Inelastic Capital in Intangible Economies*

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Abstract

Capital accumulation in modern economies increasingly takes the form of intangible investment. This form of investment relies heavily on the contributions of specialized workers—such as inventors, managers, and entrepreneurs. To examine the macroeconomic implications of the rising share of intangibles, we develop and calibrate a general neoclassical model where capital formation requires both investment goods (tangible investments) and specialized labor (intangible investments). We show that rising intangibles renders the supply of capital more inelastic owing to the limited supply of specialized labor. Rising intangibles also change the incidence of capital taxation: whereas in traditional neoclassical models the tax burden falls entirely on production workers, in intangible economies, it is borne primarily by specialized workers and capital owners.

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1 Introduction

The standard neoclassical growth model treats capital formation as a frictionless process: final goods can be converted into new capital at no cost. This assumption implies that this model predicts an infinitely elastic supply curve of capital. As a result, shifts in the supply or demand for capital — arising from changes in household saving or changes in the productivity of investment — result in large movements in quantities rather than prices.

This prediction, however, stands in stark contrast to the empirical data, as asset prices exhibit larger volatility than the capital stock. While models with capital adjustment costs yield a short-run inelastic capital supply (Hayashi, 1982), they still imply an infinitely elastic capital supply in the long run. Hence, these models cannot explain why "savings gluts" (Bernanke et al., 2007; Mian et al., 2020) tend to generate higher movements in the price than in the quantity of assets.[1]

This paper takes a different route: we emphasize that transforming final goods into productive capital requires specialized labor—entrepreneurs, inventors, and managers—whose aggregate supply is inherently limited. Because more capital cannot simply be constructed from final goods but must also be conceptualized, designed, and implemented by these scarce workers, the supply curve for capital slopes upward over any horizon.

This more realistic view of capital formation also highlights a key link between financial and labor markets. In our model, an influx of savings raises the wages of specialized investment workers relative to those of production worker. This mechanism suggests that the secular decline in interest rates has played an important role in driving the rise in wage polarization. (Autor and Dorn, 2013). Finally, this perspective on capital formation has crucial implications for the incidence of corporate taxation: while the tax burden falls entirely on production workers in the standard neoclassical model, it is borne primarily by specialized workers and capital owners in intangible economies.

Our paper proceeds as follows. We first develop a stylized extension of the standard neoclassical growth model that features a more realistic investment function. Our key departure is that the investment good is more labor intensive than the consumption good, reflecting the fact that modern capital formation requires specialized labor (i.e., inventors, entrepreneurs, financiers). In addition, we allow for accumulated capital (i.e., building on past research, adjustment frictions) to be an input to capital formation in the short-run.

We then characterize the capital supply elasticity in the model, which governs how responsive firms are to capital demand shocks.[2] In the neoclassical growth model, capital supply is infinitely elastic, since the final good can be frictionlessly transformed into capital. In that case, small demand-side shocks (e.g., interest rate) have large and immediate effect on capital accumulation. In contrast, when investment requires labor in fixed supply, the supply elasticity is finite,

[1]See, for instance, Gutiérrez and Philippon, 2017 and Crouzet and Eberly, 2023 for empirical evidence.

[2]To be precise, the supply elasticity of capital in the model is the elasticity of the capital stock (at some horizon) with respect to a change in the value of capital (i.e., the marginal q).

even in the long-run. We show that a higher degree of intangibility in capital formation leads to a lower capital supply elasticity, both in the short-run and in the long-run.

In Section 3, we map the theoretical framework to the data. Our goal is to provide systematic evidence on the evolution of the labor share in the consumption and investment good sectors over time. To do this, we combine sectoral production data from KLEMS with the BEA's input-output (I-O) make-use matrix to compute the consolidated labor share of each sector. This approach accounts for the labor embedded both directly and indirectly across industries. Our findings reveal a growing wedge in labor intensity between the investment and consumption sectors—particularly in the use of high-skilled labor. As a robustness check, we validate our labor share measure using firm-level accounting data from Compustat.

In Section 4, we explore the quantitative effect of a shift towards intangibles that raises the labor intensity in the investment good sector. We calibrate the key elasticities of the model using data on the uses of capital income and valuations (intangible share, total investment yield, and aggregate capital share), as well as a short-run tax elasticity of investment taken from Chodorow-Reich, Smith, Zidar and Zwick (2024). Using the calibrated model we simulate a large (25 pp.) rise in the share of intangible investment, consistent with our evidence from the US corporate sector over 1972-2022. We find that a shift towards intangibles of the magnitude that we have seen in the data implies a significant redistribution of aggregate income away from production labor towards investment labor (-6 pp. of GDP). The model also predicts that the economy becomes more inelastic: the supply elasticity declines by roughly half in the long-run and one-third in the short-run.

Finally, we use the calibrated model to quantify the incidence of corporate taxes in our economy. In the neoclassical growth model, the capital supply elasticity is infinite. As a result, capital tax cuts have a large effect on investment, and end up being born by production workers via higher wages. In intangible economies, however, tax cut have a weaker effect on investment, hence a lower effect on production worker wages. As a result, capitalists and investment labor absorb more of the shock via revaluation gains (for capitalists) and higher wages (for investment labor). In the calibrated model, the incidence of capital taxes is roughly half for capitalists, a quarter for production labor, and a quarter for investment labor.

Literature review. A growing literature documents a rise in intangible capital (see, e.g., Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Eisfeldt, Falato and Xiaolan, 2023; Corrado, Haskel, Jona-Lasinio and Iommi, 2022). Beyond being harder to measure, intangible capital may exhibit distinct economic properties relative to tangible capital. In Eisfeldt and Papanikolaou (2013), intangible capital is unique because it is embedded in key talents. In Crouzet, Eberly, Eisfeldt and Papanikolaou (2022), intangible capital tends to be non-rival—allowing it to be used simultaneously in different production streams—and is characterized by limited excludability, which prevents firms from capturing all the associated benefits or rents. In this paper, we shift our focus away from the economic properties of intangible capital to its distinct inputs, namely, that it is created by specialized labor in finite supply.

Luttmer (2018) studies an economy in which households supply both managerial and production labor—with managerial labor contributing to both production and investment. Like our paper, Luttmer (2018) highlights that organizational capital is produced using a specialized input by discussing the implications of a fixed supply of managerial capital. Another closely related paper is Bhandari and McGrattan (2021), which highlights that a significant portion of small firms' value stems from organizational capital accumulated through the owner's effort. The paper further explores the implications of this finding for the taxation of non-corporate businesses. Relative to these papers, we provide new data and moments to calibrate a flexible theory of capital supply.

In our model, the revenues from capital formation accrue to three distinct factors: investment goods, investment workers, and existing capital. This is similar to Kogan et al. (2020), which assume that investment workers generate blueprints, which are then implemented by firms through a decreasing return to scale function in investment goods. The key difference is that, in our model, the distribution of investment rents between investment workers and existing capital is determined by their respective Cobb-Douglas exponents in the capital formation production function, rather than being exogenous.[3]

Finally, our paper offers an alternative explanation for investment stagnation despite high Tobin's Q inferred from the data. The existing literature focuses on markups and market power (e.g., Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019; Barkai, 2020; Ball and Mankiw, 2023; De Ridder, 2024). In contrast, we emphasize the fact that capital formation requires specialized labor (i.e., entrepreneurs, inventors, etc.) which constrains the extent to which investment goods can be transformed into productive capital. Put differently, in our paper, the "fixed factor" that limits the ability (or willingness) of firms to scale is not the fact that it would lower their price, but, rather, that it would increase their labor cost.

2 Stylized model

2.1 Setup

Output production. We focus on a representative firm. Output Y_t (i.e., the final good) is produced using capital K_t and production labor $L_{Y,t}$ through the standard Cobb-Douglas production function

$$Y_t = A_{Y,t} K_t^{\alpha} L_{Y,t}^{1-\alpha}, \qquad \text{(output production)}$$

[3] This assumption ensures that factors are paid their marginal products, and so that the decentralized equilibrium implements the first best (this is similar to the Hosios (1990)'s condition in matching models).

where $\alpha \in (0,1)$ denotes the importance of capital relative to labor and $A_{Y,t}$ represent Hicks-neutral productivity in production (TFP). The output can be used to consume C_t or to invest I_t.

Capital formation. Our key departure from the standard neoclassical growth model is that the final good cannot be frictionlessly transformed into productive capital: producing and installing new productive units of capital also requires specialized labor (i.e., inventors, entrepreneurs, managers) as well as accumulated capital (i.e., building on past research, adjustment frictions). We represent this through the following capital formation function:

$$K_{t+1} = (1-\delta)K_t + H_t \quad \text{where} \quad H_t \equiv A_{H,t} K_t^{\theta} \left(I_t^{1-\chi} L_{H,t}^{\chi} \right)^{1-\theta}. \qquad \text{(capital formation)}$$

The capital formation function is a Cobb-Douglas function of three inputs: existing level of capital K_t, the final good I_t , and investment labor $L_{H,t}$. The parameter $\chi \in (0,1)$ governs the importance of specialized labor relative to the final good while the parameter $\theta \in (0,1)$ governs the importance of existing capital relative to the other two inputs. Finally, $A_{H,t}$ represent Hicks-neutral productivity in capital formation (IST shock).[4] We provide a microfoundation for this aggregate capital formation function in Appendix A.3.2.

Our specification of the investment function nests and generalizes several important models used in the literature. One such special case is where capital formation only requires investment goods ($\theta = 0$, $\chi = 0$); that is, $H_t = A_{H,t} I_t$, which corresponds to the neoclassical growth model (NGM).[5] Another important special case is where investment requires a mix of investment goods and existing capital ($\chi = 0$); that is, $H_t = A_{H,t} K_t^{\theta} I_t^{1-\theta}$, which corresponds to the q-theory of investment (e.g., Uzawa, 1969; Hayashi, 1982).[6] The parameter θ governs the curvature of the adjustment cost function.

Firm optimization. The rest of the model is standard. The representative firm takes as given the interest rate r_t, as well as wage rates for production and investment labor $(w_{Y,t}, w_{H,t})$. It chooses production labor $L_{Y,t}$, investment labor $L_{H,t}$, and a quantity of output I_t to maximize the present

[4]Because we choose a Cobb-Douglas production function, this can alternatively represent technology shocks that make it easier to install capital, shocks that make investment good more productive (as in Greenwood et al., 1997), or shocks that make investment labor more productive.

[5]Note that, even in this case, the efficiency $A_{H,t}$ at which the final good can be transformed into capital can change over time as in Greenwood et al. (1997).

[6]Hayashi (1982) assumes capital accumulation takes the form $K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t)$, where $\Phi(\cdot)$ is a function with constant return to scale; hence, our model can be interpreted as the special case $\Phi(I_t, K_t) = K_t^{\theta} I_t^{1-\theta}$. See Appendix A.3.3 for more on the connection between our model and Hayashi (1982). Since this seminal paper, the q-theory of investment has evolved away from modeling capital formation as concave in I/K (like we do) towards cost functions that are convex in I/K. The key difference is that adjustment costs appear in the budget constraint while capital formation appear in the capital accumulation equation.

value of future payouts:

$$V_0 = \max_{\{L_{Y,t}, L_{H,t}, I_t, K_{t+1}\}} \sum_{t=1}^{\infty} R_{0\to t}^{-1} D_t,$$

$$\text{s.t.} \qquad D_t = Y_t - I_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t}, \qquad \text{(firm budget constraint)}$$

where $R_{0\to t} = \prod_{s=0}^{t}(1+r_s)$ denotes the cumulative return of one dollar invested between 0 and t. This maximization is subject to the equations for production, capital formation, and capital accumulation above. The corresponding Lagrangian is

$$\mathcal{L}_0 = \sum_{t\geq 1} R_{0\to t}^{-1} \Big(Y_t - I_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t} \Big) + \sum_{t\geq 1} R_{0\to t}^{-1} q_t \Big((1-\delta) K_t + H_t - K_{t+1} \Big).$$

The Lagrange multiplier q_t can be interpreted as the marginal value of one capital unit. Given our assumption that the production and capital formation functions exhibit linear returns to scale, the market value of the representative firm is given by $V_t = q_t K_{t+1}$; that is, the marginal and average values of capital units are equal (Hayashi, 1982).

Solving the representative firm problem yields the following set of optimality conditions:

$$w_{Y,t} L_{Y,t} = (1-\alpha) \cdot Y_t, \qquad \text{(firm foc } L_{Y,t}\text{)}$$

$$w_{H,t} L_{H,t} = (1-\theta)\chi \cdot q_t H_t, \qquad \text{(firm foc } L_{H,t}\text{)}$$

$$I_t = (1-\theta)(1-\chi) \cdot q_t H_t, \qquad \text{(firm foc } I_t\text{)}$$

$$(1+r_{t+1}) q_t K_{t+1} = \alpha \cdot Y_{t+1} + q_{t+1} \left((1-\delta) K_{t+1} + \theta \cdot H_{t+1} \right), \qquad \text{(firm foc } K_{t+1}\text{)}$$

with the transversality conditions $\lim_{T\to\infty} R_{0\to t}^{-1} q_T K_{T+1} = 0$. Note that, due to our Cobb-Douglas assumption, the three inputs of capital formation — existing capital, investment goods, and investment workers — earn a constant share of the revenues from capital formation $q_t H_t$. More specifically, a share θ of revenues is paid to existing capital holders, a share $(1-\theta)\chi$ is paid to investment workers, and a share $(1-\theta)(1-\chi)$ is paid to purchase investment goods.

Note that capital owners earn two types of income: they receive a constant share α of revenues from output production Y_t and a constant share θ of revenues from capital formation $q_t K_{t+1}$.

Household optimization. Since the focus of the paper is on the production side, we only provide a stylized microfoundation of the household problem. The representative household takes as given the sequence of prices $\{w_{Y,t}, w_{H,t}, r_t\}_{t=1}^{\infty}$ and chooses a sequence of consumption and wealth

$\{C_t, W_t\}_{t=1}^{\infty}$ to maximize welfare:

$$U_0 = \max_{\{C_t, W_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}.$$
$$\text{s.t.} \quad C_t + W_t = w_{Y,t} L_{Y,t} + w_{H,t} L_{H,t} + (1 + r_t) W_{t-1},$$

where V_0, the initial level of wealth, is given. For simplicity, we assume in this section that the representative household supplies a fixed quantity of production labor $L_{Y,t} = 1 - \mu$ and investment labor $L_{H,t} = \mu$; we will consider the more general case in which labor is elastically supplied in Section 4, when taking the model to the data. The household optimality with respect to consumption gives the usual Euler equation:

$$C_t^{-\gamma} = \beta(1 + r_{t+1}) C_{t+1}^{-\gamma}. \qquad \text{(worker foc } C_t\text{)}$$

Equilibrium. An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, I_t, C_t, K_t, Y_t\}_{t\geq 1}$, and prices $\{w_{H,t}, w_{L,t}, r_t, q_t\}_{t\geq 1}$ that solve the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t, firm foc K_{t+1}), household problem (worker foc C_t), with the market clearing conditions $L_{Y,t} = 1 - \mu$, $L_{H,t} = \mu$, and $C_t + I_t = Y_t$, subject to the technological constraints for (output production) and (capital formation). Via Walras' law, we get that household wealth equals firm value: $W = V$.

2.2 Steady-state

We now characterize the steady-state of the model, assuming that the productivity of the production and capital formation functions remain fixed over time $A_{Y,t} = A_Y$ and $A_{H,t} = A_H$. We consider the more general case of a balanced growth path where the productivity of investment and production labor grow at some exogenous rate in Section 4.

