealth of the agent grows exponentially at rate $-i/q + g - \rho$. At maturity, the tree is transformed into output, and, therefore, the return of an entrepreneur is

$1/q - 1$.

When the tree matures, the entrepreneur then becomes a rentier and invests in a diversified portfolio of trees. The wealth of a rentier evolves as

$$\frac{dW_t}{W_t} = (r - \rho) dt. \quad (2.4)$$

Figure 1 plots the total wealth of an entrepreneur with a tree maturing at date $T = 15$ in a low vs high interest rate economy. It shows the heterogeneous effect of the interest rate across agents in the economy. While a lower interest rate decreases the growth rate of rentiers, it increases the growth rate of successful entrepreneurs. Lower rates, i.e. higher valuations, decrease the fraction of shares i/q that must be sold to outside investors in order to finance growth. Therefore, successful entrepreneurs get less diluted as their firms grow.

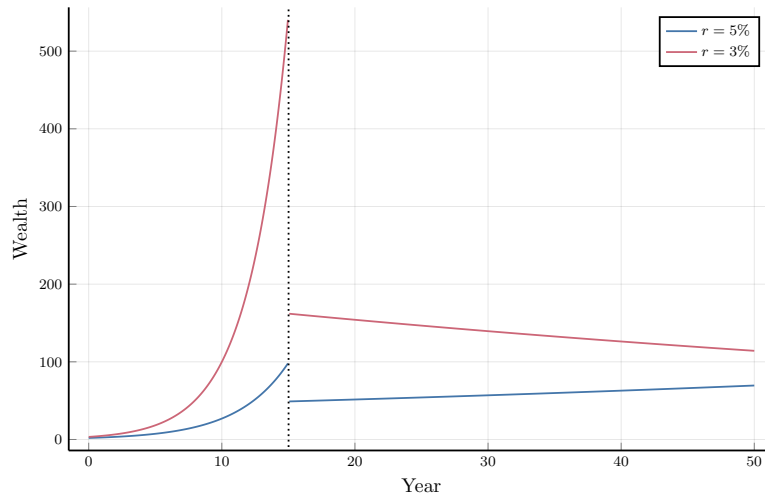


Figure 1: Realized wealth of an entrepreneur with a tree that matures at 15 years ($T = 15$)
Numerical example with $i = 0.4$, $g = 0.5$, $\delta = 0.5$, $\eta = 0.05$, $\rho = 0.04$

Discussing our assumptions. We want to discuss two key assumptions that we make the model. The first assumption is that trees require outside investment to grow, i.e. $i > 0$. This assumption captures an important characteristic of the firms owned by the most successful entrepreneurs in the U.S: they required a lot of outside funding to grow. As we will discuss in Section 4, this equity issuance took the form of VC funding, stock-based compensation, and public equity offering. The assumption that firms are systematically net equity issuers will be relaxed in the more general model, in Section 3.

The second key assumption is that entrepreneurs must remain fully exposed to their tree. This assumption is motivated by the data: most of the wealth of entrepreneurs is invested in their own firms (Quadrini, 2000; Cagetti and De Nardi, 2006; Roussanov, 2010). This type of friction can be derived from a moral hazard problem (He and Krishnamurthy, 2012; Brunnermeier and Sannikov, 2014; di Tella, 2017). In the general model, we will allow entrepreneurs to have an exposure to their firms lower or higher than one.

Lastly, the key distinction between entrepreneurs and rentiers is that entrepreneurs fully invest in one tree whereas rentiers own a diversified portfolio of trees. By “entrepreneur”, we

therefore refer to any agent that is disproportionately exposed to a growing tree. This includes founders of firms that use equity financing, but also early employees paid in stock-options or restricted stocks, or even VC investors with concentrated portfolios.

2.2 Interest rates and the wealth distribution

We now define our measure of tail inequality and examine its relationship to the interest rate.

Tail Inequality. We focus on a measure of wealth inequality that captures the right tail of the distribution.

Definition 2.1. We say that the distribution of a random variable X has a Pareto tail if there exists $\zeta > 0$ such that

$$\lim_{x \rightarrow \infty} \frac{\log \mathbb{P}(X > x)}{\log x} = -\zeta.$$

ζ is called the Pareto exponent.

We define the tail inequality index as $\theta = 1/\zeta$ for a distribution with a Pareto tail. A higher level of tail inequality θ thus imply a thicker upper tail of the wealth distribution. The following proposition characterizes steady-state tail inequality as the discount rate r varies.³

Proposition 2.2. Assume that $0 < i < g - \rho$. There exists $r^* \in (g - \delta, \rho + \eta)$ such that

$$\theta(r) = \begin{cases} \frac{-\frac{i}{q(r)} + g - \rho}{\eta + \delta} & \text{for } r \in (g - \delta, r^*) \\ \frac{r - \rho}{\eta} & \text{for } r \in (r^*, \rho + \eta) \end{cases}$$

Tail inequality $\theta(\cdot)$ is a “u-shaped” function of the interest rate: it is a strictly decreasing function for $r \in (g - \delta, r^*)$ and a strictly increasing function for $r \in (r^*, \rho + \eta)$.

When $r > r^*$, we obtain the usual result that lower rates decrease tail inequality. The reason is that for high enough rates of return, the upper tail of the wealth distribution is only populated by rentiers, and these agents benefit from higher interest rates. The “rentier regime” ($r > r^*$) is probably not the empirically relevant case. In the data, the wealthiest Americans often have most of their wealth invested in fast-growing firms that they founded.

In contrast, when $r < r^*$, the upper tail of the wealth distribution is populated by entrepreneurs and rentiers. In this case, lower rates *increase* tail inequality. This is because the right tail of the distribution is determined by the growth rate of successful entrepreneurs and those agents benefit from lower interest rates. The key idea is that a decline in the interest rate r leads to an increase of the price of share q . High valuations imply that entrepreneurs can finance the growth of their tree by issuing fewer shares, thereby leading to a lower dilution rate. Figure 2 plots the relationship between the discount rate and tail inequality in a numerical example.

³To be precise, we consider a *balanced growth path* where aggregates grow at rate η . For simplicity, we characterize steady state distribution of wealth *normalized* by the total wealth in the economy.

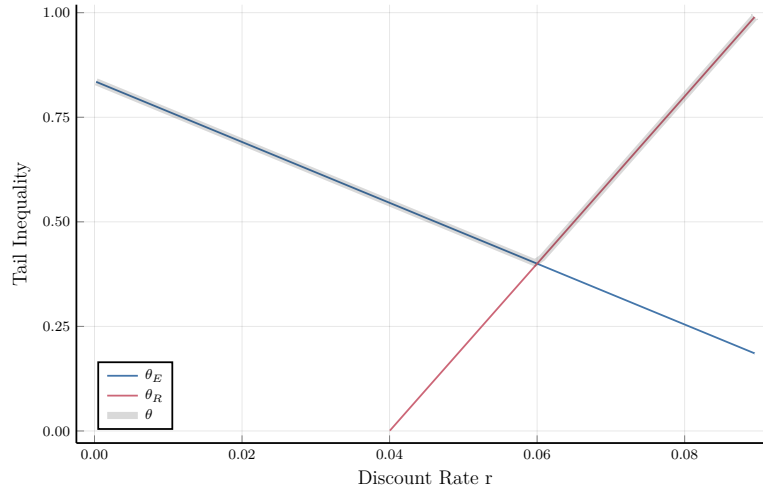


Figure 2: Tail inequality as a function of the interest rate.
Numerical example with $i = 0.4$, $g = 0.5$, $\delta = 0.5$, $\eta = 0.05$, $\rho = 0.04$

Proposition 2.2 characterizes the *long-run* effect of a change in r on tail inequality. In Appendix A.2, we also characterize the *transitory* dynamics of the wealth distribution as the interest rate decreases. We summarize our results here: due to the sudden rise in valuations, the wealth of existing fortunes initially jumps. This positive effect is short-lived, since they earn lower returns from now on. Despite these lower returns, top wealth shares rise due to a composition effect: there is a growing flow of successful entrepreneurs entering the top. This is consistent with the importance of “displacement” in the recent rise of top wealth shares, as documented in Gomez (2019), Gârleanu and Panageas (2017), and Zheng (2019).

So far, we have considered an exogenous change in discount rates. We are agnostic about the exact source of the change in the interest rate. It could come from a change in savings coming from abroad (e.g., global savings glut) or from domestic markets (e.g. population aging). For the sake of example, we extend the model to incorporate an additional group of agents (i.e., “workers”). We show in Appendix A.3 that changes in the discount rate of workers can generate the changes in interest rates considered in Proposition 2.2.

The simple model presented in this section predicts that the sensitivity of tail inequality θ to the interest rate depends on the dilution rate i/q and the sensitivity of the share price to the interest rate (i.e., the duration). This insight holds in a much more general model, as we show in the next section.

3 General model: a q-theory of inequality

We now extend the stylized model to make it more realistic. Entrepreneurs now own firms, not trees. We allow for an arbitrary degree of firm heterogeneity, i.e. firm TFP follows a continuous-time Markov Chain. Moreover, firm investment is optimally chosen to maximize the value of the firm. Despite these extensions, the model remains tractable. In particular, we are able to derive a sufficient statistic for the effect of discount rates on top wealth inequality.

3.1 Set up

Environment. The economy is populated by a continuum of entrepreneurs and rentiers that grows at exogenous rate η . Agents are born entrepreneurs and are endowed with a firm. The firm has initial capital K_0 and initial TFP a_0 is distributed according to a probability distribution ψ_0 . All risk is idiosyncratic. At rate δ , entrepreneurs sell their firm and become a rentiers who live forever and hold a diversified portfolio with return r .

Firm problem. Firms produce an homogeneous consumption good (the *numéraire*) and operate an aK technology where a denotes TFP and K denotes the capital stock. TFP evolves over time according to time-reversible Markov Chain with states $s \in \{1, \dots, N\}$.

The problem of a firm in state s is to choose a growth rate g as to maximize the present value of future payouts discounted at rate r . In order to grow its capital stock by gK , the firm must invest $i(g)K$ units of the consumption goods, where $i'(\cdot) > 0$ and $i''(\cdot) > 0$. The value of a firm $V_s(K)$ is the solution to the following *Hamilton-Jacobi* equation (HJB)

$$rV_s(K) = \max_g \left\{ (a_s - i(g))K + V'_s(K)gK + (\mathcal{T}V)_s(K) \right\}. \quad (3.1)$$

where $\mathcal{T} = (\tau_{ss'})$ denotes the transition probability matrix for the states s .⁴ Given that the value function is homogeneous in capital K , it can be expressed as $V_s(K) = q_s K$, where q_s is ratio of the market value of the firm to its book value (i.e., Tobin's q). The HJB thus simplifies to

$$rq_s = \max_g \left\{ a_s - i(g) + q_s g + (\mathcal{T}q)_s \right\}. \quad (3.2)$$

From now on, we assume that there exists a solution to (3.2). The optimal growth rate g_s satisfies the following first-order condition

$$i'(g_s) = q_s. \quad (3.3)$$

Pricing. The return of a firm is:⁵

$$\frac{dR_t}{R_t} = \underbrace{\frac{a_s - i(g_s)}{q_s} dt}_{\text{payout yield}} + \underbrace{g_s dt + \frac{dq_s}{q_s}}_{\text{capital gains}}.$$

The return is the sum of three terms: the payout yield $(a_s - i(g_s))/q_s$, the growth rate of the firm g_s , and the change in share price dq_s/q_s .

It is useful to think of a firm that never pays dividends and simply sells and repurchases shares to distribute payouts to shareholders. When the firm payout is negative (i.e., $i(g_s) > a_s$), the return is the growth rate of the number of outstanding shares g_s , minus the fraction $(i(g_s) - a_s)/q_s$ of shares sold to outside investors, plus the change in the value of a share.

⁴To be precise, \mathcal{T} is the infinitesimal generator for s defined as $\mathbb{E}_s[\frac{dq_s}{dt}] = (\mathcal{T}q)_s$. We abuse notation and treat \mathcal{T} as an $S \times S$ matrix.

⁵By abuse of notation, we denote s the state of the firm at time t .

Conversely, when the firm payout is positive, the return is the growth rate of the number of outstanding shares plus the fraction $(a_s - i(g_s))/q_s$ of shares that are repurchased from existing shareholders, plus the change in the value of a share.

The HJB ensures that the expected return of owning a share equals the interest rate r , which pins down q_s .

Wealth accumulation. Agents have log utility and discount the future with rate ρ , which implies that they optimally consume a constant fraction ρ of their wealth. We assume that entrepreneurs must hold all of their wealth in the firm equity. The growth rate of wealth for an entrepreneur is given by

$$\frac{dW_t}{W_t} = \frac{dR_t}{R_t} - \rho dt. \quad (3.4)$$

Rentiers invest in a mutual fund that holds a diversified portfolio of firms. The growth rate of wealth for rentiers is simply given by

$$\frac{dW_t}{W_t} = (r - \rho) dt. \quad (3.5)$$

3.2 Book wealth

It is useful to define the book wealth of an entrepreneur B_t as wealth divided by the share price, i.e. $B_t = W_t/q_t$. It corresponds to the amount of firm capital owned by the entrepreneur.⁶

The distribution of book wealth and market wealth for entrepreneurs share the same tail inequality index θ .⁷ This is because the ratio between the two (i.e., the price of a share q) is bounded. Intuitively, inequality between wealthy entrepreneurs is mostly driven by differences in the amount of capital that they own, rather than differences in the market value q of that capital.

Book wealth is easier to work with because, in contrast with market wealth, book wealth does not “jump” when the TFP of the firm changes. Using Proposition 3.4, the law of motion of book wealth of an entrepreneur holding a firm is given by

$$\frac{dB_t}{B_t} = \underbrace{\left(\frac{a_s - i_s}{q_s} + g_s - \rho \right)}_{\mu_s} dt \quad (3.6)$$

Return on assets. We now compare the growth rate of the capital owned by the entrepreneur μ to the profitability of the firm as measured by its *return on assets*. The accounting concept of return on assets (roa) is the ratio of income received by shareholders to the book value of assets. In our model, the roa equals the TFP adjusted for capital depreciation and adjustment

⁶We could also define the book wealth of rentiers as $B_t = W_t/\bar{q}$, where \bar{q} denotes the price of a share in the mutual fund. See Appendix B.1 for more details regarding the balance sheet of the mutual fund.

⁷See the proof of Proposition 3.1.

costs

$$\text{roa}_s \equiv a_s - (i(g_s) - g_s).$$

We can now rewrite the growth rate of book wealth (given in Equation 3.6) in terms of the roa:

$$\frac{dB_t}{B_t} = \underbrace{\left(\text{roa}_s + (g_s - \text{roa}_s) \left(1 - \frac{1}{q_s} \right) - \rho \right)}_{\mu_s} dt.$$

When there is no equity issuance, i.e. $\text{roa}_s = g_s$, the growth rate of book wealth is simply given by the difference between the return on assets roa and her consumption rate ρ . However, in presence of equity issuance, there is an additional term $(g_s - \text{roa}_s) \left(1 - \frac{1}{q_s} \right)$, which equals the proportion of new shares issued to raise capital $(g_s - \text{roa}_s)/q_s$ time the difference between the share price and one.

In short, in a model with equity issuance, there are two distinct ways by which an entrepreneur can accumulate firm capital: either by owning a firm with a high return on assets roa, or by issuing new shares with prices higher than one. It is through this second term that lower rates (higher valuations), affect the growth rate of entrepreneurs.

3.3 Tail inequality

We now characterize the steady-state wealth distribution for entrepreneurs and rentiers, as well as the effect of discount rates on tail inequality. The following proposition characterizes the tail inequality index θ .

Proposition 3.1. *Suppose that there exists $\theta_E > 0$ such that*

$$\rho_D \left(\frac{1}{\theta_E} \mathcal{D}(\mu) + \mathcal{T} \right) = \delta + \eta,$$

where $\mathcal{D}(\mu)$ is a diagonal matrix with (μ_1, \dots, μ_S) on its main diagonal and $\rho_D(\cdot)$ denotes the largest real eigenvalue of a matrix. Then the distribution of wealth has a Pareto tail and tail inequality θ is given by

$$\theta = \max \left(\theta_E, \frac{r - \rho}{\eta} \right).$$

If $\theta_E > \frac{r - \rho}{\eta}$, both types of agents have the same tail inequality $\theta = \theta_E$ and the upper tail of the wealth distribution is populated by both entrepreneurs and rentiers. Otherwise, the distribution of wealth across rentiers has a thicker tail than the distribution of wealth across entrepreneurs and the upper tail of the wealth distribution is populated only by rentiers. As discussed earlier, the first case is the empirically relevant one, so from now on we assume that θ_E exists and is greater than $\frac{r - \rho}{\eta}$.