It is useful to think of the steady-state (or long-run) quantity of capital as determined by an equilibrium between the demand and supply of capital. The demand for capital directly obtains by combining firm foc K_{t+1} with the steady state condition $H = \delta K$:

$$A_Y \alpha K^{-(1-\alpha)} (1 - \mu)^{1-\alpha} = q\left(r + (1 - \theta)\delta\right). \qquad \text{(capital demand)}$$

This equation can be seen as a market pricing equation, which pins down the price of capital q given the flow of payments to capital holders and the interest rate r. Another, equivalent, interpretation of (capital demand) is that firms demand capital until the marginal productivity of capital (the left-hand side) coincides with its user cost (the right-hand side), as in Hall and Jorgenson (1967).[7] This equation traces a downward slopping demand for capital: a higher capital price q increases its user cost (since a firm would need to borrow more to purchase one unit of capital), reducing the quantity of capital demanded by firms.

[7] The analogy provided by Hall-Jorgenson is that "the firm may be treated as accumulating assets in order to supply capital services to itself".

The supply of capital obtains by plugging optimal investment (firm foc I_t) into the steady-state version of (capital formation):

$$A_H K^\theta \Big(\left((1-\theta)(1-\chi)q\delta K\right)^{1-\chi} \mu^\chi \Big)^{1-\theta} = \delta K. \tag{capital supply}$$

This equation says that, in steady-state, the quantity of new capital goods created every period H (the left-hand side) must equal the quantity of capital goods that depreciate every period (the right-hand side). A higher shadow value of capital q pushes firms to invest more (firm foc I_t), which increases the steady-state supply of capital (capital formation). In the particular case where investment does not require specialized workers ($\chi = 0$), the supply of capital is perfectly elastic and this equation pins down q. As soon as capital formation requires investment labor ($\chi > 0$), however, this equation traces an upward slopping supply curve for capital. Solving for K in (capital supply) and (capital demand) gives the long-run demand and supply curve of capital, respectively. The capital demand curve says that a higher capital price q increases the user cost of capital, decreasing the firm's demand for capital (capital demand). The capital supply curve says that a higher capital price q increases firm investment, increasing the steady-state quantity of capital (capital supply). The equilibrium is determined by the intersection between these two curves.

2.3 Implications

2.3.1 Supply elasticity of capital

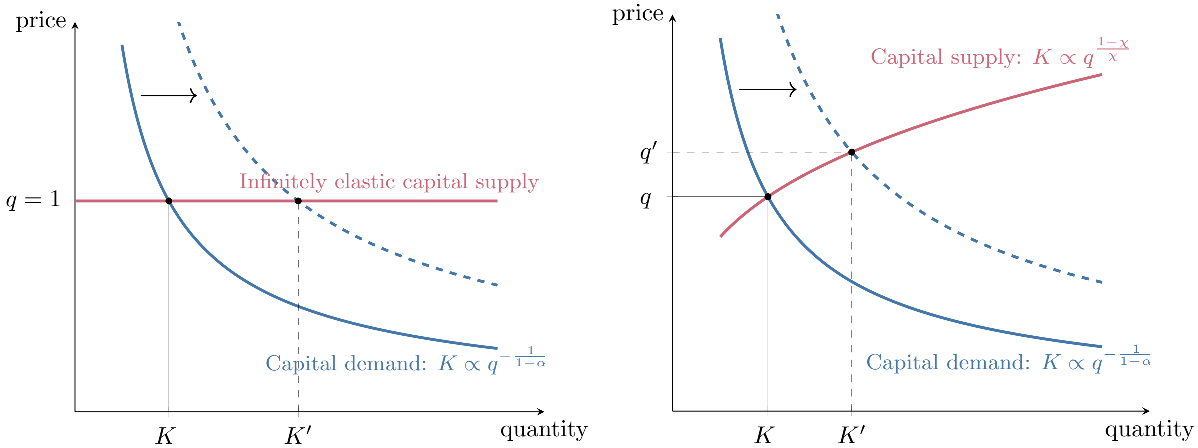
In contrast with the NGM, our model generates an inelastic supply of capital, both in the short-run and in the long-run.

Long-run capital supply. Differentiating (capital supply) implies that the long-run elasticity of capital supply to q is

$$\frac{\partial \log K}{\partial \log q} = \frac{1-\chi}{\chi}. \tag{long-run elasticity}$$

This elasticity spans values from zero (when investment only requires specialized labor) to infinity (when investment only requires the final good, as in the neoclassical growth model). Notice that the long-run capital supply elasticity does not depend on θ; as we discuss below, the parameter θ only determines the short-run capital supply elasticity.

The supply of capital is inelastic when investment requires specialized workers. This has key implications for the long-run effect of a shift in the demand of capital — say, due to a permanent increase in the household's time preference parameter β or a permanent increase in output productivity A_Y. To visualize this fact, Figure 1 plots the steady-state demand and supply for capital. Panel (a) corresponds to the neoclassical growth model ($\chi = 0$), while Panel (b) corresponds to an economy that requires specialized worker for capital formation ($\chi > 0$ in Panel b). In the neo-



(a) Neoclassical growth model ($\chi = 0$) (b) Model with specialized labor ($\chi > 0$)

Figure 1: Long-run equilibrium in the capital market

classical growth model (Panel a), the supply of capital is perfectly elastic, so an outward shift in capital demand results in a higher steady-state capital stock with no change in the rental rate of capital. In contrast, in a model with specialized workers (Panel b), the supply of capital is inelastic, so a shift in the demand for capital is only partially absorbed by a rise in capital, and part of the adjustment takes the form of a higher capital price (i.e., a higher firm value relative to its capital stock).

Short-run capital supply. Capital is also inelastic in the short-run, due to the joint presence of specialized labor ($\chi > 0$) and existing capital ($\theta > 0$) in the capital formation function. Differentiating (capital formation) and (firm foc I_t) implies

$$\frac{\partial \log K_{t+1}}{\partial \log q_t} = \delta \frac{(1-\chi)(1-\theta)}{1-(1-\chi)(1-\theta)}. \qquad \text{(short-run elasticity)}$$

While the capital share of investment θ did not matter for the long-run capital elasticity, it is the key determinant of the short-run capital elasticity. Capital being an input in production $\theta > 0$ makes capital less elastic in the short-run. This is exactly the logic behind the q-theory of investment, which emphasizes the presence of capital adjustment costs in the short-run. As in the long-run, the capital supply elasticity decreases with the importance of specialized labor in capital formation χ.

More generally, for a given sequence of perturbation $(\mathrm{d}\log q_s)_{s \leq t}$, we have:

$$\mathrm{d}\log K_{t+1} = \delta \frac{(1-\chi)(1-\theta)}{1-(1-\chi)(1-\theta)} \sum_{s=0}^{\infty} \left(1 - \delta \frac{\chi(1-\theta)}{1-(1-\chi)(1-\theta)}\right)^{s} \mathrm{d}\log q_{t-s}.$$

See Appendix A.2.2 for the derivation. For a constant price perturbation $\mathrm{d}\log q_{t-s} = \mathrm{d}\log q$, we recover the long-run supply elasticity $(1-\chi)/\chi$.

Table 1 compares the short- and long-run supply elasticity of capital in our model relative to different benchmark models. There are two key takeaways. First, in economies that do not require specialized investment labor ("NGM" and "q-Theory"), capital is infinitely elastic in the long-run. This is because the relative price of capital in terms of goods must be one since they are the same thing. Adding a positive capital share of investment ($\theta > 0$) makes the short-run elasticity finite (see "q-theory"), but does not solve the problem of infinite elasticity in the long-run. Second, the necessity of specialized labor in capital formation (i.e., $\chi > 0$) makes the supply elasticity of capital finite, both in the short- and long-run. This is because labor supply is finite (zero in our case), which makes the response of capital formation to q constrained by the fixed pool of investment worker. In other words, investment booms raise the wages of investment labor, hence dampening the size of the boom.

Table 1: Comparing the supply elasticity of capital across benchmark models

Model	Constraint		Elasticity of capital	
	θ	χ	short-run	long-run
Neoclassical growth	$\theta = 0$	$\chi = 0$	$+\infty$	$+\infty$
Q-theory	$\theta > 0$	$\chi = 0$	$\delta \frac{1-\theta}{\theta}$	$+\infty$
Stylized model	$\theta > 0$	$\chi > 0$	$\delta \frac{(1-\chi)(1-\theta)}{1-(1-\chi)(1-\theta)}$	$\frac{1-\chi}{\chi}$

Notes. The short-run supply elasticity of capital is $\partial \log K_{t+1} / \partial \log q_t$ while the long-run is $\partial \log K_t / \partial \log q$.

2.3.2 Wage gap between investment and production workers

Capital supply is inelastic in our model because capital formation requires an input that is fixed at the aggregate level: investment labor. In equilibrium, after a shift in the demand for capital, firms respond by increasing investment, which bids up the wages of investment workers who are in limited supply. Hence, the flip-side of inelastic capital is that shifts in the demand for capital will affect the wage gap between investment and production workers.

To see this, one can combine (firm foc $L_{Y,t}$) and (firm foc $L_{H,t}$) to express the ratio of payments between investment and production workers:

$$\frac{w_H L_H}{w_Y L_Y} = \frac{1-\theta}{1-\alpha} \cdot \chi \cdot \frac{q_t H_t}{Y_t} = \frac{1-\theta}{1-\alpha} \cdot \chi \cdot \frac{\delta \alpha}{r + \delta(1-\theta)}. \tag{2.1}$$

This first equality reflects that, due to our Cobb-Douglas assumptions for our production functions, investment and production workers receive a fixed share of the revenues from their respective sectors. The second equality uses (capital demand), which pins down the ratio between output and the value of the capital stock in steady state.

Therefore, shocks that disproportionately affect the investment sector will increase the wage gap w_H/w_Y. Consider, for instance, an increase in the household's time preference β generating a decline in r. As shown in (2.1), the decline in r increases the steady-state value of capital relative to output, which increases the wage gap between investment and production workers. Similar, a rise in the importance of specialized workers in capital formation (a rise in χ) increases the wage gap between the two type of workers.

Put differently, while all workers are paid their marginal products, investment workers produce cash-flows that have a longer duration than production workers. As a result, as interest rates decline, the marginal product of investment workers increases relative to production workers, and so the wage gap between the two workers increases as a result.

Our theory for the wage gap between investment and production workers provides an alternative view of the "skill-biased" view of technical change (see, for instance, Griliches, 1969, Krusell et al., 2000, Berlingieri et al., 2024). In our model, investment workers benefit from investment booms because they are key inputs in capital formation, not because they are complements with capital in production. One key difference between the two theories is that, in our model, any shock that make capital more valuable increases the wage of investment workers — whether or not this higher capital value ultimately translates into a higher capital stock. We will return to this idea when quantifying the effect of the rise in intangible investment on the wage gap between investment and production workers in Section 4.

3 Empirical evidence

The central idea of the stylized model is that producing capital goods is more labor intensive—particularly in terms of specialized labor—than producing consumption goods. We now provide empirical evidence that this pattern increasingly holds in the U.S. economy, driven in part by the rising importance of intangible investment.

3.1 Measuring the labor share of investment versus consumption

3.1.1 Evidence from national accounts

Datasets. We begin with the BEA's input-output matrix, available from 1963 to 2010, which provides a detailed decomposition of expenditures across 71 industries (U.S. Bureau of Economic Analysis, 2024a). This matrix tracks how each industry allocates its spending across intermediate inputs, labor, and capital. We further disaggregate the labor component into payments to college-educated and non-college-educated workers using data from the Integrated Industry-Level Production Account (U.S. Bureau of Economic Analysis, 2024b). In addition, the input-output matrix reports how each industry's output is allocated—whether to other industries as intermediate goods, or to final uses in consumption and investment.

Investment in organizational services. In a series of revisions to the National Income and Product Accounts in 1999, 2013, and 2018, the U.S. Bureau of Economic Analysis reclassified firm expenditures on intellectual property products—software, research and development, and artistic originals—as investments. Corrado et al., 2009 argues for a much broader definition of intangible capital. Following their lead, we reclassify certain firm expenditures on professional services as investments.

Specifically, we capitalize 60% of firm spending on the following professional service categories: "Administrative and support services", "Miscellaneous professional, scientific, and technical services", and "Management of companies and enterprises".[8] The first category reflects spending on outsourced business support services, such as personnel administration and training. The second includes externally purchased services like accounting, consulting, design, and computer services. The third captures the service output of establishments that manage other units within a company. Purchases of these services plausibly represent investment in what the literature refers to as organization capital (Atkeson and Kehoe, 2005; Eisfeldt and Papanikolaou, 2013; Crouzet and Eberly, 2021; Crouzet et al., 2022)—expenditures related to workforce human capital, distribution systems, logistics, product design, and customer and brand capital. We group all of these under the umbrella term "Organizational services."

Figure 2 shows the composition of investment as a share of GDP, after this reclassification exercise. We observe a decline in investment in structures and equipment, and a corresponding rise in investment in intellectual property products and organizational services.

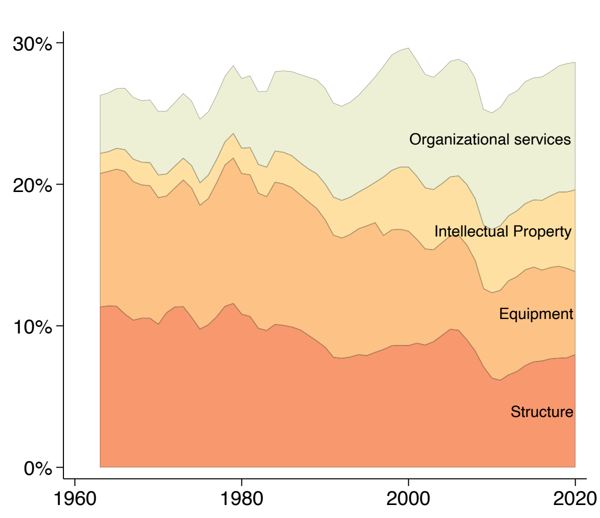


Figure 2: Types of investment expenditures as a fraction of GDP

Methodology. Our goal is to compute the labor share of value added required to produce one unit of a consumption good versus one unit of an investment good.

[8] Ideally, we would use a more detailed industry breakdown, but the BEA input-output matrix is available only at the level of 71 industries for a time series beginning in 1963.

We proceed in two steps. First, we compute the consolidated labor share of each industry. This measure accounts not only for the share of the industry's own value added paid to labor, but also for the labor embodied in all intermediate inputs used in its production. Formally, we use the Leontief inverse of the input-output matrix to capture both direct and indirect contributions of labor throughout the production network.

Second, we compute the labor share of the consumption sector as the weighted average of industry labor shares, where the weights correspond to the share of final consumption expenditures coming from each industry. Similarly, the labor share of the investment sector is calculated using the share of final investment expenditures as weights.

Note that the weighted average labor shares of the consumption and investment sectors sum to the average labor share of the aggregate economy, up to a wedge driven by the difference between labor embodied in exports and imports.

Results. Data from the BEA show that most of the output of each industry is used either for consumption or for investment purposes. Appendix Figure A1 illustrates this pattern: most industries produce primarily either consumption goods or investment goods. This supports our assumption that an industry's income shares are similar regardless of whether its output is used for consumption or investment.

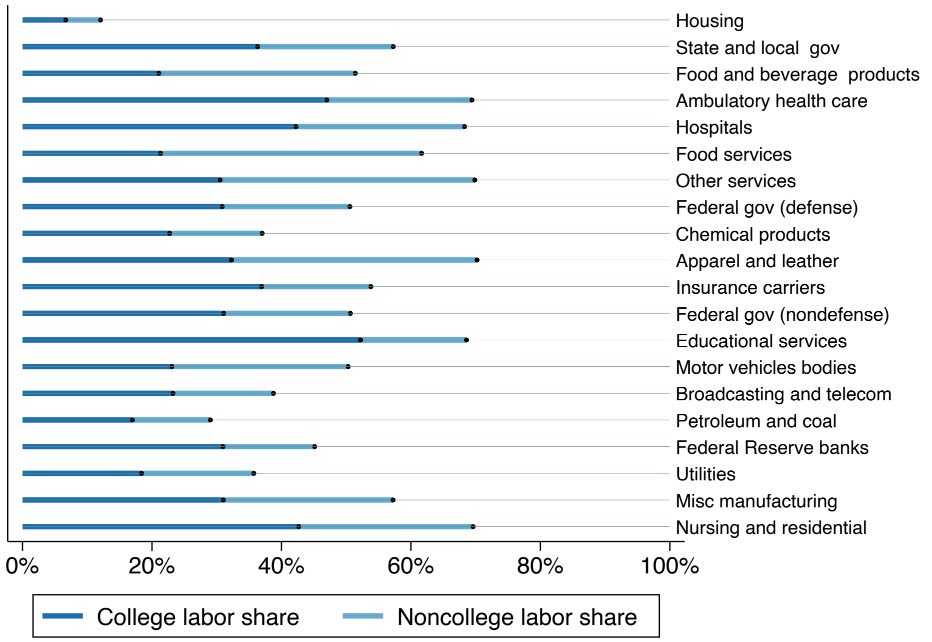
To understand the variation across industries, Figure 3a presents data for the twenty largest industries ranked by their contribution to final consumption expenditures (by households or government). Housing stands out as a major consumption industry and is composed almost entirely of capital services, primarily due to imputed rents from owner-occupied housing. Other significant consumption sectors include food and healthcare.

Figure 3b presents the corresponding data for the twenty largest industries ranked by their contribution to investment expenditures. Industries providing organizational services are particularly important and tend to have high labor shares—especially sectors such as miscellaneous professional and scientific services, administrative and support services, and management of companies. Tangible investment sectors, such as construction, also exhibit high labor shares.

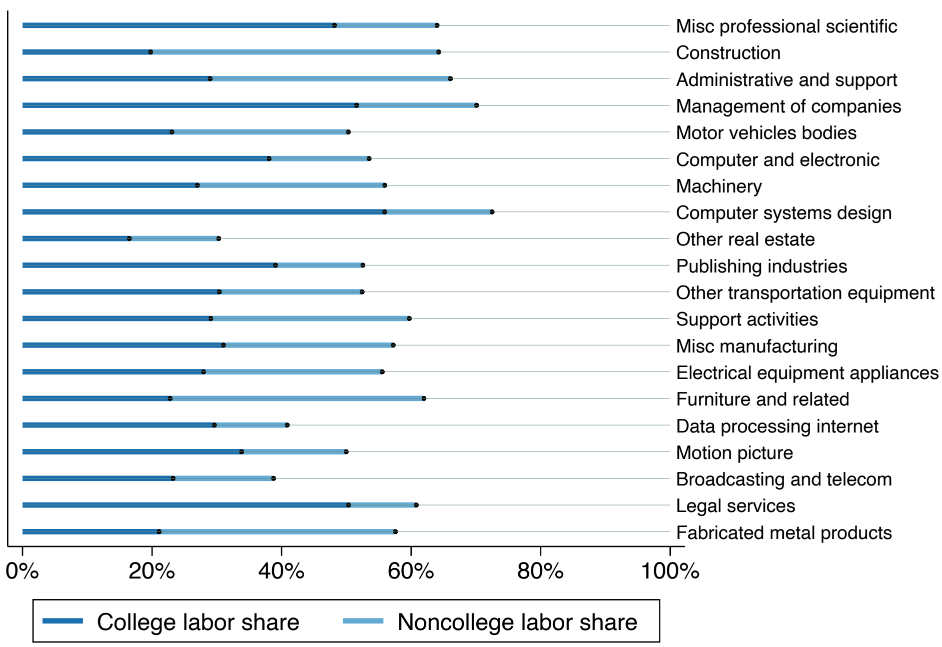
We also report labor shares separately for college- and non-college-educated workers. Consistent with intuition, intangible capital is particularly intensive in college labor, while construction is especially intensive in non-college labor.

Given the consolidated labor share of each industry, we compute the labor share of the aggregate consumption sector by taking the weighted average of industry labor shares, using the share of final consumption expenditures coming from each industry. We apply the same approach to compute the labor share of the aggregate investment sector:

$$LS_C = \sum_i w_{Ci} LS_i \quad ; \quad LS_I = \sum_i w_{Ii} LS_i \tag{3.1}$$



(a) Labor share for largest consumption-producing industries



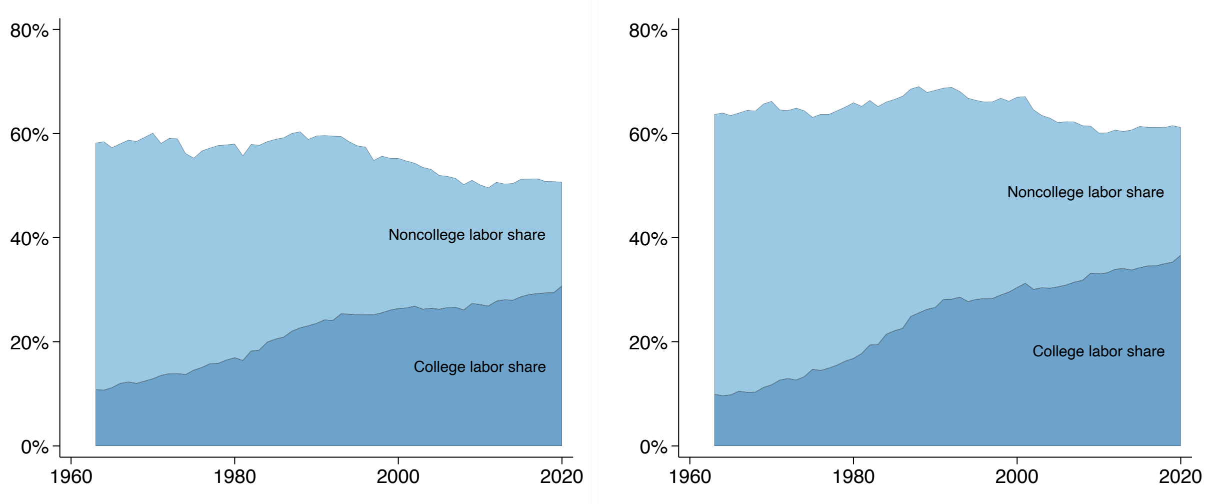
(b) Labor share for largest investment-producing industries

Figure 3: Labor shares by industry in 2017

where w_{Ci} denotes the share of final consumption expenditures in the output of industry i, and w_{Ii} denotes the share of final investment expenditures in the output of industry i.

Figure 4 presents the results. Both sectors begin with a similar labor share in 1962. Over time, however, the labor share in the aggregate consumption sector declines, while the labor share in the investment sector remains relatively stable.

In both sectors, a growing share of labor compensation goes to college-educated workers relative to non-college workers. However, this shift is more pronounced in the investment sector, where the share of payments to college workers has increased more rapidly than in the consumption sector.



(a) Aggregate consumption sector

(b) Aggregate investment sector

Figure 4: Labor income share for consumption and investment sectors

How can we explain the rise in the labor income share of the investment sector? One part of the story is the shift from tangible to intangible investment (Figure 2). Intangible investment is significantly more labor intensive—especially college—labor intensive—than tangible investment (Figure 4). However, another possibility is that all forms of investment have become more labor intensive over time. For instance, automation may have progressed more rapidly in the consumption sector than in the investment sector.

To systematically assess these explanations, we use a shift-share decomposition to analyze the rising difference in labor intensity between the investment and consumption sectors.

Taking the difference of (3.1) over time, denoted Δ, we have:

$$\Delta LS_I - \Delta LS_C = \underbrace{\sum_i (w_{Ii} - w_{Ci})\Delta LS_i}_{\text{Within}} + \underbrace{\sum_i (\Delta w_{Ii} - \Delta w_{Ci})(LS_i + \Delta LS_i)}_{\text{Between}} \tag{3.2}$$

The first term (the within component) reflects the difference between the weighted average covariance of changes in investment and consumption expenditures and the change in the labor share. It captures the idea that consumption industries may have become more automated over time. The second term (the between component) captures the covariance between the change in sectoral expenditure shares and the labor share. It reflects the shift in the investment sector toward intangible forms of capital, which tend to have a higher labor share.

Table 2 reports the decomposition. We find that roughly half of the rise in labor intensity is accounted for by the within component, and half by the between component. Hence, both channels play a significant role.

Table 2: Decomposing the rise in labor intensity in investment versus consumption sector

	Total	Within	Between
Δ College labor share I vs C	6.8%	5.5%	1.3%
Δ Noncollege labor share I vs C	−1.8%	−2.9%	1.1%
Δ Total labor share I vs C	5.0%	2.6%	2.4%

Robustness. In the appendix, we show that our results are robust to a range of checks: (i) using a more detailed input-output matrix when available; (ii) not capitalizing organizational services; (iii) excluding certain sectors—housing, farms, and government—from the analysis; (iv) removing all government final uses from both the consumption and investment sectors; and (v) using BEA input-output data after the industry redefinition, which reclassifies the secondary products of industries to their primary industries. Across all these robustness checks, we find the same stylized fact: there is a growing wedge in the labor share of the investment sector, especially in the share of payments going to college-educated workers.

3.1.2 Evidence from Compustat

The previous section uses data from the BEA to compute the labor share of the investment sector. We now show that our results remain the same if we use investment expenditures reported by Compustat instead (e.g., Peters and Taylor, 2017, Eisfeldt and Papanikolaou, 2013, He et al., 2024).

Methodology. We use Compustat-CRSP merged data covering the period from 1972–2022. We use usual screens to focus on the nonfinancial corporate sector. Our goal is to construct industry-level series for tangible investments, investment in R&D, and investment in organizational services.

We start from the statement of cashflows, as reported in Compustat. The key accounting identity is the cashflows from operations, the cashflows from financing activities, and cashflows for investing activities sum up to zero. We use the convention that changes in cash balances represent net payments to debt holders. Notice that this cashflow identity is the financial accounting counterpart of the firm budget constraint in the model. These equations allow us to account for 100% of the cash that comes in due to profits (cashflows from operations), the net cash that comes out to pay owners (cashflows from financing activities), and the cash that comes out due to investment (cashflows from investment activities).

Adjustments. Building on the existing literature, we conduct some data imputations and adjustments. We make four main adjustments to ensure that our measures accurately capture both tangible and intangible investments. We provide a brief description below, with the detailed methodology in Appendix B.1.

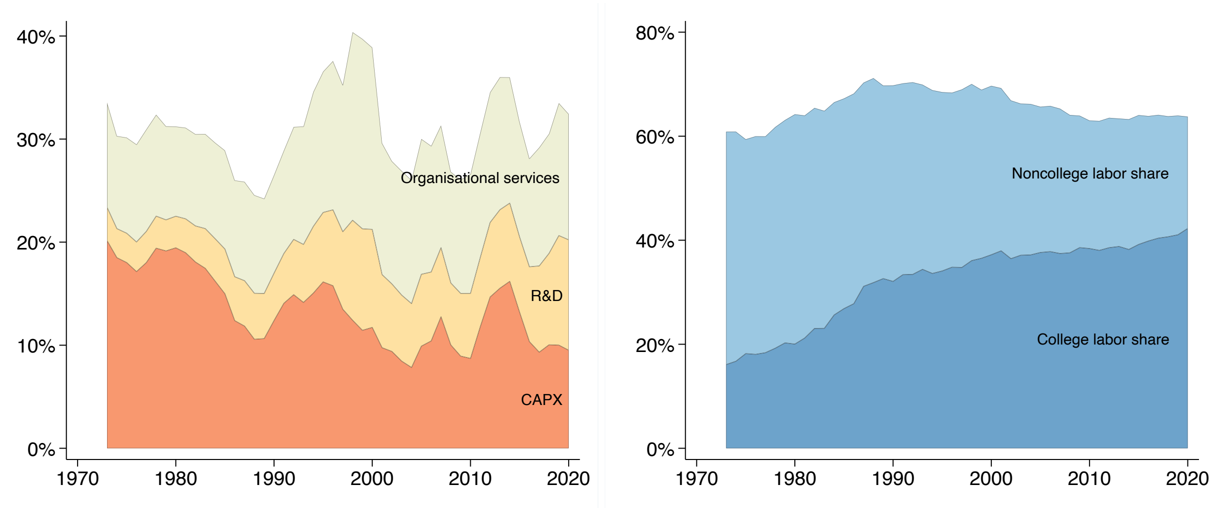
Our first adjustment corrects the undercounting of intangible investments by reclassifying a portion of SG&A (selling, general, and administrative expenses) and all R&D expenditures from operating expenses to capital expenditures. The reason is that thes line items typically include labor costs associated with intangible capital formation. Expensing these investment labor costs (as is currently done) lowers measured operating cashflows and overstates current profits. Our second adjustment reflects net entry of firms into and out of the public market. We account for the cash outflows (for instance due to new listings) and inflows (for instance due to acquisitions of public targets) experienced by passive investors who rebalance their portfolio to maintain a market-neutral investment. Our third adjustment deals with payments made in the form of new shares rather than cash by splitting the growth in share count into a cash component and a "noncash" component, thereby correctly incorporating stock-based compensation and stock-financed acquisitions as economic costs. The breakdown between stock compensation and acquisitions is only available after 2011. In prior years, we assume that acquisition is a constant share $\omega \in (0, 1)$ of noncash payments, where ω is computed at the industry year on post 2011 data. Finally, our fourth adjustment imputes the intangible share of "ambiguous" investments, such as acquisitions and IPOs, by assuming they contain the same intangibility, $\tilde{\chi}$, as other forms of investment within each industry and year.

Results. Figure 5a shows the distribution of investment as a proportion of value added over time. Three patterns stand out. First, the share of tangible investment declines. Second, the share of intangible investment rises, driven by increases in both R&D and organization capital—as proxied by the SG&A component in Compustat. Third, this shift reflects a broader rise in investment intangibility: from around 10% in the 1970s to 20% after 2000.

To explore the implications of these changing investment expenditures on the labor component of investment, we combine these investment shares with data on the labor component of tangible investment (equipment and structures), R&D, and organization capital from the BEA. We then construct the labor share of investment as the weighted average labor share across these three components. Note that both the weights and the labor shares within each investment type change over time. The resulting labor share, shown in Figure 4b, evolves similarly to the BEA series. Notably, we find that the labor share of investment increases—a contrast to the well-documented decline in the aggregate labor share of the economy.

Specifically, Figure 5a shows a large rise in the intangibility of investment, from roughly 10% in the 1970s to 20% post-2000. We then combine these changing investment expenditures with the labor component of tangible investment (equipment and structure), R&D, and Organization capital from the BEA. The result, plotted in Figure 4b show a similar evolution of the labor share relative to the BEA. Actually, we find that the labor share of investment *increases*, which contrasts with the well-known decline in the labor share in the aggregate economy.

Overall, these results show that results form the BEA and Compustat show similar trends regarding the rising importance of intangibles in investment, and, as a result, the reliance of capital formation on high-skill labor.



(a) Composition of investment

(b) Labor share of investment sector

Figure 5: Labor income share for consumption and investment sectors

3.2 Testing the model mechanism

We now use this data on the uses of capital income to test some implications of our model.

Time series evidence. One important prediction of our model is that an increase in the valuation of the corporate sector relative to output should be associated with a rise in the wage differential between investment and production workers (2.1). We now test this time-series prediction.

Figure 6a plots the ratio between the valuation of public firms in Compustat and their capital income. We find a rising disconnect between the market value of the firms and their revenues, which is consistent with a vast array of evidence (see, e.g., Gomez and Gouin-Bonenfant, 2024).