Local expression. One can show that the tail inequality θ can be expressed as the ratio of the average growth rate of entrepreneurs in the right tail divided by the sum of population growth

η and the entrepreneurship exit rate δ

$$\theta = \frac{\lim_{b \rightarrow +\infty} \mathbb{E}[\mu | B = b]}{\eta + \delta}. \quad (3.7)$$

In the case of homogenous growth (i.e., μ is the same for all agents), the above expression coincides with the one in [Wold and Whittle \(1957\)](#). Using this expression, we can write the derivative of (log) tail inequality with respect to the interest rate as the change in the (proportional) change in the average growth rate of agents in the right tail

$$\partial_r \log \theta = \frac{\partial_r \lim_{b \rightarrow +\infty} \mathbb{E}[\mu | B = b]}{\lim_{x \rightarrow +\infty} \mathbb{E}[\mu | B = b]}. \quad (3.8)$$

However, the right hand side in this expression cannot be measured in the data, since it requires to know how the composition of households in the right tail reacts to the interest rate. This is because a change in the interest rate changes the average growth rate of entrepreneurs “at the top” of the wealth distribution, but also change the composition of agents “reaching the top”.

What we need is to obtain the derivative of tail inequality as an expectation of derivatives rather than a derivative of an expectation. We now derive an expression for the derivative of tail inequality θ with respect to the interest rate r . This is the main theoretical result of this paper.

Proposition 3.2 (Sufficient statistic). *Let τ denote the age of an entrepreneur, and s_t the state of her firm at age t . The derivative of (log) tail inequality θ with respect to the interest rate is given by*

$$\partial_r \log \theta = \lim_{b \rightarrow +\infty} \mathbb{E} \left[\frac{\frac{1}{\tau} \int_0^\tau \partial_r \mu_{s_t} dt}{\frac{1}{\tau} \int_0^\tau \mu_{s_t} dt} \middle| B = b \right]. \quad (3.9)$$

This equation says that the effect of the interest rate on tail inequality is given by its effect on the *past* growth rate of entrepreneurs in the right tail. This form of ex-post conditioning is key: what matters is not the effect of interest rates on the average growth rate of entrepreneurs already at the top, it is its effect on on the growth rate of the entrepreneurs that *are going to reach the top*.

What is remarkable is that the right hand side of this expression can be directly measured in the data. Using Equation 3.6, the derivative of the growth rate of book wealth μ_s can be written as

$$\partial_r \mu_s = \underbrace{\frac{a_s - i(g_s)}{q_s}}_{\text{payout yield}} \underbrace{\partial_r \log q_s}_{\text{duration}} + \underbrace{\left(1 - \frac{i'(g_s)}{q_s}\right)}_{=0} \partial_r g_s. \quad (3.10)$$

It is the sum of a term that is due to the effect of r on the share price, and a term that is due to the effect of r on the optimal growth rate of the firm. Because the optimal growth rate ensures that the firm invests up to the point where the marginal cost of capital equals its marginal value, the second term is zero (an application of the envelope theorem).

Plugging the expression for the derivative of μ (Equation 3.10) into Proposition 3.2, we obtain a sufficient statistic for the effect of interest rates on tail inequality

$$\partial_r \log \theta = \lim_{b \rightarrow +\infty} \mathbb{E} \left[\frac{\frac{1}{\tau} \int_0^\tau \frac{a_{s_t} - i(g_{s_t})}{q_{s_t}} \partial_r \log q_{s_t} dt}{\frac{1}{\tau} \int_0^\tau \mu_{s_t} dt} \middle| B = b \right]. \quad (3.11)$$

Lower interest rates increase tail inequality as long as entrepreneurs reaching the top are on average net equity issuers (i.e., $\frac{a-i(g)}{q} < 0$). Intuitively, when interest rates are low, valuations are high, which means that fewer shares need to be sold to outside investors to raise funds. This increases the rate at which successful entrepreneurs accumulate firm capital.

We now show how to extend our sufficient statistic approach to account for firm leverage. In Appendix D, we also extend the approach to allow agents's savings rates to react to changes in interest rates, which happens when the elasticity of intertemporal substitution is different from one.

3.4 Leverage

So far, we have considered the case of all-equity firms. In reality, firms issue both equity and debt claims. We now introduce debt issuance in a tractable way. While the capital structure of the firm has no effect on the growth rate of the firm, it has implications for the return process for equity-holders (including the founder). We now describe how accounting for leverage changes the effect of interest rates on tail inequality.

Book leverage. Let λ denote the book leverage of a firm (the ratio of capital to equity) and assume that the entrepreneur must invest all of her wealth in equity shares of the levered firm. Let $q_{\lambda s}$ be the price of an equity share for a firm in state s with book leverage λ

$$q_{\lambda s} = 1 + \lambda(q_s - 1). \quad (3.12)$$

The return of a levered firm in state s is the sum of the equity payout, growth rate of book equity, and change in equity price

$$\frac{dR_{\lambda t}}{R_{\lambda t}} = \frac{\lambda(a_s - i(g_s)) + (1 - \lambda)(g_s - r)}{q_{\lambda s}} dt + g_s dt + \frac{dq_{\lambda s}}{q_{\lambda s}}.$$

We now describe how leverage affects wealth accumulation as well as its sensitivity with respect to the interest rate.

The law of motion of wealth for an entrepreneurs is

$$\frac{dW_t}{W_t} = \frac{dR_{\lambda t}}{R_{\lambda t}} - \rho dt.$$

As before, we define book wealth as $B_t = W_t/q_{\lambda_s}$. The growth rate of book wealth is given by

$$\frac{dB_t}{B_t} = \underbrace{\left(\frac{\lambda(a_s - i(g_s)) + (1 - \lambda)(g_s - r)}{q_{\lambda_s}} + g_s - \rho \right)}_{\mu_{\lambda_s}} dt.$$

Tail Inequality. The sufficient statistic becomes

$$\partial_r \log \theta = \lim_{b \rightarrow +\infty} \mathbb{E} \left[\frac{\frac{1}{\tau} \int_0^\tau \partial_r \mu_{\lambda_{s_t}} dt}{\frac{1}{\tau} \int_0^\tau \mu_{\lambda_{s_t}} dt} \middle| B = b \right]. \quad (3.13)$$

where the derivative of the growth rate of an entrepreneur holding a levered firm with respect to the interest rate is

$$\partial_r \mu_{\lambda_s} = \underbrace{\frac{\lambda(a_s - i(g_s)) + (1 - \lambda)(g_s - r)}{q_{\lambda_s}} \partial_r \log q_{\lambda_s}}_{\text{dilution effect}} + \underbrace{1 - \lambda \frac{q_s}{q_{\lambda_s}}}_{\text{leverage effect}}. \quad (3.14)$$

Notice that there are two differences with the formula that does not account for leverage (Equation 3.10). First, the expression for the payout yield is now the one of the levered equity claim $\lambda(a_s - i(g_s)) + (1 - \lambda)(r - g_s)$. Second, leverage has a direct effect on the growth rate of the entrepreneur. For example, consider a firm with a zero payout yield. Whenever “market leverage” exceeds one, the leverage effect implies that a lower interest rate implies a higher growth rate.⁸

4 Empirics

In this section, we use data from various sources to estimate the effect of discount rates on top wealth inequality. We then use this estimate to quantify how much of the rise in top wealth inequality in the U.S. over the post 1980 period can be explained by the decline in discount rates.

4.1 Measuring the sufficient statistic

In presence of leverage, as derived in Equation 3.13, the derivative of tail inequality with respect to the interest rate is:

$$\partial_r \log \theta = \lim_{b \rightarrow +\infty} \mathbb{E} \left[\frac{\frac{1}{\tau} \int_0^\tau \left(\frac{a_{s_t} - i(g_{s_t})}{q_{s_t}} \partial_r \log q_{s_t} + 1 - \lambda \frac{q_{s_t}}{q_{\lambda_{s_t}}} \right) dt}{\frac{1}{\tau} \int_0^\tau \mu_{s_t} dt} \middle| B = b \right]. \quad (4.1)$$

To estimate it in the data, we need to measure the lifetime average payout yield, duration, market leverage, and growth rate of wealth for individuals in the right tail of the distribution. Given a sample of N individuals in the right tail of the wealth distribution, our estimator for

⁸Market leverage is defined as the ratio of firm assets and equity both at market value, i.e. $\lambda \frac{q_s}{q_{\lambda_s}}$.

the derivative of tail inequality with respect to the interest rate is:

$$\widehat{\partial_r \log \theta} = \frac{1}{N} \sum_{i=1}^N \frac{\text{payout yield}_i \times \text{duration} + 1 - \text{market leverage}_i}{\text{growth rate}_i}. \quad (4.2)$$

As we discuss shortly, we calibrate “duration” rather than trying to estimate it separately for each individual.

Forbes 400. We identify individuals in the right tail of the wealth distribution using the list of the wealthiest 400 Americans constructed by Forbes Magazine. The list is created by a dedicated staff of the magazine, based on a mix of public and private information.⁹ Figure 3 plots our measure of tail inequality θ for the sample of Forbes 400 from 1985 to 2015. Using two alternative methods (i.e., “top share” and “log rank regression” approaches), we find that θ increased from roughly 0.55 to 0.7 over the time period, which corresponds to a relative increase of 24 log points.