We then measure the evolution of the wage differential between investment and production workers $\log(w_{H,t}/w_{Y,t})$ using data from the Current Population Survey. We define "investment labor" as occupations in managerial and professional specialty occupations, whereas"production labor" includes occupations in technical, sales, and administrative support occupations, service occupations, and production occupations. We focus on the annual labor income reported in the March survey. We compute the growth in the average wage within occupation class — investment or production labor — with and without adjusting for composition changes. To control for composition changes within each class, we first residualize wages across a set of interacted worker characteristics and construct the composition-adjusted wage as the average residualized wage adjusted for the average wage within the occupation across years.[9]

Figure 6b shows the evolution of the wage differential over time. We find that the wage premium increases by 10 log points on an unadjusted basis, and by 25 log points after accounting for composition changes within occupations over time. The discrepancy between the two numbers suggests that characteristics associated with lower wages have become relatively more common in investment occupations than in production occupations. One natural explanation is that, as the demand for investment labor expands, workers switching from production to investment roles are, on average, less skilled than the workers already in these positions. Note that, while our estimates control for changes in observed characteristics, similar dynamics may be at play for unobservable ones, suggesting that our results may underestimate the actual rise in the wage differential on a fixed-composition basis.

Our finding of a rising wage differential between investment and production workers aligns with the literature on U.S. wage inequality, which document that occupations requiring "abstract" skills have seen a relative increase in wages compared to those relying on "routine" skills since the 1980s (see, e.g., Autor and Dorn, 2013). Our finding is also also related to the literature on CEO wage differentials over the same period (see, e.g., Gabaix and Landier, 2008), given that managers are a subset of investment workers.

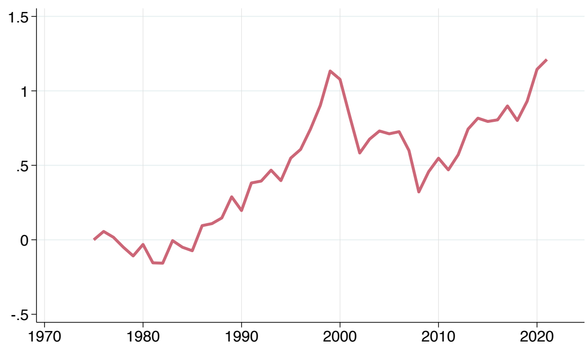
We can more formally test the relationship between the two quantities by estimating local projections of the wage differential between production and investment workers on shocks in corporate sector valuations:

$$\log\left(\frac{w_{H,t+h}}{w_{Y,t+h}}\right) = \alpha_h + \beta_h \Delta \log\left(\frac{V_t}{\Pi_t}\right) + \gamma_h \log\left(\frac{w_{H,t-1}}{w_{Y,t-1}}\right) + \epsilon_{t+h} \tag{3.3}$$

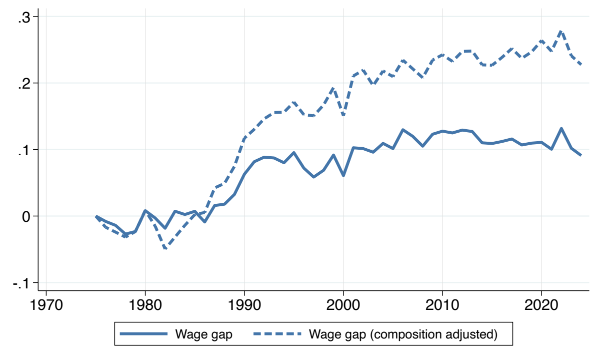
Figure 6c presents the estimates $\{\widehat{\beta}_h\}_{h=1}^{10}$ with a 95% confidence interval using robust standard errors. We find that a rise in corporate valuations predicts an increase in the wage differential over the following years. The response of wages is gradual, consistent with the presence of adjustment costs in the short-run (i.e., $\theta > 0$ in the model). Note that the coefficient β_h is not estimated from the overall trends in the wage differential and in valuations, but, rather, from the deviations of these two series around their respective trends.

Cross-sectional evidence. Our model predicts that more intangible economies have a lower elasticity of capital — see the expression (short-run elasticity). A similar implication would arise in the cross-section in a multi-sector model. As long as there is an imperfectly elastic supply of labor across industries and occupations, we should expect that industries with a higher share of

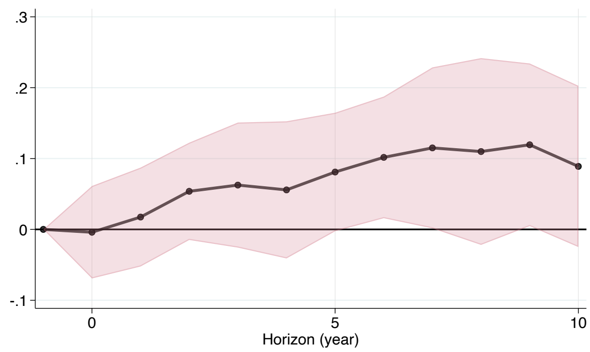
[9]The set of characteristics includes sex (two groups: male and female), race (two groups: White and non-White), age (five groups: 16–25, 26–35, 36–45, 46–55, and 56–64 years), and education (five groups: some high school, high school diploma, some college, bachelor's degree, and more than a bachelor's degree).



(a) Enterprise value / capital income (log)



(b) Investment-production wage differential (log)



(c) Local projection of wage ratio on valuation shock

Figure 6: Relation between corporate valuations and the investment-production wage differential

intangibility tend to have a lower capital supply elasticity.

As a model validation exercise, we now examine the relationship between short-run capital elasticity and the tangibility at the industry-level (since it is related to the importance of specialize labor). To estimate the short-run investment-q elasticity $\frac{\partial \log(I_t + w_{H,t} L_{H,t})}{\partial \log q_t}$, we leverage cross-industry variation in total investment and valuations. We focus on $J = 33$ broad industry groups over 1972-2022. As an empirical proxy for $\log q_{j,t}$ we use log enterprise value $\log V_{j,t}$. The idea is that annual industry-level fluctuations in market values are mostly driven by changes in the valuation of capital q_t, rather than changes in capital K_t. We estimate an industry-specific "investment-q elasticity" β_j via:

$$\Delta \log(I_{j,t} + w_{j,t} L_{H,j,t}) = \alpha_j + \beta_j \Delta \log V_{j,t} + \gamma_j \log(I_{j,t-1} + w_{j,t-1} L_{H,j,t-1}) + \epsilon_t. \tag{3.4}$$

Figure 7 plots the average tangibility of investment over the sample for each industry, as well as its investment-q elasticity. Despite the limited sample, we find a significant positive relationship, consistent with the prediction from the model.

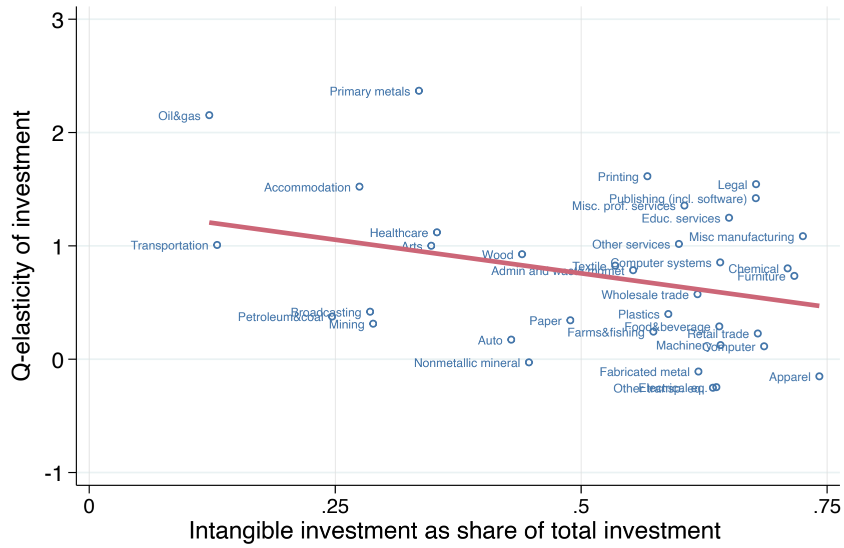


Figure 7: Investment-q elasticity and tangibility of investment.

Notes: On the x-axis, the industry-level tangibility of investment is the (unweighted) 1972-2022 average χ_j. On the y-axis, we report estimated coefficient φ_j, which captures the elasticity of total investment to firm value (a proxy for q). The red line reports the predicted relationship using all data points (unweighted).

4 Quantitative analysis

We now develop a more quantitative model that we calibrate using our micro-data on the uses of capital income. We then use the model for two experiments (i) the effect of a shift towards intangible capitals on the economy, and (ii) the effect of a corporate tax cut.

4.1 Full model

We first extend our stylized model to make it more realistic. We add four ingredients : (i) corporate taxes (ii) an elastic labor supply in and out of the labor force and between between production and investment occupations, and (iii) an exogenous growth π in the productivity of production and investment labor.

Production side. The production side is the same as in the stylized model except we now add corporate taxes, that are rebated to the household The firm budget constraint then becomes:

$$D_t = (1-\tau_{K,t})(Y_t - w_{Y,t}L_{Y,t}) - I_t - w_{H,t}L_{H,t}, \qquad \text{(firm budget constraint')}$$

where $\{\tau_{K,t}\}_{t=1}^{\infty}$ is a sequence of capital income tax rates, or more precisely taxes on payouts.

Household side. On the household side, we now allow the quantity of production and investment labor to be elastically supplied. More precisely, we assume that the representative household chooses a sequence of consumption, production labor, and investment labor to maximize welfare:

$$U_0 = \max_{\{C_t, L_{Y,t}, L_{H,t}, V_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \left\{ \frac{1}{1-\gamma} \left(C_t - \frac{L^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} \right)^{1-\gamma} \right\}$$

$$\text{where } L_t \equiv \left((1-\mu)^{-\frac{1}{\phi}} L_{Y,t}^{\frac{1+\phi}{\phi}} + \mu^{-\frac{1}{\phi}} L_{H,t}^{\frac{1+\phi}{\phi}} \right)^{\frac{\phi}{1+\phi}}$$

The quantity σ represents the elasticity of aggregate labor supply while ϕ represents the elasticity of substitution between production and investment labor. The stylized model presented in Section 2 corresponds to the special case $\sigma = \phi \to 0$, in which case the supply of production and investment labor is fixed. The household first-order conditions with respect to $L_t, L_{Y,t}$ and $L_{H,t}$ gives:

$$L_t = w_t^{\sigma}; \qquad L_{Y,t} = (1-\mu) \left(\frac{w_{Yt}}{w_t} \right)^{\phi} L_t; \qquad L_{H,t} = \mu \left(\frac{w_{H,t}}{w_t} \right)^{\phi} L_t, \qquad \text{(worker foc } L_t\text{)}$$

where w_t is defined similarly to L_t. In the case $\sigma = \phi \to 0$ (inelastic labor supply) we get $L_t = 1, L_{Y,t} = 1 - \mu, L_{H,t} = \mu$ as in the stylized model from Section 2. Conversely, in the case $\phi \to \infty$ (perfect substitution), the wages between production and investment workers are equalized, $w_{H,t} = w_{Y,t}$.

Equilibrium. An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, L_t, I_t, K_t\}_{t \geq 1}$, and prices $\{w_{H,t}, w_{L,t}, w_t, r_t, q_t\}_{t \geq 1}$ that solve the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t, firm foc K_{t+1}) and household problem (worker foc C_t, worker foc L_t).

4.2 Calibration

We now calibrate the model. First, we set two parameters externally. We set $\sigma = 0.5$, consistently with the macro evidence at the business cycle frequency. We then set $\phi = \sigma$, which corresponds to a linearly separable labor supply for production workers and investment workers. The parameter β is chose to match a roughly 7% log return (unlevered), which is the average over our sample. In Appendix C.3, we show that, along a balanced growth path with growth rate π, the log return is constant at $\log R = -\log \beta + \gamma\pi$, where γ is the household's EIS. Assuming $\gamma = 1$ and $\pi = 2\%$, we get $\beta = 0.95$. Second, we set the parameter $\chi = 0.59$, which governs the importance of intangible inputs in capital formation.

Third, we calibrate the remaining parameters four parameters (α, θ, δ) internally by targeting two long-run moments (the total investment yield and the labor income share) and one short-run moment (the tax elasticity of investment). The total investment yield is the sum of tangible and intangible investments, as a share of enterprise value. In Appendix A.1, we discuss how this moment has the interpretation of a dilution rate: it measures how much a capitalists that would consume all the capital income would get diluted over time. The second moment is the labor share, which is defined as total payments to labor (to both production and investment labor) as a share of aggregate income.

Table 3: Internally calibrated parameters

Moment	Notation	Formula	Target
Total investment yield	$\frac{I+w_H L_H}{V}$	$(1-\theta)\delta$	13%
Capital share	$\frac{\Pi}{GDP}$	$\frac{\alpha}{1+\chi\alpha\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)}}$	33%
Tax elasticity of investment	$\frac{\partial\log(I_0+w_{H,0}L_{H,0})}{\partial\log(1-\tau_K)}$	$\frac{r+(1-\theta)\delta}{r+\rho}\frac{1}{\theta}$	4

Notes. α is the capital share in production; θ is the production share in investment; χ is the intangibility of investment; δ is the depreciation rate; r is the return; ρ is the annual decay of the tax cut.

Note that, together, these two long-run moments can not separately identify $1-\theta$ from δ. The reason is that a high investment yield can be due to the fact that capitalists extract little rents from investment (θ is low) or that the quantity of investment is high (δ is high). Income shares alone can not distinguish between these two cases. Guided by the earlier intution that θ effectively governs the elasticity of capital supply in the short-run (as in the q-theory of investment), we use a short-run moment for our last empirical target.

We draw on evidence from Chodorow-Reich, Smith, Zidar and Zwick (2024) on the empirical response of US firms to the 2017 Tax Cuts and Jobs Act. We focus on their result that uses tax files for a sample of roughly $N = 7000$ (nonfinancial, non-passthrough domestic firms). The variable of interest is log investment change (post-TCJA versus pre-TCJA). The authors estimate that a 1% decline in the corporate income tax rate leads to a roughly 4% decline in investment (see their Table 3, columns 2 and 3). We interpret their estimates as a partial equilibrium short-run response in the model (i.e., holding prices w_Y, w_H, R constant).

In Appendix 4, we map their regression coefficient to a simple formula that is the product of two terms. We show that a tax cut (with an annual decay rate of τ) implies a short run response of log total investment of $\frac{r+(1-\theta)\delta}{r+\rho}\frac{1}{\theta}$ (see Table 3). The first terms accounts for the fact that (i) capital payout is levered to the tax rate and (ii) the tax cut is not fully permanent. The second term is $\frac{1}{\theta}$, which governs the short-run elasticity of capital supply in partial equilibrium. Table 4 reports the model calibration using these short-run and long-run targets.

Given the Cobb-Douglas assumption on capital formation, our calibration implies that the revenues from capital formation are distributed as follows: a share $\theta = 30\%$ is paid to capital owners, a share $(1-\theta)(1-\chi) = 29\%$ is spent on investment goods and a share $(1-\theta)\chi = 41\%$ is spend on investment workers.[10]

[10]Using an indirect inference method, Kogan et al. (2020) estimates that the revenues from capital formation are

Table 4: Calibrated parameters

Parameters	Description	Value
σ	Aggregate labor supply elasticity	0.5
ϕ	Sectoral labor supply elasticity	0.5
β	Subjective discount factor	0.95
α	Capital share in production	0.38
θ	Capital share in investment	0.3
δ	Capital depreciation rate	0.19
χ	Intangibility index	0.59

4.3 Quantifying the effect of a rise in intangibles

We now conduct an experiment where we quantify the long-run effect of a rise in the intangibility of investment χ by 25 pp.

Table 5: Distributional effect of a rise in intangibles

Variable	Symbol	Baseline (pp.)	Low $\chi \to$ high χ
Panel A – Expenditures			
Consumption	$Y - I$	76	-0
Tangible investments	I	10	-6
Intangible investments	$w_H L_H$	14	6
Panel B – Income			
Capital income	$Y - w_Y L_Y$	33	0
Labor income (production)	$w_Y L_Y$	53	-6
Labor income (investment)	$w_H L_H$	14	6

First, Table 5 reports the effect of rising intangibility on the composition of GDP (see Section 3.1.2 for national accounting definitions). On the expenditure side (Panel A), there is a rise in intangible investments (6 pp. of GDP) offset by a decline in tangible investments (-6 pp. of GDP). On the income side (Panel B), we see a decline in the production labor income share 53 pp. compensated by a rise in the investment labor income share (53 pp.).

Effect on capital elasticity. Figure 8 shows the long-run effect of rising intangibles on the capital supply curve, which amounts to a counter-clockwise rotation of the supply curve.[11] Notice that capital supply becomes much less elastic, due to the growing importance of imperfectly-elastic

distributed as follows: a share $\alpha = 45\%$ is paid to investment goods, a share $(1-\eta)(1-\alpha) = 13\%$ is paid to capital owners, and a share $\eta(1-\alpha) = 42\%$ is paid to investment workers. Hence, compared to this paper, our estimates imply that a higher share of investment rents accrue to capital owners relative to investment workers.

[11] Note that a change in χ affects the long-run equilibrium Y due to an endowment effect. For visual purposes, we construct Figure 8 by showing a rotation around the initial equilibrium, which amounts to consider a joint change in χ and A_H that ensure no change in the long-run (q, K).

labor in capital formation.

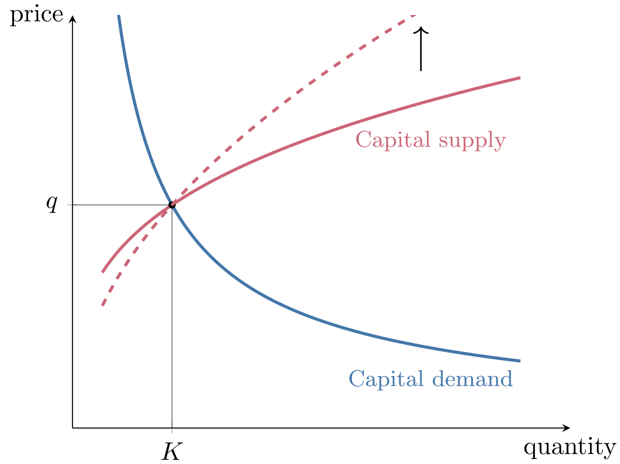


Figure 8: Effect of a rise in intangibles on the market for capital

Quantitatively, varying the value of χ in the model, while holding all the other parameters constant, has a large effect on both the short- and long-run capital supply elasticities. Table 6 reports the model-implied elasticities. The baseline model has low elasticities compared to the case of full tangibility (i.e., setting $\chi = 0$ as in the standard q-theory of investment). In the short-run it is 0.14 versus 0.43, and in the long-run it is 1.54 versus $+\infty$. We obtain a decline in both the long-run elasticity by more than half when χ is increased by 25pp.

Table 6: Capital supply elasticity.

	Baseline	Q-theory ($\chi = 0$)	Low χ ($\chi = 0.47$)	High χ ($\chi = 0.72$)
Short-run	0.14	0.43	0.11	0.17
Long-run	1.54	$+\infty$	1.1	2.23

Notes. The short-run supply elasticity of capital is $\partial \log K_{t+1} / \partial \log q_t$ while the long-run is $\partial \log K_t / \partial \log q$.

Effects on the wage gap. A rise in χ corresponds to a rise in labor demand for one type of labor (i.e., investment labor). How does that translate into changes in relative wages? In the counterfactual, we obtain a large response of the wage premium by $\Delta(w_H/w_Y)/(w_H/w_Y) = +33\%$. To understand the sources of the rise in the wage premium, we use the following decomposition:

$$\mathrm{d}\log \frac{w_H}{w_Y} = \frac{1}{1+\phi}\,\mathrm{d}\log \frac{w_H L_H}{w_Y L_Y} = \frac{1}{1+\phi}\frac{\mathrm{d}\chi}{\chi}.$$

The first equality uses labor supply (worker foc L_t) to express the change in relative wage in terms of the change in wage bills, which depends on the occupation-level labor supply elasticity ϕ. If it was perfectly elastic ($\phi = \infty$), we would see no change in relative wages. In the baseline calibration, labor is moderately elastic, with $\frac{1}{1+\phi} = 0.67$. The second equality uses the long-run relative marginal products of labor (2.1) $\frac{d\chi}{\chi} = 54\%$. Quantitatively, the rise in the wage gap implied by the model is consistent with the actual increase in the wage gap between investment and production workers observed in the data since 1970, as plotted in Figure 6.

4.4 Quantifying the effect of a corporate tax cut

We now use the calibrated model to consider the allocative and redistributive effects of a corporate tax cut.

Theory. A corporate tax cut can be seen as a shift in the demand for capital, as represented in Figure 8. As usual in supply-demand systems, a positive demand shock will increase prices and quantities. When the supply curve is horizontal (as in the NGM), then the demand shock is entirely absorbed by quantities K. But in the baseline, long-run capital supply is imperfectly elastic due to finite labor supply. As a result, a demand shock, in this case a tax cut, leads to a rise of both prices q_t and quantities K_t.

To set ideas, consider a permanent decline in τ_K. Using a standard comparative statics approach and the long-run demand and supply elasticities in the model, we obtain

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau_K)} = \frac{1}{\text{demand elasticity} + \text{supply elasticity}} = 0.9,$$
$$\frac{\mathrm{d}\log q}{\mathrm{d}\log(1-\tau_K)} = \frac{\text{supply elasticity}}{\text{demand elasticity} + \text{supply elasticity}} = 0.6.$$

The shock is absorbed through both a higher valuation of capital q *and* a rise in capital formation K. As a benchmark, in the NGM, the response of capital would be roughly twice as high (1.9) with no response of valuations.

More generally, a shift towards intangibles means that the market increasingly clears via higher valuations rather than actual capital formation. As we discuss next, this imperfect pass-through of demand shocks to quantities has important effects on the incidence of capital taxes (i.e., who wins and loses in terms of welfare). Intuitively, a jump q induced by a tax cut will benefit investment labor (via higher wages) and initial capitalists (via wealth effect).

We first consider an arbitrary perturbation $\{\mathrm{d}\tau_{K,t}\}_{t=0}^{\infty}$ around the undistorted steady-state. Which factors of production win and/or lose in response to this shock? We follow Fagereng et al. (2024) and apply the envelope condition on the household value function to obtain the total welfare effect of the change in prices induced by the tax shock.

Proposition 4.1. *The equilibrium welfare effect, in units of $t = 0$ consumption, associated with the per-*

turbation $\{\mathrm{d}\tau_{K,t}\}_{t=1}^{\infty}$ *around the undistorted steady-state, is given by*

$$\textit{Welfare Gain} = \underbrace{\sum_{t=1}^{\infty} R^{-t} L_Y \cdot \mathrm{d}w_{Y,t}}_{\textit{welfare gain (production workers)}} + \underbrace{\sum_{t=1}^{\infty} R^{-t} L_H \cdot \mathrm{d}w_{H,t}}_{\textit{welfare gain (investment workers)}} \underbrace{+\,\mathrm{d}V_0 + \sum_{t=1}^{\infty} R^{-t} V \cdot \mathrm{d}r_t}_{\textit{welfare gain (capitalists)}}.$$

Proposition (4.1) decomposes the welfare effect into the contribution of changes in prices for the three factors of production: production labor, investment labor, and capital. For labor, the welfare effect captures the fact that the tax shock affects the path of wages $\{\mathrm{d}w_{Y,t}, \mathrm{d}w_{H,t}\}_{t=1}^{\infty}$. For capitalists, there is an initial wealth gain $\mathrm{d}V_0$ as well as the contribution of changes in forward returns $\{\mathrm{d}r_t\}_{t=1}^{\infty}$. The total (private sector) welfare effect is equal to the present value of the change in the tax rate times capital income (i.e., the mechanical effect of taxes on profits holding everything else constant): Welfare Gain $= \sum_{t=1}^{\infty} R_{0\to t}^{-1} \Pi_t \cdot \mathrm{d}\tau_{K,t}$.

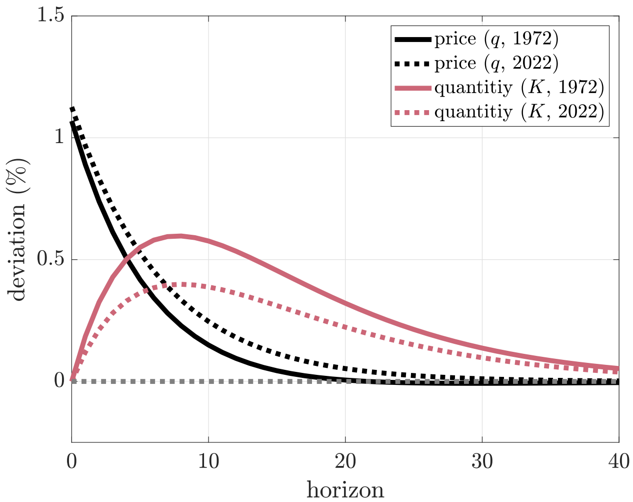
Quantitative experiment. We now simulate a 1% cut around the steady-state, which decays at an annual rate of $1-\rho = 0.9$. We consider the case of a small-open economy, where forward returns are constant (i.e., $\mathrm{d}r_t = 0$). Throughout this section, our baseline environment is a small open economy, where we simulate deviations induced by taxes while imposing $\mathrm{d}r_t = 0$ (the interest rate is not affected by the shock, but wages are).[12] This simplifies the formula for the welfare effect for capitalists in Proposition (4.1), which becomes only the initial wealth gain $\mathrm{d}V_0$ (i.e., the rise in their wealth due to the fact that future post-tax capital income has increased).

Capital accumulation. Figure 9a plots the response of capital accumulation and valuations over a 40 year period after the shock. We show the dynamics implied by the model using the low versus high χ calibrations to describe how the shift towards capital affects the response of the economy. In both cases, the value of capital jumps on impact, but it takes several years for the stock of capital to peak. Notice, however, that in the intangible economy (high χ calibration), the stimulative effect of the tax cut is much lower, consistent with the earlier calculations regarding the supply elasticity of capital (see Table 6).

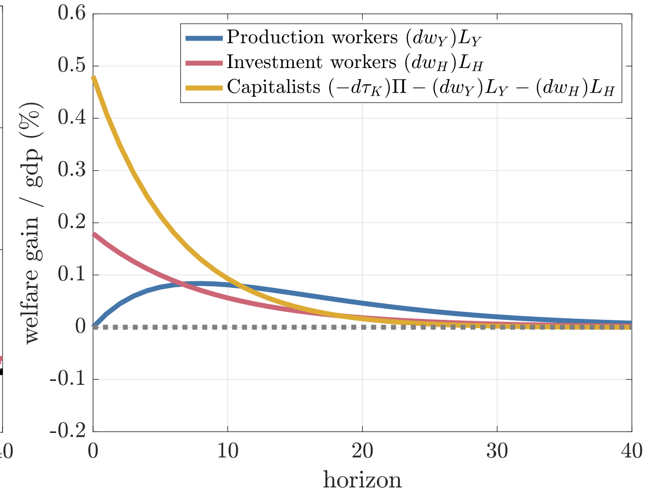
How does the general equilibrium (GE) response of investment respond to the partial equilibrium (PE) target used for calibration? In PE, we target a high elasticity, but in a GE experiment where the tax applies to all of capital income (which represents 4% of GDP in the baseline), we expect a much weaker response due to rising wages and interest rates.

Table 7 reports the elasticity of total investment to the tax shock in PE (our calibration target), as well as the corresponding elasticity in a small open economy (the baseline), and in GE. We find that rising wages reduce the tax elasticity of investment nearly fourfold. The second column reports the tax elasticity of investment wages: compared to the PE environment (where wages

[12] In terms of the implied dynamics $\mathrm{d}q_t, \mathrm{d}K_t$, assuming a "small open economy" is this is equivalent to setting the EIS to $\gamma = 0$, which implies $1 + r_t = \beta^{-1}$.



(a) Capital and valuations



(b) Welfare effects

Figure 9: Model-implied effect of a cut in corporate taxes

Table 7: Short-run response to a tax shock.

Tax elasticity of:	Investment $(I + w_H L_H)$	Investment wages (w_H)	Valuation (q)
Partial equilibrium	4	0	1.1
Small open economy (baseline)	0.96	0.64	0.55
General equilibrium	0.36	0.24	0.21

Notes. The short-run tax elasticity of a variable X_t is the response $\mathrm{d}\log X_1$ in response to a tax shock $\mathrm{d}\tau_{k,t} = (1-\rho)^t\,\mathrm{d}\tau_K$ ($\rho = 0.1$ annually). "Partial equilibrium" is the equilibrium deviation with the restriction $\mathrm{d}w_Y = \mathrm{d}w_H = \mathrm{d}R = 0$; "small open economy" is the equilibrium deviation with the restriction $\mathrm{d}w_Y = \mathrm{d}w_H = 0$; "general equilibrium" is the equilibrium deviation where all prices adjust (EIS is set to $\gamma = 1$)).

are fixed), in a small open economy the elasticity is large at 0.64. Finally, the GE case, where interest rates $\mathrm{d}r_t$ also respond, has an even lower tax elasticity of investment. The response of interest rates is governed by the household EIS $\frac{1}{\gamma}$. Intuitively, in the case where $\gamma \to \infty$ (Leontief preferences), aggregate capital does not adjust because household are not willing to substitute across time periods. We report the case $\gamma = 1$ (log utility) and the resulting tax elasticity of investment nearly ten times lower than in PE.

Welfare effects. The welfare effect for workers works through changes in their wage, which are themselves fully pinned down by the path of the state and co-state variables $\{q_t, K_t\}_{t=0}^{\infty}$. A rise in capital K benefits both types of workers, via their complementarity with existing capital, but the value of capital q directly affects the marginal product of investment labor. Figure 9b plots the resulting path of (undiscounted) welfare effects for both types of labor, expressed as a share of steady-state GDP. While the tax shock appears to benefit both types of labor equally, it is worth pointing out that production labor accounts for roughly 4 times more labor income than investment labor in steady-state (see Table 5).

The welfare effect for capitalists can be decomposed as the present value of higher future payouts:

$$\mathrm{d}V_0 = \sum_{t=1}^{\infty} R^{-t}\Big(-\Pi \cdot \mathrm{d}\tau_{K,t} - L_Y \cdot \mathrm{d}w_{Y,t} - L_H \cdot \mathrm{d}w_{H,t}\Big).$$

Notice that those higher payouts are very front-loaded, especially compared to the back-loaded increase in production worker wages (see Figure 9b). This is because wages rise slowly with capital accumulation, while the benefit for capitalists on post-tax capital income is immediate.

Table 8 reports the share of private welfare gains that accrue to each factor of production. In the baseline, 20% of the tax incidence falls on production workers, while 26% falls on investment labor and 54% on capitalists. Note that this stands in sharp contrast with the NGM, where 100% of the incidence falls on productions workers. This is because, when capital is fully elastic ($\theta = 0$), there is no wealth effect since $q = 1$ at all times.