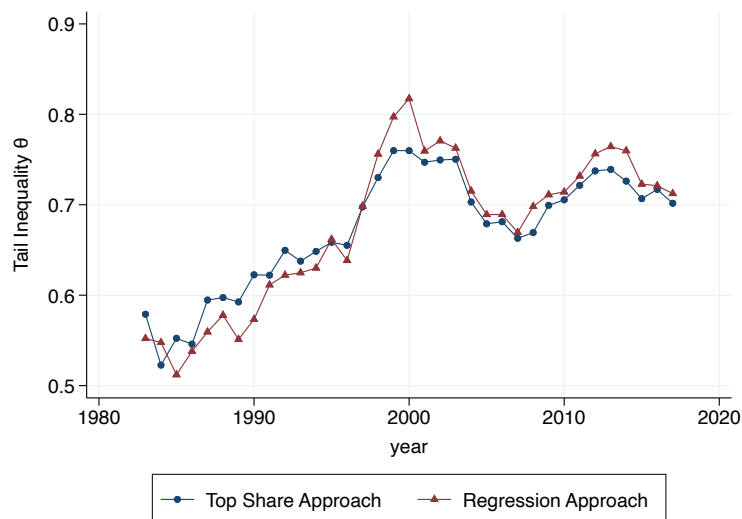


Figure 3: Tail inequality 1985-2015

Notes. This figure plots the estimate of the tail inequality θ using data from Forbes 400. In blue, we estimate θ as 1 minus the wealth of 400th individual divided by the average wealth of an individual in Forbes 400 (see Jones and Kim, 2016). In red, we estimate θ as the opposite of the slope in a regression of log networth on log rank (see Gabaix and Ibragimov, 2011).

We now focus on the year 2015 and define the right tail as the top 100, a group for which information is more widely available. Out of this set of individuals, we remove 31 individuals who inherited all their wealth (they correspond to rentiers in our framework). We also remove 19 individuals who own financial firms (for instance hedge fund managers), since our framework does not directly apply to them. We are left with 50 individuals for which we have detailed information (age, wealth, source of wealth, firms that they founded).

⁹Forbes Magazine reports: “We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don’t pretend to know what is listed on each billionaire’s private balance sheet, although some candidates do provide paperwork to that effect.”

Growth rate of wealth. We approximate the cumulative growth rate of wealth for individuals at the top of the wealth distribution as the log of the ratio between current wealth and an (imputed) initial wealth of \$100,000 in 2015 dollars. We obtain the growth rate by dividing the cumulative growth rate by the age minus 18. We reports summary statistics on the average payout yield in Table 1. We estimate an average growth rate of 23.6%, with big outliers corresponding to Mark Zuckerberg and Dustin Moskovitz (Facebook).

Payout yield. We use the following methodology to compute the average payout yield of firms owned by individuals in Forbes 100. We first describe our methodology for firms for which accounting statements are available (SEC filings). Following the methodology of [Fama and French \(2005\)](#), we estimate the yearly equity issuance rate as cash dividends minus the yearly change in the book value of stockholders’ equity in excess of the yearly change in retained earnings, divided by the average book equity during the period. Importantly, this measure of equity issuance includes all forms of equity issuances: IPO, seasoned equity offering, but also stock issued in firm acquisition, as well as stocks granted to employees.

Because firms typically include past accounting statements in their SEC S-1 registration form, this method captures a significant portion of equity issuance pre-IPO. For the period preceding the first year reported in the S-1 filing form, we compute the yearly equity issuance rate since founding date consistent with the book equity and retained earnings reported in that year.¹⁰ Finally, we obtain the firm average payout yield by dividing the equity issuance rate by the market value of the firm, assuming a constant market-to-book equity in years prior to the IPO. (More details are given in Appendix C).

For firms that have issued equity but for which we do not have accounting statements (for instance because they were bought before getting public such as WhatsApp), we use publicly available information on funding rounds. This method is imperfect because it makes it hard to account for stock granted to employees, and, therefore, it tends to underestimate the dilution rate. Finally, we set a net payout yield equal to zero for private firms that are fully owned by their founder.

We reports summary statistics on the average payout yield in Table 1. We find that firms owned by individuals in the right tail in 2015 have had an average annual payout yield of -3.3% since they were founded. Notice that there is substantial variation ranging from -16% for Travis Kalanick (Uber) to 2.5% for Les Wexner (L brands).

Table 1: Summary Statistics

	Obs.	Average	Percentiles				
			Min	p25	p50	p75	Max
Growth Rate (%)	50	23.6	11.7	15.5	17.6	25.0	88.4
Payout Yield (%)	50	-3.3	-16.0	-5.3	-1.8	0.0	2.5
Market Leverage	50	1.5	1.0	1.1	1.5	1.5	3.1

¹⁰We obtain founding date from Ritter’s website as described in [Ritter \(2016\)](#).

Duration. The effect of the interest rate on tail inequality depends on the average duration of the firms owned by individuals that reach the top of the wealth distribution. Duration is defined as the derivative of the log value of the firm equity with respect to the interest rate. This quantity is hard to measure quantitatively, since it needs to be an ex-ante measure of the cash-flow duration of the firm, rather than an ex-post.

In our preferred calibration we assume a duration of 30 years, which we see as conservative.¹¹ It implies that if the interest rate permanently decreases by 1 percentage point, the valuation of firms would decrease by 30%. We also explore two alternative hypothesis: a duration of 20 years and a duration of 40 years.

Market leverage. We define the market leverage of a company as the value of the firm (i.e. sum of the market value of the equity plus debt) divided by the market value of the equity in years for which these variables are observables. We set the market leverage of private firms to 1.5, in line with the average leverage of public firms in our sample.

Results. We use Equation 4.2 to combine our estimates for the average payout yield, market leverage, and growth rate of wealth of individuals reaching the top of the wealth distribution. Table 2 contains the predicted effect of the interest rate on tail inequality $\partial_r \log \theta$ in our preferred calibration as well as using a alternative hypotheses about duration. To describe the variation in the data, we also compute the sufficient statistic for each individual and report summary statistic. In our favorite calibration, we obtain a value for $\partial_r \log \theta$ of -7.0. The interpretation is that for a 1 percentage point decrease in the interest rate, tail inequality increases by 7.0%. Alternative calibrations imply values ranging between -4% and -8% (i.e., in the same order of magnitude).

Table 2: Measuring the effect of interest rate on tail inequality $\partial_r \log \theta$

	Obs.	Average	Percentiles				
			Min	p25	p50	p75	Max
$\partial_r \log \theta$ (Duration = 30 years)	50	-7.0	-25.7	-11.0	-4.1	-2.7	1.6
Duration = 20 years	50	-5.7	-21.6	-7.7	-3.7	-2.7	0.0
Duration = 40 years	50	-8.4	-29.8	-14.4	-4.4	-3.0	3.3

4.2 Contribution of lower discount rates to rising inequality

To quantify the importance of our proposed mechanism, we compare the actual increase of tail inequality between 1985 and 2015 to the one we can predict on the basis of the long run decline of real interest rates in the period. To be precise, using our sufficient statistic we compute the counter-factual tail inequality in 2015 assuming that discount rates had remained at their 1985

¹¹One reference is [Gormsen and Lazarus \(2019\)](#), who finds in one exercise that the duration of the top 20% of the firms in CRSP is 39 years.

level

$$\widehat{\log \theta}_{2015} = \log \theta_{2015} + \widehat{\partial_r \log \theta} \times (r_{1985} - r_{2015}). \quad (4.3)$$

Our benchmark values are $\theta_{2015} = 0.75$ for the actual increase in tail inequality (as per Figure 3) and $\partial_r \log \theta = -7.0$. The only remaining object to quantify is the change in (real) discount rates $r_{1985} - r_{2015}$. In our model, the discount rate is equal to the risk-free interest rate given that there is no aggregate risk. In reality, it should be measured as the discount rate for an all equity firm (i.e., the sum of the risk-free rate and the equity risk premium)

One reference for the change in the real interest rate is the Cleveland Fed’s real rate model, which implies a decline in the 10-year yield of roughly 5 percentage points from 1985 to 2015. Many explanations have been proposed for the decline in interest rates including: a shortage of safe assets (Caballero et al., 2008); changing demographics (Carvalho et al., 2016); a shift in monetary policy regime (Lettau et al., 2018); secular stagnation (Eggertsson et al., 2019). We do not take a stand on the reason for this phenomenon: we only require it to be something that does not come from change in preferences of entrepreneurs and rentiers.