Table 8: Incidence of capital taxes (welfare gain as a share of private sector welfare gains).

Factor	Baseline	NGM ($\chi=\theta=0$)	Q-Theory ($\chi=0$)	Low χ ($\chi=0.47$)	High χ ($\chi=0.72$)
Production workers	20	100	50	17	26
Investment workers	26	0	0	30	22
Capitalists	54	0	50	53	52

To understand the difference between the baseline and the NGM, it is useful to consider an intermediary model (i.e., the q-theory special case), where we impose full tangibility $\chi = 0$ while keeping a positive capital share $\theta = 0.3$ as in the baseline calibration. In that case, we a slightly lower capitalist share. We also run the same model experiment in the low- and high-intangible calibrations. Notice that the shift towards intangibles has led to a reallocation of the incidence of capital taxes, away from production labor and towards investment labor and capitalists.

5 Conclusion

In this paper, we enrich the standard neoclassical growth framework by introducing a more realistic investment process that pairs tangible inputs—machinery, computers, plants—with both accumulated capital and specialized labor (such as inventors, entrepreneurs, and financiers). This key departure recognizes that the formation of new capital hinges on skilled workers who are in finite supply. As a result, the supply curve for capital slopes upward even in the long run, providing a new mechanism for the empirical observation of large disconnect between corporate sector valuations and investment quantities.

After calibrating our model to U.S. data, we show that a growing importance of intangible investment can explain several key macroeconomic trends over the past two decades: persistently weak tangible investment despite high valuations, increasing factorless income, and widening wage disparities across occupations.

For tractability, our model parametrizes capital formation using a Cobb-Douglas function. This specification implies that tangible and intangible investments are neither complements or substitutes, and so they each represent a constant expenditure share of capital formation. Alternatively, we could consider an investment function in which tangible and intangible investments have an arbitrary elasticity of substitution. In the realistic case in which the two inputs are complement, the secular decline of equipment prices (investment goods) could have contributed to the rise in the expenditure share of intangible investment — a version of Baumol's cost disease. Estimating the elasticity of substitution between tangible and intangible investments is an important avenue for future research.

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Appendix

A Appendix for Section 2

A.1 Capital rents

We now describe the income (or rents) associated with capital ownership. We use this income to write the value of firms in different ways.

Rents. Capital extracts *Ricardian* rents in the model (i.e., workers are paid their marginal product but below their average product). We define production and investment rents associated to capital ownership as:

$$\text{production rents} \equiv L_{Y,t} \cdot \left(\frac{Y_t}{L_{Y,t}} - \frac{\partial Y_t}{\partial L_{Y,t}} \right) = \alpha Y_t$$
$$\text{investment rents} \equiv \left(I_t^{1-\chi} L_{H,t}^{\chi} \right) \cdot \left(\frac{H_t}{I_t^{1-\chi} L_{H,t}^{\chi}} - \frac{\partial H_t}{\partial (I_t^{1-\chi} L_{H,t}^{\chi})} \right) = \theta q_t H_t.$$

Production rents are the difference between the average and marginal product of production labor. Because of the Cobb-Douglas assumption for the production function, production rents represent a constant share α of gross output Y_t, as in the NGM. We define investment rents similarly, but where the input to investment is the bundle of consumption good and specialized labor. Because of the Cobb-Douglas assumption for the capital formation function, investment rents represent a constant θ of the revenues from capital formation $q_t H_t$.

Present-value of capital. The previous paragraph says that capital owners earn rents from both the production and the capital formation processes. In a present-value sense, this implies that the value of capital can be decomposed into the present value of production rents and the present value of investment rents. To see this formally, re-arrange (firm foc K_{t+1}) to obtain:

$$(1 + r_{t+1}) q_t K_{t+1} = \underbrace{\alpha \cdot Y_{t+1}}_{\text{production rents}} + \underbrace{\theta \cdot q_{t+1} H_{t+1}}_{\text{investment rents}} + (1 - \delta) q_{t+1} K_{t+1}$$

Solving forward gives the following present value equation for the value of a unit of capital q_0:

$$q_0 = \sum_{t=1}^{\infty} R_{0 \to t}^{-1} (1 - \delta)^{t-1} \left(\alpha \cdot \frac{Y_t}{K_t} + \theta \cdot \frac{q_t H_t}{K_t} \right).$$

Re-arranging gives the value of a unit of capital q_0 as the sum of the present-value of production rents associated to that unit of capital and the present-value of its investment rents:[13]

$$q_0 = \underbrace{\sum_{t=1}^{\infty} R_{0 \to t}^{-1} (1 - \delta)^{t-1} \alpha \cdot \frac{Y_t}{K_t}}_{\text{present value of production rents}} + \underbrace{\sum_{t=1}^{\infty} R_{0 \to t}^{-1} (1 - \delta)^{t-1} \theta \cdot \frac{q_t H_t}{K_t}}_{\text{present-value of investment rents}}$$

[13] Kogan et al., 2020 calls this second term the present value of future growth opportunities.

Present-value of firm. We now examine another present-value identity, which focuses on the cash flows associated with owning a financial claim on the firm, rather than owning a unit of capital (in our model, the two approaches are equivalent because of constant return to scale). The valuation equation for the representative firm is

$$(1 + r_{t+1})V_t = D_{t+1} + V_{t+1}.$$

This equation says that the return of owning a firm is the sum of a flow component — the firm payout $D_{t+1} = Y_{t+1} - w_{Y,t+1}L_{Y,t+1} - I_{t+1} - w_{H,t+1}L_{H,t+1}$ — and a capital gain component — the value of the firm next period V_{t+1}. We can re-express this equation as:

$$(1 + r_{t+1})V_t = \Pi_{t+1} + \left(1 - \frac{I_{t+1} + w_{H,t+1}L_{H,t+1}}{V_{t+1}}\right) V_{t+1},$$

where $\Pi_t \equiv Y_{t+1} - w_{Y,t+1}L_{Y,t+1}$ denotes capital income (see Section 3). Relative to the previous equation, the flow component is now the total amount of capital income Π_t while the capital gain component is the "diluted" value of next-period firm value $\left(1 - \frac{I_{t+1}+w_{H,t+1}L_{H,t+1}}{V_{t+1}}\right) V_{t+1}$. The intuition is that, while a firm could pay out all of its capital income to capital owners, it would have to finance its investment through share issuance: the dilution rate $(I_t + w_{H,t}L_{H,t})/V_t$ then corresponds to the rate at which existing owners get diluted over time.[14]

Solving this equality forward and using the transversality condition gives that the value of the firm can be written as the present value of future dividends, or, alternatively, as the present value of diluted capital income:

$$V_0 = \sum_{t=1}^{\infty} R_{0\to t}^{-1} D_t = \sum_{t=1}^{\infty} R_{0\to t}^{-1} \left(\prod_{s=1}^{t-1} \left(1 - \frac{I_{t+1} + w_{H,t+1}L_{H,t+1}}{V_{H,t+1}}\right)\right) \Pi_t.$$

It says that existing capitalists (those that own capital at $t = 0$) do not own all of future capital income; rather, they own a share that decays over time because they must continuously re-invest to maintain their stocks of capital. This is an accounting identity implied by the definition of a firm return.

Using the firm first-order condition for (firm foc I_t), we can rewrite the dilution rate as:

$$\frac{I_{t+1} + w_{H,t+1}L_{H,t+1}}{V_{H,t+1}} = (1 - \theta) \cdot \frac{H_t}{K_{t+1}}.$$

In steady-state, $H_s = \delta K_{s+1}$ and so the dilution rate is exactly $(1 - \theta)\delta$. Note that the dilution rate decreases with θ: the fact that capital owners earn some rents associated with capital formation ($\theta > 0$) makes capital formation less dilutive for capital holders relative to the neoclassical model.

[14] Our concept of dilution rate is related to Gârleanu et al. (2016), who emphasize the gap between the growth of aggregate dividends distributed by public firms and the growth of dividends per share as a result of IPOs.

A.2 Log-linearizing

A.2.1 Long-run changes

The steady-state of the model is given by the intersection of the following capital demand and supply curves:

$$K = (1-\mu)\left(\frac{r+(1-\theta)\delta}{A_Y \alpha} q\right)^{-\frac{1}{1-\alpha}} \qquad \text{(capital demand')}$$

$$K = \mu \left(\left(\frac{A_H}{\delta}\right)^{\frac{1}{(1-\chi)(1-\theta)}} (1-\theta)(1-\chi)\delta q \right)^{\frac{1-\chi}{\chi}}, \qquad \text{(capital supply')}$$

where $r = 1/\beta - 1$ (worker foc C_t)

We now log-linearize the supply and demand curves of capital to estimate the long-run effect of changes in A_Y, A_H an β on capital:

$$\begin{aligned}
\mathrm{d}\log K &= -\frac{1}{1-\alpha}\left(\mathrm{d}\log q + \frac{r}{r+(1-\theta)\delta}\,\mathrm{d}\log r - \mathrm{d}\log A_Y\right) \\
\mathrm{d}\log K &= \frac{1-\chi}{\chi}\left(\frac{1}{(1-\chi)(1-\theta)}\,\mathrm{d}\log A_H + \mathrm{d}\log q\right).
\end{aligned}$$

Combining the two equations allows us to solve for $\mathrm{d}\log K$:

$$\begin{aligned}
\mathrm{d}\log K &= -\frac{1}{1-\alpha}\left(\frac{\chi}{1-\chi}\,\mathrm{d}\log K - \frac{1}{(1-\chi)(1-\theta)}\,\mathrm{d}\log A_H + \frac{r}{r+(1-\theta)\delta}\,\mathrm{d}\log r - \mathrm{d}\log A_Y\right) \\
\implies \mathrm{d}\log K &= \frac{\frac{1}{1-\alpha}}{1+\frac{1}{1-\alpha}\frac{\chi}{1-\chi}}\left(\mathrm{d}\log A_Y - \frac{r}{r+(1-\theta)\delta}\,\mathrm{d}\log r + \frac{1}{(1-\chi)(1-\theta)}\,\mathrm{d}\log A_H\right).
\end{aligned}$$

This last equation summarizes the long-run elasticity of capital K to exogenous shocks in A_Y, A_Y, and β — since $\mathrm{d}\log(1+r) = -\mathrm{d}\log\beta$). Take, for instance the case of a productivity shock in the output production function. In the case $\chi = 0$ (consumption and investment share the same inputs), we get $\mathrm{d}\log K = \frac{1}{1-\alpha}\,\mathrm{d}\log A_Y$, which is the classical elasticity of capital to the interest rate obtained in the standard neoclassical growth model. In the polar case with $\chi \to 1$, however, (capital formation only requires specialized labor), we get $\mathrm{d}\log K \to 0$: capital is fully inelastic.

A.2.2 Short-run changes

We know log-linearize the stylized model around its steady-state to obtain the log deviation in capital and prices as a function of the sequence of productivity shocks.

We first solve for the perturbation in K_{t+1} in terms of backward-looking variables. Log-linearizing (capital formation) and (firm foc I_t) gives

$$\begin{aligned}
\mathrm{d}\log K_{t+1} &= (1-\delta)\,\mathrm{d}\log K_t + \delta\,\mathrm{d}\log H_t \\
\mathrm{d}\log H_t &= \mathrm{d}\log A_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)(1-\chi)\,\mathrm{d}\log I_t \\
\mathrm{d}\log I_t &= \mathrm{d}\log q_t + \mathrm{d}\log H_t.
\end{aligned}$$

Combining the last two equations gives

$$\mathrm{d}\log H_t = \frac{1}{1-(1-\theta)(1-\chi)}\left(\mathrm{d}\log A_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)(1-\chi)\,\mathrm{d}\log q_t\right). \tag{A.1}$$

Plugging into the first equation gives the following log-linearized backward-looking equation for K_t

$$\begin{aligned}\mathrm{d}\log K_{t+1} &= (1-\delta)\,\mathrm{d}\log K_t + \frac{\delta}{1-(1-\theta)(1-\chi)}\left(\mathrm{d}\log A_{H,t} + \theta\,\mathrm{d}\log K_t + (1-\theta)(1-\chi)\,\mathrm{d}\log q_t\right) \\ &= \left(1-\delta\frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)}\right)\mathrm{d}\log K_t + \delta\frac{\theta}{1-(1-\theta)(1-\chi)}\,\mathrm{d}\log A_H + \delta\frac{(1-\theta)(1-\chi)}{1-(1-\theta)(1-\chi)}\,\mathrm{d}\log q_t.\end{aligned}$$

We now find the log-linearized forward equation for q_t. Start from (firm foc K_{t+1})

$$(1+r_t)K_{t+1}q_t = \alpha Y_{t+1} + ((1-\delta)K_{t+1} + \theta H_{t+1})q_{t+1},$$

which gives

$$\begin{aligned}\mathrm{d}\log(1+r_t) + \mathrm{d}\log q_t + \mathrm{d}\log K_{t+1} &= \frac{r+(1-\theta)\delta}{1+r}\,\mathrm{d}\log Y_{t+1} + \frac{1-(1-\theta)\delta}{1+r}\,\mathrm{d}\log q_{t+1} \\ &+ \frac{1-\delta}{1+r}\,\mathrm{d}\log K_{t+1} + \frac{\theta\delta}{1+r}\,\mathrm{d}\log H_{t+1}.\end{aligned}$$

Substituting out $\mathrm{d}\log Y_{t+1}$ and $\mathrm{d}\log H_{t+1}$ using (output production) and (A.1), respectively, yields

$$\begin{aligned}\mathrm{d}\log(1+r_t) + \mathrm{d}\log q_t + \mathrm{d}\log K_{t+1} = &\frac{r+(1-\theta)\delta}{1+r}\left(\mathrm{d}\log A_{Y,t+1} + \alpha\,\mathrm{d}\log K_{t+1}\right) + \frac{1-(1-\theta)\delta}{1+r}\,\mathrm{d}\log q_{t+1} \\ &+ \frac{1-\delta}{1+r}\,\mathrm{d}\log K_{t+1} \\ &+ \frac{\theta\delta}{1+r}\frac{1}{1-(1-\theta)(1-\chi)}\left(\mathrm{d}\log A_{H,t+1} + \theta\,\mathrm{d}\log K_{t+1} + (1-\theta)(1-\chi)\,\mathrm{d}\log q_{t+1}\right).\end{aligned}$$

Log-linearizing (worker foc C_t) with a potentially time varying impatience parameter β_t, we get

$$\mathrm{d}\log(1+r_{t+1}) = -\,\mathrm{d}\log\beta_{t+1} + \gamma\,\mathrm{d}\log C_{t+1}/C_t.$$

We consider the case $\gamma \to 0$ to simplify (or open economy). Re-arranging give the forward-looking equation for q_t:

$$\begin{aligned}\mathrm{d}\log q_t = &\,\mathrm{d}\log\beta_{t+1} + \frac{r+(1-\theta)\delta}{1+r}\,\mathrm{d}\log A_{Y,t+1} + \frac{\theta\delta}{1+r}\frac{1}{1-(1-\theta)(1-\chi)}\,\mathrm{d}\log A_{H,t+1} \\ &- \left(\frac{r+(1-\theta)\delta}{1+r}(1-\alpha) + \frac{\theta\delta}{1+r}\left(1-\frac{\theta}{1-(1-\theta)(1-\chi)}\right)\right)\mathrm{d}\log K_{t+1} \\ &+ \frac{1-\delta\frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)}}{1+r}\,\mathrm{d}\log q_{t+1}.\end{aligned}$$

Hence, the log-linearized equilibrium can be written as a system of ODEs:

$$\begin{pmatrix} \mathrm{d}\log K_{t+1} \\ \mathrm{d}\log q_t \end{pmatrix} = (I+A)\begin{pmatrix} \mathrm{d}\log K_t \\ \mathrm{d}\log q_{t-1} \end{pmatrix} + B \begin{pmatrix} \mathrm{d}\log \beta_t \\ \mathrm{d}\log A_{Y,t} \\ \mathrm{d}\log A_{H,t} \end{pmatrix},$$

where the matrix A and B are defined as follows

$$A \equiv \begin{pmatrix} -\delta \frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)} & \delta \frac{(1-\theta)(1-\chi)}{1-(1-\theta)(1-\chi)} \\ \frac{(r+(1-\theta)\delta)(1-\alpha)+\theta\delta\left(1-\frac{\theta}{1-(1-\theta)(1-\chi)}\right)}{1-\delta\frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)}} & \frac{1+r}{1-\delta\frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)}} - 1 \end{pmatrix}$$

$$B \equiv \begin{pmatrix} 0 & 0 & \delta\frac{\theta}{1-(1-\theta)(1-\chi)} \\ -\frac{1+r}{1-\delta\frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)}} & -\frac{r+(1-\theta)\delta}{1-\delta\frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)}} & -\frac{\frac{\delta\theta}{1-(1-\theta)(1-\chi)}}{1-\delta\frac{(1-\theta)\chi}{1-(1-\theta)(1-\chi)}} \end{pmatrix}.$$

The eigenvalues of the matrix A discipline the speed at which the system converge to its new steady-state after a permanent change in the time preference parameter β, or TFP shocks A_Y and A_H.