Estimating the equity risk premium (ERP) is notoriously difficult. Duarte and Rosa (2015) estimate the one-year ahead S&P 500 ERP using the principal component of 20 different models.¹² Their ERP series is very volatile but exhibits somewhat of an increase over the post 1980 period. Overall, their ERP is roughly 3 percentage points higher in 2015 compared to 1985. Assuming an average market leverage for the S&P 500 of roughly 1.5 (as in Barro, 2009), it corresponds to a change in the ERP of un-levered equity of $3/1.5 = 2$.

We now proceed with the assumption that the discount rate r has decreased by 3 percentage points from 1985 to 2015, which corresponds to a 5 percentage points decline in the risk-free rate and a 2 percentage points increase in the ERP. Our framework implies that the contribution of declining discount rates on tail inequality is

$$\widehat{\log \theta}_{2015} - \log \theta_{2015} = \underbrace{\widehat{\partial_r \log \theta}}_{=-7.0} \times \underbrace{(r_{2015} - r_{1985})}_{=-0.03} = -0.21. \quad (4.4)$$

In words, this says that tail inequality would decrease by 0.21 log points if discount rates were to revert to their 1985 level. Because the overall change in tail inequality during the period was roughly 0.3 log points (see Figure 3), we conclude that the decline in discount rates can explain up to two-thirds of the rise in tail inequality during the time period.

5 Conclusion

In this paper, we have studied the effect of discount rates on inequality in an economy where successful entrepreneurs continuously displace existing fortunes. In our model, we show that the effect of discount rates on inequality depends on its effect on arriving fortunes, not existing fortunes. Informed by the fact that, in the data, arriving fortunes tend to net equity issuers, we

¹² “[The models can be categorized] into five groups: predictors that use historical mean returns only, dividend-discount models, cross-sectional regressions, time-series regressions and surveys.”

conclude that low discount rates cause wealth inequality to be high.

We develop a sufficient statistic to quantify the effect of discount rates on inequality. It depends on three key moments: the average growth rate of individuals reaching the top, the average payout yield of the firms that they own, as well as their leverage. After measuring these moments using a combination of Compustat and SEC filings, we conclude that the effect of discount rates on inequality is large. According to our best estimates, the 3% decline in discount rates from 1985 to 2015 can explain two-thirds of the rise in top wealth inequality.

A growing literature argues, conversely, that high wealth inequality causes discount rates to be low (see, for instance, [Mian et al., 2019](#) for interest rates or [Gollier, 2001](#); [Toda and Walsh, 2019](#); [Gomez, 2016](#) for the equity premia). Joining these two mechanisms could give rise to interesting amplification effects. For instance, a rise in wealth inequality and a decline in discount rates could be mutually reinforcing. We leave this topic for future research.

A Appendix for Section 2

A.1 Proof for Proposition 2.2

Proof. Denote

$$\begin{aligned}\theta_R(r) &= \frac{r - \rho}{\eta} \\ \theta_E(r) &= \frac{-\frac{i}{q(r)} + g - \rho}{\eta + \delta}\end{aligned}$$

Because, $r > g - \delta$ the price of the tree is not infinite. Because $r < \rho + \eta$, the distribution of wealth has a finite average, i.e. $\theta_R(r) < 1$.

Note that we can rewrite θ_E in terms of the interest rate r :

$$\theta_E(r) = \frac{r + \delta \left(1 - \frac{1}{q}\right) - \rho}{\eta + \delta}$$

Using this formula, we get:

$$\begin{aligned}\theta_E(\rho + \eta) &< \theta_R(\rho + \eta) = 1 \\ \theta_R(g - \delta) &< \theta_E(g - \delta) < 1\end{aligned}$$

There is a unique r^* at which the two functions intersect, which is given by

$$\begin{aligned}\theta_E(r^*) &= \theta_R(r^*) \\ \Leftrightarrow 1 - \frac{1}{q(r^*)} &= (r^* - \rho) \frac{\delta}{\eta} \\ \Leftrightarrow \eta(-i - r^* - g) &= (r^* - \rho)(-i + \delta) \\ \Leftrightarrow r^* &= \frac{\eta}{-i + \delta + \eta}(g - \delta) + \frac{-i + \delta}{-i + \delta + \eta}(\rho + \eta)\end{aligned}$$

r^* is between the lower bound $g - \delta$ and the upper bound $\rho + \eta$.

Finally, the distribution has a thick tail if $\max(\theta_E(r), \theta_R(r)) > 0$. A necessary and sufficient condition is $\theta_E(\rho) > 0$, which is equivalent to $i < g - \rho$ \square

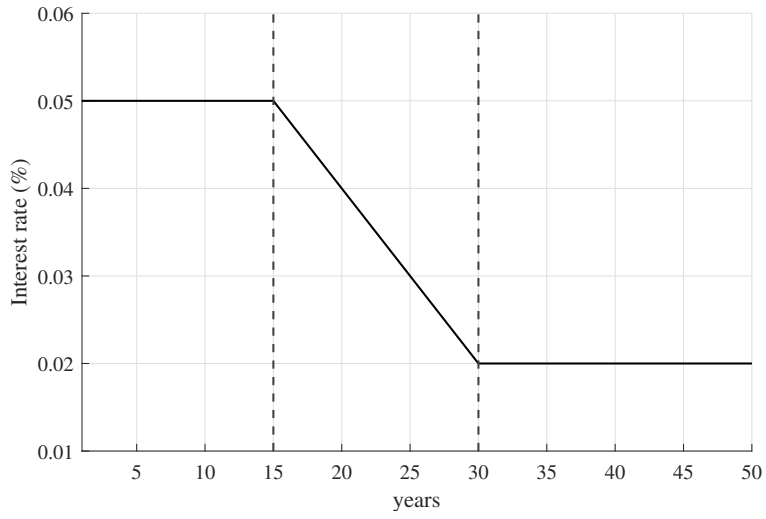
A.2 Transition dynamics

Proposition 2.2 characterizes the *long-run* effect of a change in r on tail inequality. To understand the mechanism, we find it useful to also study the *transitory* dynamics of the wealth distribution as the interest rate decreases.

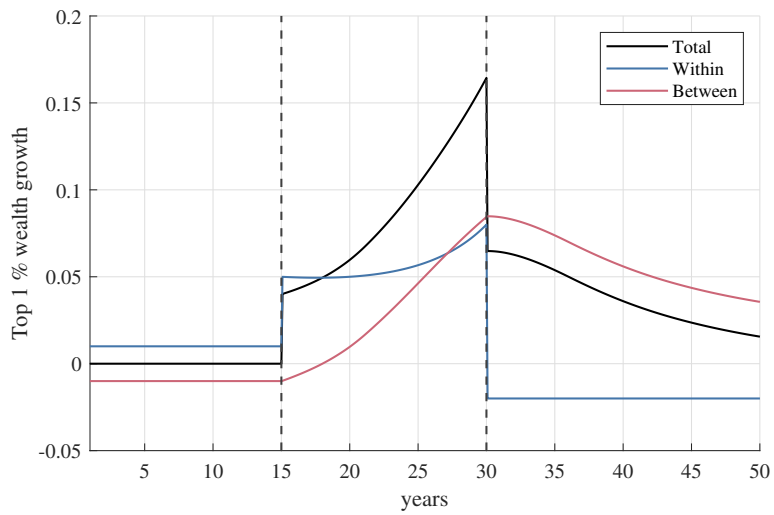
We start in an economy in steady state with an interest rate equals to 5%, using the same calibration as 2. We consider the effect of a unanticipated and permanent decline in the interest rate by 3 percentage points. We assume that the decline is gradual, taking place over the course of fifteen years (see Figure 4a). We plot the evolution of total wealth owned by the top 1% during this time in Figure 4b. We obtain it by computing the evolution of the wealth density using the Kolmogorov Forward equation.¹³

The growth rate of the total wealth in the top 1% is initially zero since the initial distribution is in a steady state. It then becomes positive during the transition period, consistent with the fact tail inequal-

¹³We time discretize the model at the monthly frequency and solve for the transition dynamics over a finite grid using ‘‘Pareto extrapolation’’ (see [Gouin-Bonenfant and Toda, 2019](#)).



(a) Interest rate.



(b) Top 1% wealth growth rate.

Numerical example with $i = 0.4$, $g = 0.5$, $\delta = 0.5$, $\eta = 0.05$, $\rho = 0.04$

ity increases when the interest rate decreases. In the long run, it converges back to zero, reflecting the fact that the wealth distribution reaches its new steady state.

Following Gomez (2019), we decompose the growth rate of wealth held by the top 1% in two terms: a “within” term that is equal to the average wealth growth of *existing* agents in the top 1% and a “between” term that arises due to changes in the composition of agents in the top 1%.¹⁴

As reported in Figure 4b, the within term decreases by 3 percentage points from one steady state to the other. This reflects the fact that the interest rate decreases by 3 percentage points during the period. Interestingly, the within term initially increases during the transition period. The reason is that the average realized return on wealth $r_t dt + dq_t/q_t$ increases due to the unexpected increase in the value of the tree q_t . It is worth emphasizing that these (transitory) capital gains have no effect on the long run level of top wealth shares.

In contrast, the “between” term increases over time. As discussed above, a lower interest rate increases the value of the tree q , which increases the growth rate of entrepreneurs whose tree is still growing. Therefore, there is a growing flow of entrepreneurs reaching the top 1%, which pushes up the total wealth owned by the top 1% as the result of a composition effect. In the long run, the between term exactly compensates the within term: the wealth distribution adjusts so that the flow of new fortunes exactly compensates the decrease in the growth rate of existing fortunes.

A.3 Closing the model

Agents. The full economy now also includes workers. Workers have log utility with a subjective discount factor $\bar{\rho}$. Like entrepreneurs, workers are also born with firms, but they can immediately sell them to the market. Denote π the proportion of agents that are born workers.

Stationary equilibrium. The steady-state share of aggregate wealth owned by workers, denoted x , is given by:

$$0 = (r - \bar{\rho})x + \eta\left(\pi\frac{1}{S} - x\right) \quad (\text{A.1})$$

where S denotes the steady-state average size of a tree:

$$0 = g - \delta + \eta\left(\frac{1}{S} - 1\right). \quad (\text{A.2})$$

Joining the two equations gives x as a function of the interest rate r :

$$(r - \bar{\rho} - \eta)x = \pi(g - \delta - \eta) \quad (\text{A.3})$$

Market clearing. Market clearing for goods requires the amount of goods consumed and invested to be equal to the output of maturing trees:

$$(x\bar{\rho} + (1 - x)\rho)q + i = \delta. \quad (\text{A.4})$$

¹⁴Formally, applying the results in Gomez (2019), we have

$$\frac{dS_t}{S_t} = \underbrace{(r_t - \rho) dt + \frac{dq_t}{q_t}}_{\text{within}} + \underbrace{\left(\int_{x=Q_t}^{Q_t q_t} (x - Q_t)\pi(x)g(x)dx\right) \delta dt}_{\text{between}} + \underbrace{\left(1 - \frac{pQ_t}{S_t}\right) \eta dt}_{\text{demography}}$$

where g_t denotes the wealth density, Q_t denotes the top 1% quantile (i.e. $\int_{Q_t}^{+\infty} g_t(x)dx = p$), S_t denotes the total wealth in the top 1% (i.e. $S_t = \int_{Q_t}^{+\infty} xg_t(x)dx$), and π denotes the proportion of entrepreneurs.

The left-hand side is aggregate demand (consumption and investment), while the right-hand side is aggregate supply. Substituting out q in terms of r (Equation 2.2), we obtain an equation that gives the market clearing interest rate r as a function of x :

$$r = x\bar{\rho} + (1 - x)\rho + g - \delta \quad (\text{A.5})$$

Combining Equation A.3 and Equation A.5 gives a system of two equations and two unknowns x and r . Therefore, we can solve for the equilibrium interest rate.

The next proposition shows that, when π is close enough to one, changes in $\bar{\rho}$ can generate the full spectrum of interest rates considered in Proposition 2.2.

Proposition A.1. Denote $r_\pi(\bar{\rho})$ the interest rate as a function of the subjective discount factor of workers:

1. $r_\pi(\cdot)$ is an increasing function of $\bar{\rho}$. Moreover, as π tends to one, $r_\pi(\cdot)$ spans the interval $(g - \delta, \rho + \eta)$, i.e.

$$\lim_{\pi \rightarrow 1} \lim_{\bar{\rho} \rightarrow 0} r_\pi(\bar{\rho}) = g - \delta$$

$$\lim_{\pi \rightarrow 1} \lim_{\bar{\rho} \rightarrow +\infty} r_\pi(\bar{\rho}) = \rho + \eta$$

2. As long as $i < \delta - \frac{\rho}{1 - \frac{g-\delta}{\eta}}$, the distribution of workers always has a thinner tail than the distribution of entrepreneurs or rentiers. Therefore, tail inequality θ is given by Proposition 2.2.

Therefore, the proposition says that, when the proportion of workers is high enough in the economy, changes in their subjective discount factor generate changes in the interest rate in the economy

Proof of A.1. There exists one and only one solution $x_\pi(\bar{\rho}) \in (0, 1)$ that solves the system given by Equation A.3 and A.5:

$$x_\pi(\bar{\rho}) = \begin{cases} \frac{1 + \vartheta(\bar{\rho}) - \sqrt{(1 + \vartheta(\bar{\rho}))^2 - 4\vartheta(\bar{\rho})(1 - \pi)}}{2} & \text{if } 0 < \bar{\rho} < \rho \\ \pi & \text{if } \bar{\rho} = \rho \\ \frac{1 + \vartheta(\bar{\rho}) + \sqrt{(1 + \vartheta(\bar{\rho}))^2 - 4\vartheta(\bar{\rho})(1 - \pi)}}{2} & \text{if } \bar{\rho} > \rho \end{cases}$$

where $\vartheta(\bar{\rho}) = (\eta - (g - \delta)) / (\rho - \bar{\rho})$.

Moreover, we have

$$\begin{aligned} \lim_{\bar{\rho} \rightarrow 0} r_\pi(\bar{\rho}) &= g - \delta + \rho \frac{1 + \vartheta(0) - \sqrt{(1 + \vartheta(0))^2 - 4\vartheta(0)(1 - \pi)}}{2} \\ \lim_{\bar{\rho} \rightarrow +\infty} r_\pi(\bar{\rho}) &= \rho + \eta - (1 - \pi)(\eta - (g - \delta)) \end{aligned}$$

Therefore

$$\begin{aligned} \lim_{\pi \rightarrow 1} \lim_{\bar{\rho} \rightarrow 0} r_\pi(\bar{\rho}) &= g - \delta \\ \lim_{\pi \rightarrow 1} \lim_{\bar{\rho} \rightarrow +\infty} r_\pi(\bar{\rho}) &= \rho + \eta \end{aligned}$$

Finally, denote $\bar{\theta}(r) = \frac{r - \bar{\rho}}{\eta}$. To ensure that the tail of the wealth distribution is not dominated by workers, a sufficient condition is that

$$\theta_E(r_\pi(\rho)) > \bar{\theta}(r_\pi(\rho))$$

Since $r_\pi(\rho) = \rho + g - \delta$, the condition can be rewritten as

$$\begin{aligned} \frac{g - \delta + \delta \left(1 - \frac{1}{q}\right)}{\delta + \eta} &> \frac{g - \delta}{\eta} \\ \Rightarrow i &< \delta - \frac{\rho}{1 - \frac{g - \delta}{\eta}} \end{aligned}$$

□

B Appendix for Section 3

B.1 Mutual fund balance sheet

Let $K = (K_1, \dots, K_S)'$ be the (normalized) stock of capital in firm of state $s = 1, \dots, S$. The law of motion for K is given by

$$\dot{K} = (\mathcal{D}(g) + \mathcal{T})K + \eta(\psi_0 K_0 - K). \quad (\text{B.1})$$

Let W_{Bs} be the book wealth of entrepreneurs in states $s = 1, \dots, S$ and W_B^R is the book wealth of rentiers.

$$\dot{W}_B = (\mathcal{D}(\mu) + \mathcal{T})W_B + \eta(\psi_0 K_0 - W_B) - \delta W_B \quad (\text{B.2})$$

$$\dot{W}_B^R = (r - \rho)W_B^R + \delta \sum_{s=1}^S \frac{q_s}{\bar{q}} W_{Bs} - \eta W_B^R \quad (\text{B.3})$$

Let W_{Bs}^M be the book wealth in state $s = 1, \dots, S$ that is owned by the mutual fund

$$\dot{W}_B^M = (\mathcal{D}(g) + \mathcal{T})W_B^M + (\mathcal{D}(g) - \mathcal{D}(\mu))W_B^M + \delta W_B - \eta W_B^M \quad (\text{B.4})$$

Notice that $W_B^M + W_B = K \implies \dot{W}_B^M + \dot{W}_B^E = \dot{K}$. The mutual fund q is given by

$$\bar{q} \equiv \frac{\sum_s q_s W_{Bs}^M}{\sum_s W_{Bs}^M} \quad (\text{B.5})$$