A.3 Alternative formulations of the model

A.3.1 Equivalence to semi-endogenous growth model

The simplest semi-endogenous growth (seg) model without population growth

$$Y_t = A_t^\alpha \overline{L_Y}, \qquad \text{(production)}$$
$$A_{t+1} = A_t + A_t^\theta \overline{L_H}. \qquad \text{(innovation)}$$

The parameters α and θ govern the increasing return to scale in the model due to the nonrivalry of ideas A. When $\alpha > 0$, production has IRS in (A, L_Y). When $\theta > 0$, ideas production has IRS in (A, L_H).

Out stylized model has the following equations instead:

$$Y_t = K_t^\alpha (1-\mu)^\alpha, \qquad \text{(production)}$$
$$K_{t+1} = (1-\delta)K_t + K_t^\theta \mu^\alpha. \qquad \text{(capital accumulation)}$$

Notice that, with inelastic labor, the seg model has the same dynamic as the baseline model in the case $\delta = 0$.

A.3.2 A microfoundation for aggregate capital formation

We now discuss one particular way of micro-founding the stylized model in Section 2 along the lines of Kogan et al. (2020). Assume that the representative firm can hire investment workers to create blueprints according to a Cobb-Douglas production function $N_t = K_t^{1-u} L_{H,t}^u$, where N_t denotes the number of blueprints, K_t denotes the amount of capital, and $L_{H,t}$ denotes the amount of investment workers. The parameter $u < 1$ encapsulates the idea that the creation of new blueprints exhibits diminishing returns to scale holding constant the amount of existing capital.

The firm then implements new blueprints using investment goods. For each new blueprint j, the firm can purchase a quantity of investment goods $I_{j,t}$ to create a quantity of project-specific capital $K_{j,t} = A_{H,t} I_{j,t}^{v}$. The parameter $v < 1$ encapsulates the idea that investment in new project is subject to decreasing returns to scale. Once implemented with a quantity of project-specific capital $K_{j,t}$, the project yields an output $Y_{j,t} = A_{Y,t} K_{j,t}^{\alpha} L_{Y,j,t}^{1-\alpha}$ every period, where $L_{Y,j,t}$ denotes the amount of project-specific production workers. Finally, the project-specific capital $K_{j,t}$ depreciates at rate δ.

Denote $Y_t = \int Y_{j,t}\,\mathrm{d}j$, $K_t = \int K_{j,t}\,\mathrm{d}j$ and $L_{Y,t} = \int L_{j,t}\,\mathrm{d}j$ the aggregate amount of output, capital, and production workers across existing projects in the economy. Since all projects can hire production workers at the same wage, total output in the economy can be represented using the aggregate production function $Y_t = A_{Y,t} K_t^{\alpha} L_{Y,t}^{1-\alpha}$. The total amount of capital in the economy then evolves as

$$K_{t+1} = (1-\delta)K_t + N_t \cdot A_{H,t} \left(\frac{I_t}{N_t}\right)^{v},$$

where $N_t = K_t^{1-u} L_{H,t}^{u}$ denotes the aggregate number of new blueprints and I_t denotes the aggregate quantity of investment goods used to implement new blueprints — so that $A_{H,t}\left(\frac{I_t}{N_t}\right)^{v}$ represents the quantity of capital per blueprint.

Note that this system of equations for output production and capital formation in this economy is equivalent to the one discussed in Section 2, with $\theta \equiv (1-u)(1-v)$ and $\chi \equiv u(1-v)/(v+u(1-v))$, so the two economies are isomorphic.

A.3.3 Formulation using intermediary capital inputs

We now provide an alternative formulation of our model that is useful to clarify its link with Hayashi (1982). We decompose our capital formation function (capital formation) in two steps. First, an intermediate firms uses investment goods and investment labor to produce "intermediary" capital goods

$$G_t = A_{G,t} I_t^{1-\chi} L_{H,t}^{\chi}$$

The representative firm then installs these intermediary capital goods using a capital installment function:

$$H_t = \mathcal{A}_{H,t} K_t^{\theta} G_t^{1-\theta}.$$

Combining these two equations yields our stylized model with $A_{H,t} = \mathcal{A}_{H,t} A_{G,t}^{1-\theta}$. In the rest of this section, we will pick $\mathcal{A}_H = \delta^{\theta}/(1-\theta)^{1-\theta}$, which ensures that the derivative of adjustment costs with respect to G are equal to one at the steady-state.

The fact that intermediary capital goods are produced using a constant return to scale production function implies that the price of the intermediary capital p_G is equal to its marginal cost, which gives:

$$p_{G,t} = \frac{1}{A_{G,t}} \left(\left(\frac{1-\chi}{\chi}\right)^{\chi} + \left(\frac{\chi}{1-\chi}\right)^{1-\chi} \right) w_{H,t}^{\chi}.$$

The representative's firm problem for capital installment is the same as in Hayashi (1982):

$$
\begin{aligned}
p_{G,t} G &= (1-\theta) q_t H_t \\
\implies p_{G,t} \frac{G_t}{H_t} &= (1-\theta) q_t \\
\implies p_{G,t} \left(\frac{H}{K}\right)^{\frac{\theta}{1-\theta}} \mathcal{A}_H^{-\frac{1}{1-\theta}} &= (1-\theta) q_t
\end{aligned}
$$

In steady-date, we have $H = \delta K$ so, together with $\mathcal{A}_H^{-\frac{1}{1-\theta}} = \delta^{-\frac{\theta}{1-\theta}}(1-\theta)$, we obtain $p_G = q$.

This derivation shows that our model is similar to Hayashi (1982) in that the ratio between the market value of installed capital and its book value, q/p_G, is equal to one in steady-state. Put differently, the long-run response of q to aggregate shocks in our model is entirely driven by the long-run response of p_G, the price of the intermediary capital output constructed from a mix of good and investment labor, as the ratio q/p_G remains constant in the long-run.

A.3.4 Formulation using two sectors

The production technology (see Section 2.1) can be re-written as a multi-sector economy that produces that has three factors of production (K, L_Y, L_H) and two sectors that produce, respectively, a final good consumption C and a capital good H. We focus on the case where $\theta = 0$ (long-run):

$$
\max_{K_Y, L_Y} Y - w_K K_Y - w_Y L_Y \quad \text{s.t.} \quad Y = A_Y K_Y^{\alpha} L_Y^{1-\alpha} \qquad \text{(final good sector)}
$$

$$
\max_{K_H, L_H} qH - w_K K_H - w_H L_H \quad \text{s.t.} \quad H = A_H K^{\theta} \left(I^{1-\chi} L_H^{\chi}\right)^{1-\theta} \qquad \text{(capital good sector)}
$$

where q is the relative price of capital in terms of consumption (the numéraire). The long-run equilibrium is characterized by 9 unknowns (allocation K_Y, L_Y, K_H, L_H, I + prices w_K, w_Y, w_H, q) and 9 equations:

$$
\begin{aligned}
\alpha Y &= w_K K_Y && \text{(foc } K_Y\text{)} \\
(1-\alpha) Y &= w_Y L_Y && \text{(foc } L_Y\text{)} \\
\theta q H &= w_K K_H && \text{(foc } K_H\text{)} \\
(1-\theta)\chi q H &= w_H L_H && \text{(foc } L_H\text{)} \\
(1-\theta)(1-\chi) q H &= I && \text{(foc } I\text{)} \\
\delta(K_Y + K_H) &= H && \text{(steady-state } K\text{)} \\
L_Y &= 1-\mu && \text{(market clearing } L_Y\text{)} \\
L_H &= \mu && \text{(market clearing } L_H\text{)} \\
(1+r) q &= w_K + (1-\delta) q && \text{(arbitrage equation)}
\end{aligned}
$$

Capital supply. We now consider a PE experiment with 8 unknowns (drop q) and 8 equations (drop arbitrage equation). The thought experiment is to quantify movement along the supply curve.

In the stylized model, we only derive the capital supply elasticity of production in the capital goods sector as a result of a hypothetical relative price q using only market clearing for specialized labor (market clearing L_H) and optimal input choice as a function of q (foc I):

$$
\begin{aligned}
H &= A_H K^{\theta}\left(I^{1-\chi}\mu^{\chi}\right)^{1-\theta}, \\
&= A_H\left((1-\theta)(1-\chi)q\right)^{(1-\theta)(1-\chi)}\mu^{\chi(1-\theta)}K^{\theta}H^{(1-\chi)(1-\theta)}.
\end{aligned}
$$

The first equality uses the definition of H and (market clearing L_H), the second uses (foc I). Note that the we did not use the demand side. In the short-run (fixed K), we have that

$$\frac{\mathrm{d}\log H}{\mathrm{d}\log q} = \frac{(1-\theta)(1-\chi)}{1-(1-\theta)(1-\chi)}.$$

In the long-run, we use (steady-state K) and obtain

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log q} = \frac{1-\chi}{\chi}.$$

Capital demand. Note that the first order conditions and market clearing for labor is the same in the sectoral representation than in the stylized model. We now combine (foc K_Y)+(foc K_H)+(arbitrage equation) to obtain the capital demand equation in the stylized model (i.e., first-order condition for K_{t+1}).

$$\alpha Y + \theta q H = \omega_K K \implies (1+r)q = \alpha\frac{Y}{K} + \left(1-\delta+\theta\frac{H}{K}\right)q, \tag{capital demand}$$

which concludes the proof.

B Appendix for Section 3

Table A2 lists all industries included in our empirical analysis. For each industry, we report three categories: (i) the value of the industry's final use as a share of GDP; (ii) the composition of this final use, broken down into consumption, tangible investment, intangible investment, and net exports (exports minus imports); and (iii) the consolidated labor share of the industry's output, which accounts for the labor embodied in all intermediate inputs used by the industry.

Table A1: Full list of use shares and income shares by industry (2017)

Industry	Uses/GDP	Final use shares				Consolidated labor shares	
		C	I_{tan}	I_{intan}	$X-M$	College	Noncollege
Farms	1%	72%	0%	0%	28%	20%	35%
Forestry fishing	0%	50%	0%	0%	50%	20%	54%
Mining except oil	0%	2%	1%	0%	98%	17%	24%
Support activities	0%	0%	97%	0%	3%	29%	31%
Utilities	1%	99%	0%	0%	1%	18%	17%
Construction	6%	0%	100%	0%	0%	20%	44%
Wood products	0%	28%	42%	0%	30%	19%	41%
Nonmetallic mineral products	0%	64%	0%	0%	36%	19%	29%
Primary metals	0%	4%	0%	0%	96%	20%	31%
Fabricated metal products	0%	41%	19%	0%	40%	21%	36%
Machinery	2%	6%	68%	0%	26%	27%	29%
Computer and electronic	3%	25%	55%	0%	20%	38%	15%
Electrical equipment appliances	1%	47%	31%	0%	22%	28%	28%
Motor vehicles bodies	3%	43%	41%	0%	16%	23%	27%
Other transportation equipment	1%	14%	43%	0%	44%	30%	22%
Furniture and related	1%	67%	29%	0%	3%	23%	39%
Misc manufacturing	2%	68%	19%	0%	13%	31%	26%
Food and beverage products	5%	93%	0%	0%	7%	21%	30%
Textile mills	0%	77%	11%	0%	12%	24%	32%
Apparel and leather	2%	98%	0%	0%	2%	32%	38%
Paper products	0%	70%	0%	0%	30%	22%	30%
Printing and related	0%	77%	0%	0%	23%	24%	34%
Petroleum and coal	2%	78%	0%	0%	22%	17%	12%
Chemical products	3%	74%	0%	0%	26%	23%	14%
Plastics and rubber	0%	69%	1%	0%	30%	23%	27%
Other retail	0%	100%	0%	0%	0%	27%	35%
Air transportation	1%	75%	0%	0%	25%	27%	22%
Rail transportation	0%	52%	0%	0%	48%	18%	31%
Water transportation	0%	82%	0%	0%	18%	24%	24%
Truck transportation	0%	88%	0%	0%	12%	16%	43%
Transit and ground	0%	100%	0%	0%	0%	22%	36%
Other transportation	0%	64%	0%	0%	36%	24%	41%
Warehousing and storage	0%	60%	0%	0%	40%	19%	47%
Publishing industries	1%	44%	0%	43%	13%	39%	13%
Motion picture	0%	32%	0%	42%	26%	34%	16%
Broadcasting and telecom	2%	87%	2%	8%	3%	23%	16%
Data processing internet	0%	48%	0%	46%	6%	30%	11%
Federal Reserve banks	1%	82%	0%	0%	18%	31%	14%
Securities	1%	76%	0%	0%	24%	58%	13%
Insurance carriers	2%	94%	1%	0%	5%	37%	17%
Funds trusts	1%	100%	0%	0%	0%	48%	13%
Housing	8%	100%	0%	0%	0%	7%	5%
Other real estate	1%	3%	95%	0%	2%	17%	14%
Rental and leasing	1%	55%	0%	0%	45%	17%	16%
Legal services	1%	74%	16%	0%	10%	50%	10%
Computer systems design	1%	0%	13%	78%	9%	56%	17%
Misc professional scientific	7%	4%	3%	84%	9%	48%	16%
Management of companies	2%	0%	0%	99%	1%	52%	19%
Administrative and support	3%	9%	0%	90%	0%	29%	37%
Waste management	0%	99%	0%	0%	1%	20%	35%
Educational services	2%	99%	0%	0%	1%	52%	16%
Ambulatory health care	4%	100%	0%	0%	0%	47%	22%
Hospitals	4%	100%	0%	0%	0%	42%	26%
Nursing and residential	1%	100%	0%	0%	0%	43%	27%
Social assistance	1%	100%	0%	0%	0%	38%	35%
Performing arts spectator	0%	93%	0%	6%	2%	44%	24%
Amusements gambling	1%	100%	0%	0%	0%	28%	32%
Accommodation	1%	100%	0%	0%	0%	24%	31%
Food services	3%	100%	0%	0%	0%	21%	40%
Other services	3%	100%	0%	0%	0%	31%	39%
Federal gov (defense)	3%	100%	0%	0%	0%	31%	20%
Federal gov (nondefense)	2%	100%	0%	0%	0%	31%	20%
Federal gov enterprises	0%	91%	0%	0%	9%	30%	20%
State and local gov	7%	100%	0%	0%	0%	36%	21%
State and local gov enterprises	0%	100%	0%	0%	0%	32%	23%

Figure A1 visualizes one component of Table A2: the share of each industry's final use that is allocated to investment rather than consumption. The key takeaway from this figure is that most industries are used *either* for consumption or for investment, but not both. This is reassuring, as our calculation of the labor share for the aggregate consumption and investment sectors relies on the assumption that an industry's labor share is the same whether its output is used for consumption or investment. Since most industries are predominantly allocated to one category, this assumption is less restrictive in practice. The issue becomes even less significant when using the detailed input-output table with 402 industries, where an even larger fraction of industries is used primarily for either consumption or investment.