The market clearing interest rate ensures that the market value of the capital held by the mutual fund is equal to the wealth of rentiers

$$\sum_s q_s W_B^M = \bar{q} W_B^R. \quad (\text{B.6})$$

B.2 Proof of Proposition 3.1

Proof. First, consider the distribution of book wealth for entrepreneurs. The expression of the inequality index θ_E as the solution to

$$\rho_D \left(\frac{1}{\theta} \mathcal{D}(\mu) + \mathcal{T} - \mathcal{I}(\eta + \delta) \right) = 0$$

is a direct application of application of Theorem 4.1 in [Beare et al. \(2019\)](#) where the “exit rate” is constant across states and given by $\eta + \delta$. Second, consider the wealth distribution for rentiers. Since the growth rate of capital is $r - \rho$ and the “exit rate” is η , then the usual formula for the inequality index is $\frac{r-\rho}{\eta}$ as long as $r > \rho$. But since the initial distribution of capital for rentiers is the same as the distribution of capital for entrepreneurs, then we have that the higher inequality index dominates in the upper tail, and therefore $\theta_R = \max \left\{ \theta_E, \frac{r-\rho}{\eta} \right\}$. Since there are both entrepreneurs and rentiers in the economy, the inequality index is $\theta = \max \left\{ \theta_E, \theta_R \right\}$. Finally, Proposition B.1 establishes the fact that if book wealth W_b has a Pareto tail with inequality index θ , then W also has a Pareto tail with inequality index θ . \square

We now provide an explanation of why the solution to $\rho_D \left(\frac{1}{\theta} \mathcal{D}(\mu) + \mathcal{T} - \mathcal{I}(\eta + \delta) \right) = 0$ is the inequality index of the distribution of book wealth for entrepreneurs. First, consider the Kolmogorov Forward Equation (KFE) for the (normalized) book wealth $p_t(x) \equiv (p_{0t}(x), \dots, p_{St}(x))'$ across entrepreneurs

$$\partial_t p_t(x) = -\mathcal{D}(\mu) \partial_x p_t(x) x + \mathcal{T}' p_t(x) + (\eta + \delta) \left(\mathcal{D}(\psi_0) K_0 - p_t(x) \right).$$

First, conjecture that the stationary distribution satisfies $\lim_{t \rightarrow \infty} p_t(x) \sim u x^{-(1+1/\theta)}$, where $u = (u_1, \dots, u_S)'$ is the asymptotic distribution of types s . Using the Kolmogorov Forward Equation to characterize the tail of the capital distribution in the long-run, we substitute the conjecture in the KFE and obtain

$$0 = \left(\frac{1}{\theta} \mathcal{D}(\mu) + \mathcal{T}' - \mathcal{I}(\eta + \delta) \right) u \iff 0 = u' \left(\frac{1}{\theta} \mathcal{D}(\mu) + \mathcal{T} - \mathcal{I}(\eta + \delta) \right).$$

So the vector u is the left eigenvector of the matrix $\mathcal{A}(\theta) \equiv \frac{1}{\theta} \mathcal{D}(\mu) + \mathcal{T} - \mathcal{I}(\eta + \delta)$ associated with eigenvalue 0. Notice that the existence of a stationary distribution implies that the matrix $\mathcal{A}(\theta)$ has eigenvalues with non-positive real parts. Therefore, the inequality index θ is such that the spectral radius of $\mathcal{A}(\theta)$ is zero. Finally, notice that

$$\rho_D \left(\frac{1}{\theta} \mathcal{D}(\mu) + \mathcal{T} - \mathcal{I}(\eta + \delta) \right) = \rho_D \left(\frac{1}{\theta} \mathcal{D}(\mu) + \mathcal{T} \right) - (\eta + \delta).$$

Proposition B.1. *If B has a Pareto tail with exponent ζ , then W also has a Pareto tail with exponent ζ .*

Proof. Assume that B has a Pareto tail, which implies that

$$\lim_{x \rightarrow \infty} \frac{\log P(B \geq x)}{\log x} = -\zeta.$$

Since the state space is finite and we assumed that there exists a solution to the HJB (3.2), then there exists $0 < a < b < \infty$ such that

$$a < q_s < b$$

for all $s = 1, \dots, S$. We thus have that

$$P(B \geq b^{-1}x) \leq P(W \geq x) \leq P(B \geq a^{-1}x).$$

Therefore, this gives

$$\lim_{x \rightarrow \infty} \frac{\log P(B \geq x)}{\log x} \leq \lim_{x \rightarrow \infty} \frac{\log P(W \geq x)}{\log x} \leq \lim_{x \rightarrow \infty} \frac{\log P(B \geq x)}{\log x},$$

which implies

$$\lim_{x \rightarrow \infty} \frac{\log P(W \geq x)}{\log x} = -\zeta.$$

□

B.3 Proof of Proposition 3.2

Proof. The proof is in two parts

1. First let us prove that

$$\frac{\partial_r \theta}{\theta} = \frac{u' \mathcal{D}(\partial_r \mu) v}{u' \mathcal{D}(\mu) v},$$

where u, v are respectively the left and right eigenvectors of the matrix $\mathcal{A}(\zeta) \equiv \zeta \mathcal{D}(\mu) + \mathcal{T} - \mathcal{I}(\eta + \delta)$ evaluated at $\zeta = 1/\theta$. Let u, v be normalized so that they sum to one. For $\zeta = 1/\theta(r)$, we have that

$$u(\zeta)' \mathcal{A}(\zeta) = 0.$$

Differentiating by r we obtain

$$\partial_\zeta u(\zeta) \partial_r \zeta \mathcal{A}(\zeta) + u(\zeta)' (\partial_\zeta \mathcal{A}(\zeta) \zeta_r + \partial_g \mathcal{A}(\zeta) \partial_r \mu) = 0.$$

Multiplying by the right eigenvector $v(\zeta)$ we obtain

$$\partial_\zeta u(\zeta) \zeta_r \mathcal{A}(\zeta) v(\zeta) + u(\zeta)' \partial_\zeta \mathcal{A}(\zeta) v(\zeta) \partial_r \zeta + u(\zeta)' \partial_g \mathcal{A}(\zeta) \partial_r \mu v(\zeta) = 0.$$

Using the fact that $\mathcal{A}(\zeta) v(\zeta) = 0$, $\partial_\zeta \mathcal{A}(\zeta) = \mathcal{D}(\mu)$, and $\partial_g \mathcal{A}(\zeta) \partial_r \mu = \zeta \mathcal{D}(\partial_r \mu)$, we obtain

$$\partial_r \zeta u(\zeta)' \mathcal{D}(\mu) v(\zeta) + \zeta u(\zeta)' \mathcal{D}(\partial_r \mu) v(\zeta) = 0.$$

Re-arranging and suppressing the dependence of u, v on ζ , we obtain

$$\partial_r \log \zeta = -\frac{u' \mathcal{D}(\partial_r \mu) v}{u' \mathcal{D}(\mu) v}.$$

Since $\log \theta = -\log \zeta$, we have that

$$\partial_r \log \theta = \frac{u' \mathcal{D}(\partial_r \mu) v}{u' \mathcal{D}(\mu) v}$$

2. Second, let us prove that for any stochastic process $\{x_t\}$ such that x_t depends only on the Markov state s , we have:

$$\lim_{b \rightarrow +\infty} \frac{E[\bar{x} | \log B = b]}{b} = \frac{u' \mathcal{D}(x) v}{u' \mathcal{D}(\mu) v},$$

where $\bar{x}_t \equiv \int_{t_0}^t x_{t'} dt'$. From now on define $b = \log B$. We now characterize the stationary density of (\bar{x}, b, s) which we define as $p(\bar{x}, b)$ (i.e., a $S \times 1$ vector). Notice that $d\bar{x}_t = x_s dt$ and $db_t = g_s dt$, so the KFE implies that

$$0 = -\mathcal{D}(x) \partial_{\bar{x}} p(\bar{x}, b) - \mathcal{D}(\mu) \partial_b p(\bar{x}, b) + \mathcal{T}' p(\bar{x}, b) + (\eta + \delta)(p_0(\bar{x}, b) - p(\bar{x}, b)),$$

where $p_0(\bar{x}, b)$ is the distribution of (\bar{x}, b) at birth. We abuse notation and treat x as an $S \times 1$

vector. Let $m(\lambda, b) \equiv \int e^{\lambda \bar{x}} p(\bar{x}, b) d\bar{x}$ be the Laplace transform of $p(\bar{x}, b)$. Multiplying the KFE by $e^{\lambda \bar{x}}$ and integrating over \bar{x} , we obtain

$$0 = -\mathcal{D}(x) \int e^{\lambda \bar{x}} \partial_{\bar{x}} p(\bar{x}, b) d\bar{x} - \mathcal{D}(\mu) \partial_b m(\lambda, b) + \mathcal{T}' m(\lambda, b) + (\eta + \delta)(m_0(\lambda, b) - m(\lambda, b)),$$

where $m_0(\lambda, b) \equiv \int e^{\lambda \bar{x}} p_0(\bar{x}, b) d\bar{x}$. Using integration by part, we have that

$$\int e^{\lambda \bar{x}} \partial_{\bar{x}} p(\bar{x}, b) d\bar{x} = -\lambda m(\lambda, b),$$

which implies that

$$0 = \lambda \mathcal{D}(x) m(\lambda, b) - \mathcal{D}(\mu) \partial_b m(\lambda, b) + \mathcal{T}' m(\lambda, b) + (\eta + \delta)(m_0(\lambda, b) - m(\lambda, b)).$$

Let us now guess that $m(\lambda, b) \sim u_m(\lambda) e^{-b\zeta(\lambda)}$, where $u_m(\lambda)$ is an $S \times 1$ vector that satisfies $\sum_s u_{ms} = 1$. Substituting in the KFE, we get the following characterization

$$0 = u_m(\lambda) \left(\lambda \mathcal{D}(x) + \zeta(\lambda) \mathcal{D}(\mu) + \mathcal{T} - (\eta + \delta) \mathcal{I} \right).$$

Let us define the operator $\mathcal{B}(\lambda) \equiv \lambda \mathcal{D}(x) + \zeta(\lambda) \mathcal{D}(\mu) + \mathcal{T}' - (\eta + \delta) \mathcal{I}$. Notice that the distribution of (b, s) is given by $p(b) = \int p(\bar{x}, b) d\bar{x} = m(0, b)$. Define $\mathcal{L}(\lambda, b) = E[e^{\lambda \bar{x}} | \log B = b]$ and notice that

$$\mathcal{L}(\lambda, b) = \frac{\sum_s \int e^{\lambda \bar{x}} p_s(\bar{x}, b) d\bar{x}}{\sum_s \int p_s(\bar{x}, b) d\bar{x}} = \frac{\sum_s m_s(\lambda, b)}{\sum_s m_s(0, b)}.$$

We have that

$$E[\bar{x} | \log B = b] = \mathcal{L}'(0, b) = \frac{\sum_s \partial_\lambda m_s(0, b)}{\sum_s m_s(0, b)}.$$

Using the asymptotic guess for $m(\lambda, b)$, we have that

$$\begin{aligned} E[\bar{x} | \log B = b] &\sim \frac{\sum_s \partial_\lambda u_{ms}(0) e^{-b\zeta(0)} + \sum_s u_{ms}(0) e^{-b\zeta(0)} w \zeta'(0)}{\sum_s u_{ms}(0) e^{-b\zeta(0)}} \\ &\sim b \zeta'(0). \end{aligned}$$

Finally, using the fact that $u_m(\lambda) \mathcal{B}(\lambda) = 0$ and defining $v_m(\lambda)$ to be the right eigenvector of $\mathcal{B}(\lambda)$ associated with eigenvalue zero, we obtain (following the same steps in part one), that

$$\zeta'(0) = \frac{u_m(0)' \mathcal{D}(x) v_m(0)}{u_m(0)' \mathcal{D}(\mu) v_m(0)}.$$

The operator $\mathcal{B}(\lambda)$ evaluated at $\lambda = 0$ is equal to the operator $\mathcal{A}(\zeta)$ in part one evaluated at $\zeta = \zeta(0)$, so we have that $u_m(0), v_m(0) = u, v$. Putting the pieces together, we have that

$$\lim_{b \rightarrow \infty} \frac{E[\bar{x} | \log B = b]}{b} = \frac{u' \mathcal{D}(x) v}{u' \mathcal{D}(\mu) v}$$

Putting the results from part (1) and (2), we obtain

$$\begin{aligned}\partial_r \log \theta &= \frac{u' \mathcal{D}(\partial_r \mu) v}{u' \mathcal{D}(\mu) v} \\ &= \lim_{b \rightarrow \infty} \frac{E \left[\overline{\partial_r \mu} \mid \log B = b \right]}{b} \\ &= \lim_{b \rightarrow \infty} E \left[\frac{\overline{\partial_r \mu}}{\bar{\mu}} \mid \log B = b \right]\end{aligned}$$

□

C Appendix for Section 4

In this section, we describe our methodology to measure the net payout yield of firms with SEC filings.

Between two accounting statements, we follow the methodology in [Fama and French \(2005\)](#):

$$\text{net payout yield} = \underbrace{\frac{\text{div}_t}{0.5 \times (\text{me}_t + \text{me}_{t-1})}}_{\text{dividends}} + \underbrace{\frac{-(\text{seq}_t - \text{seq}_{t-1} - (\text{re}_t - \text{re}_{t-1}))}{0.5 \times (\text{me}_t + \text{me}_{t-1})}}_{\text{net share repurchases}}.$$

The market value of equity me_t is only available for years after the IPO. For pre-IPO years, we proxy the market value of equity as the book value of equity seq_t multiplied by the market-to-book ratio in the year of the IPO.

For years preceeding the first accounting statement, we measure the net payout yield as

$$\text{net payout yield} = \frac{1}{q_E} \frac{1}{T} \log(\text{seq}_T) \left(1 - \frac{\text{re}_T}{\text{seq}_T} \right).$$

This formula gives the net payout yield under the assumptions of (i) constant issuance rate χ (ii) constant ROE r_E (iii) constant market-to-book ratio q_E (iii) no dividend distribution, between the funding date of the firm and the first year in which balance sheet data is available. Denoting B_t book equity and RE_t retained earnings, clean surplus accounting gives:

$$\begin{aligned}dB_t &= (r_E - \chi) B_t dt \\ \Rightarrow B_t &= B_0 e^{(r_E - \chi)t}.\end{aligned}$$

Because we assume that the firm does not pay dividends, the law of motion of retained earnings RE_t is

$$\begin{aligned}dRE_t &= r_E B_t dt \\ \Rightarrow RE_t - RE_0 &= \frac{r_E}{r_E - \chi} B_0 e^{(r_E - \chi)t}\end{aligned}$$

We can therefore obtain the ratio of issuance to book equity χ as

$$\chi = \frac{1}{T} \log \left(\frac{B_T}{B_0} \right) \left(1 - \frac{RE_T - RE_0}{B_T} \right).$$

This gives the measure of net payout yield given above assuming an initial book equity of one million.

Note that for $T \rightarrow 0$ it is asymptotically equivalent to

$$\chi \approx \frac{B_t - B_0 - (RE_t - RE_0)}{0.5 \times (B_0 + B_t)},$$

which is the measure we use for yearly equity issuance.

D Saving Response

So far we have assumed log utility, which implies that the consumption rate of agents does not react to the interest rate.

We now extend the model with Epstein-Zin utilities, with arbitrary relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS). The effect of interest rate on the consumption rate is given by the difference between 1 and the EIS:

$$\partial_r c = \underbrace{1}_{\text{income effect}} - \underbrace{\text{EIS}}_{\text{substitution effect}},$$

If $\text{EIS} > 1$, the substitution effect is more important than the income effect: households react to a decrease in the interest rate by increasing their consumption rate. If $\text{EIS} < 1$, the income effect is more important than the substitution effect: households react to an decrease in the interest rate by decreasing their consumption rate.

Formally, with an arbitrary EIS, the derivative of the growth rate of book wealth with respect to the interest rate is:

$$\partial_r \mu = \frac{a_s - i(g_s)}{q_s} \partial_r \log q_s + \text{EIS} - 1$$

A EIS higher than one would reduce the effect of lower rates on inequality, because entrepreneurs will start consuming less. In contrast, a EIS lower than one would amplify the effect of lower rates on inequality, because entrepreneurs will start consuming less.

Table 3 reports the results of the sufficient statistics for $\partial_r \log \theta$ for various calibrations of the EIS. The first line reports the case with $\text{EIS} = 1$, which corresponds to the baseline explored in main text. This calibration consistent with evidence from [Vissing-Jørgensen \(2002\)](#) for top stockholders in the U.S. using survey data. We also report the result for $\text{EIS} = 1.5$ and $\text{EIS} = 0.5$. As predicted, the lower the EIS, the more negative the effect interest rates on tail inequality. Importantly, even for a EIS as high as 1.5, we find that a decrease in interest rates still has a negative effect on tail inequality. Therefore, our mechanism is robust to the exact value of the EIS.

Table 3: Measuring the effect of interest rate on tail inequality $\partial_r \log \theta$

	Obs.	Average	Percentiles				
			Min	p25	p50	p75	Max
$\partial_r \log \theta$ (EIS = 1)	50	-7.0	-25.7	-11.0	-4.1	-2.7	1.6
EIS = 1.5	50	-4.4	-22.2	-7.3	-1.4	0.0	4.9
EIS = 0.5	50	-9.7	-29.2	-13.2	-7.5	-5.2	-1.7

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