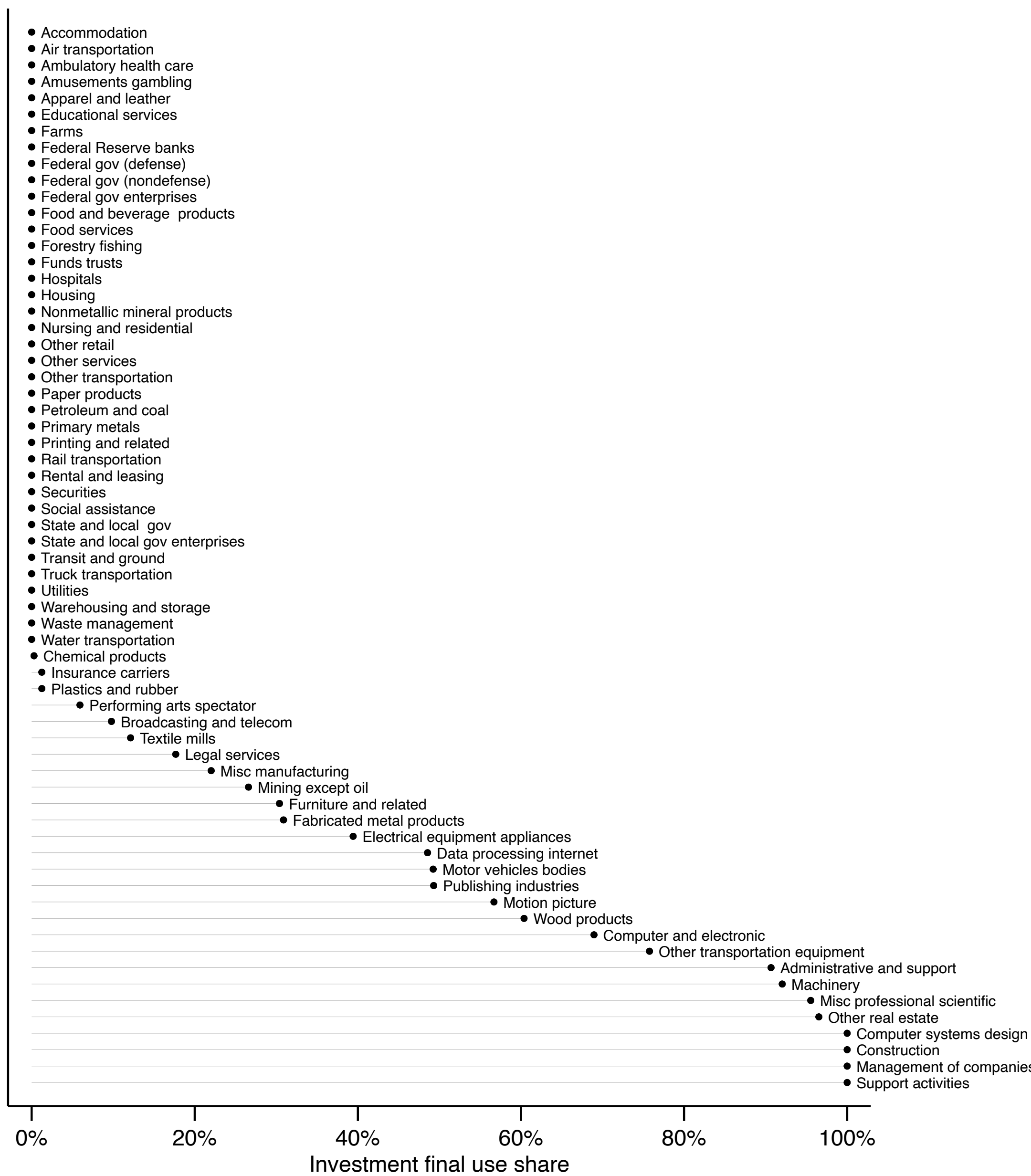

Figure A1: Investment as fraction of final use by industry

Table A2 compares the labor share computed using our baseline industry classification with the version based on the more detailed 402-industry classification for the year 2017. The results are very similar, indicating that using a coarser industry breakdown—necessary to extend the analysis back to 1963—does not materially affect our findings.

Table A2: Robustness of labor income shares

	College labor share	Noncollege labor share
Panel A: Consumption sector		
Detailed	28.2%	21.7%
Summary	29.3%	22.1%
Panel B: Investment sector		
Detailed	33.2%	23.6%
Summary	34.6%	26.6%

B.1 Compustat methodology

We now describe the list of adjustments we make to the cash-flow accounting statements of firms in Compustat in order to estimate the uses of capital income.

Adjustment #1: expensed payments to investment labor. One issue with accounting data is that the distinction between an expense (which are subtracted from sales to obtain cashflows from operation) and investment (which are not) can be arbitrary, especially in the case of intangibles.

The existing literature shows that this issue leads to an understatement of cashflows from operations and a corresponding understatement of (minus) cashflows from investment (e.g., Peters and Taylor (2017) for evidence from firm-level data and Koh et al. (2020) from aggregate data). A usual rule of thumb in the literature is to reallocate 30% of SG&A and 100% of R&D from expenses to intangible capital expenditures. This is meant to account for the incorrect expensing of labor expenses that contributes to organizational capital production. Recently, He et al. (2024) shows that salaries in sales and marketing alone account for

$$\text{cf adjustment (investment labor)} \equiv 0.3 \cdot \text{SG\&A} + \text{R\&D}.$$

Adjustment #2: net entry in sample. When a private firm becomes public or goes public, it effectively represents a negative cashflow for passive capitalists, who re-balance their portfolio to always own the market. Similarly, when a firm exits or gets acquired, it represents a positive cashflow. We now describe how we measure the contribution of net firm entry to market capitalization growth.

First, we introduce a decomposition of the total market value growth (at the aggregate or within an industry) between time t and $t+1$ as the sum of three components due to stayers, entrants, and exits. Formally, we have that

$$g_{\text{total}} = g_{\text{stayer}} + g_{\text{entry}} + g_{\text{exit}}, \tag{B.1}$$

where the components are defined in this footnote.[15]

Using these definitions, we define the net cashflow due to net firm entry as

$$\text{cf adjustment (net entry)} \equiv -(g_{\text{entry}} + g_{\text{exit}}) \cdot \text{market capitalization.} \tag{B.2}$$

It corresponds to the net cash that a passive capitalists who owns the market receives due to net firm entry during a period.

Adjustment #3: payments in stocks. How should we record a payment when it is the form of stocks? First, we can decompose the growth in market value of a stayer into growth of price per share and the growth in number of shares. To match the timing of the payment to the timing of the stock issuance, we use the fully-diluted share count (which includes outstanding shares, as well as all possible sources of potential shares such as stock options and reserved shares).

Formally, we can decompose the growth of the market capitalization of stayers (i.e., firms who do not exit the sample) defined in (B.1) into a price and number of shares component:

$$g_{\text{stayer}} = g_{\text{price}} + g_{\text{shares}},$$

where the components are defined in this footnote.[16]

Moreover, we observe the contribution of the growth in the number of shares issued for cash $g_{\text{shares, cash}}$, and the remainder $g_{\text{shares}} - g_{\text{shares, cash}}$ is due to a combination of stock compensation and stock-financed acquisitions (i.e., "noncash payments"). Using the same logic as for firm entry (see equation B.2), we define noncash payments as

$$\text{cf adjustment (noncash payments)} \equiv (g_{\text{shares}} - g_{\text{shares, cash}}) \cdot \text{market capitalization.}$$

The breakdown between stock compensation and acquisitions is only available after 2011. Therefore, we assume that acquisition is a constant share $\omega \in (0, 1)$ of noncash payments (stock issuance not associated with cash). We compute ω at the industry level after 2011, and use it to split stock compensation before 2011.

Adjustment #4: imputation of the tangible share of ambiguous investments. How should one proceed to allocate the sum of cashflows due to mergers, acquisitions, and IPOs into tangible versus intangible investment? To do so, we use the "ambiguous income" approach introduced in Cooley et al. (1995).

[15] The formulas are:

$$g_{\text{total}} = \frac{\sum_{i\in\mathcal{F}_{t+1}} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}} - 1; \qquad g_{\text{stayer}} = \frac{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t}} - 1,$$

$$g_{\text{entry}} = \frac{\sum_{i\in(\mathcal{F}_{t+1}\setminus\mathcal{F}_t)} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}}; \qquad g_{\text{exit}} = -\frac{(1+g_{\text{stayer}})\sum_{i\in(\mathcal{F}_t\setminus\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}},$$

where $\mathcal{F}_t$ is the universe of firms at time t and V_i is the market capitalization of firm i.

[16] The definitions of g_{stayer}, g_{price}, and g_{shares} are

$$g_{\text{stayer}} = \frac{P_{i,t+1}N_{i,t+1}}{P_{i,t}N_{i,t}} - 1; \qquad g_{\text{price}} = \frac{P_{i,t+1}}{P_{i,t}} - 1; \qquad g_{\text{shares}} = \left(\frac{N_{i,t+1}}{N_{i,t}} - 1\right)(1 + g_{\text{price}}),$$

where $P_{i,t}$ is the price per share and $N_{i,t}$ is the number of shares outstanding at firm i period t.

In our setup, the idea will be to assume that acquisitions have the same intangible content as other forms of investment. Denoting $\widetilde{\chi} \in (0,1)$ the intangibility of investment in the rest of investment, we have that

$$\widetilde{\chi} \equiv \frac{\text{cf adjustments}\left(\text{investment labor} + \psi \cdot (1-\omega) \cdot \text{noncash payments}\right)}{\text{cf from investing (capex)} + \text{cf adjustments}\left(\text{investment labor} + \psi \cdot (1-\omega) \cdot \text{noncash payments}\right)},$$

where we discuss the role of ψ shortly. We compute $\widetilde{\chi}$ at the industry year-level. Equipped with this last estimate, we are ready to construct our final variables.

Formulas. Having defined the key variables, we now write down the final formulas:

$$\begin{aligned}
\text{tangible investments} =& -\text{cf from investing (capex)}, \\
& -(1-\widetilde{\chi}) \cdot \text{cf from investing (acquisition)}, \\
& -(1-\widetilde{\chi}) \cdot \text{cf adjustments}\left(\text{net entry} + \omega \cdot \text{noncash payments}\right), \\
\text{intangible investments} =& -\text{cf adjustments}\left(\text{investment labor} + \psi \cdot (1-\omega) \cdot \text{noncash payments}\right), \\
& -\widetilde{\chi} \cdot \text{cf from investing (acquisition)}, \\
& -\widetilde{\chi} \cdot \text{cf adjustments}\left(\text{net entry} + \omega \cdot \text{noncash payments}\right), \\
\text{payments to capitalists} =& -\text{cf from financing} \\
& + \text{cf adjustments}\left(\text{net entry} + \text{noncash payments}\right).
\end{aligned}$$

Combining these formulas, we obtain an expression for capital income

$$\begin{aligned}
\text{capital income} =& \text{tangible investments} + \text{intangible investments} + \text{payments to capitalists} \\
=& \text{cf from operations} + \text{cf adjustments (investment labor)} \\
& -(1-\psi) \cdot (1-\omega) \cdot \text{noncash payments},
\end{aligned}$$

where the second equality uses the accounting identity of the cashflow statement. The number $\psi(1-\omega)$ corresponds to the share of noncash payments that is unreported in R&D and SG&A. Following changes in accounting standards in 2006, we set $\psi = 0.5$ before 2006 and $\psi = 0$ afterwards.

A few remarks are in order. First, that payments to capitalists are invariant to the calibration $(\widetilde{\chi}, \omega)$. This is because they only represent assumptions on how cashflows are distributed between tangible investments and intangible investments. Second, capital income is invariant not only to the calibration $(\widetilde{\chi}, \omega)$, but also the adjustments for noncash payments and net entry. This is because noncash payments and net entry are purely redistributive flows (from capitalists to tangible investments and to innovators). The only adjustment that affects the level of capital income is the adjustment for expensed investment labor. The idea is that we reclassify line items as capital expenditures, not expenses.

Table A3 reports the average distribution of capital income over the 1972–2022 period, including a detailed breakdown, and robustness checks. Tangible investments account for 42% of capital over the period, mostly due to direct capital expenditures, but also in part due to acquisition of existing businesses (who themselves are partial tangible). Intangible investments account for 44%. Cash and noncash compensation to investment labor accounts for most of it, but almost 20% comes from payouts associated with cash and noncash payout that arise in acquisitions. The most important line item is the component of sg&a that we attribute to management, marketing, and management labor (see He, Mostrom and Sufi (2024) for recent evidence).

Finally, payments to capitalists account for a mere 14% of capital income over the sample. While net cash payouts to equity holders account for 19% of capital income, they were offset by a 12% equity dilution (i.e., noncash payments). This sample average is skewed by the tech bubble of the 1990s, where payments to capitalists were negative for a few years (i.e., acquisitions and IPOs exceeded dividends and interest payouts, see Fried and Wang, 2019 for a discussion of this fact).

Table A3: Distribution of capital income (1972–2022 average).

Uses of capital income (%)	Baseline	Robustness			
		$\tilde{\chi}=0$	$\tilde{\chi}=1$	$\omega=0$	$\omega=1$
Tangible investments	36	45	37	37	42
Tangible capital expenditures	37	37	37	37	37
Mergers, acquisitions, and IPOs	–1	8	0	0	5
$(1-\tilde{\chi})\cdot$ cash acquisitions	3	8	0	3	3
$(1-\tilde{\chi})(1-\omega)\cdot$ non-cash payments	2	5	0	0	5
$(1-\tilde{\chi})\cdot$ net entry in public universe	–6	–6	0	–3	–3
Intangible investments	46	38	46	42	46
Intangible capital expenditure	38	38	38	39	35
$0.3\cdot$ selling, general, and admin. expenses	24	24	24	24	24
research and development expenses	12	12	12	12	12
$\psi\cdot\omega\cdot$ non-cash payments	2	2	2	4	0
Mergers, acquisitions, and IPOs	8	0	8	3	10
$\tilde{\chi}\cdot$ cash acquisitions	5	0	8	5	5
$\tilde{\chi}(1-\omega)\cdot$ non-cash payments	3	0	5	0	7
$\tilde{\chi}\cdot$ net entry in public universe	1	0	–6	–2	–2
Payout to capitalists	18	18	18	18	18
Net cash equity payout	19	19	19	19	19
− Non-cash payments	–13	–13	–13	–13	–13
− Net entry in public universe	6	6	6	6	6
Net debt payout	6	6	6	6	6

C Appendix for Section 4

C.1 Production network microfoundation.

The representative firm decides how much of final goods to produce every periods. There is a single consumption good C and N_K differentiated investment goods I_n. To produce these final goods, the firm operates I industries that produce intermediary goods Y_i. These intermediary goods are produced using a combination of intermediary goods, N_L labor types, and N_K different capital types. The firm maximizes the present value of payouts:

$$
\begin{aligned}
V_0 = & \max_{\{Y_{i,t}, L_{n,t}, K_{n,t+1}, C_t, I_{n,t},\}} \sum_{t=1}^{\infty} R_{0\to t}^{-1} D_t \\
\text{s.t.} \quad & D_t = C_t - \sum_{i=1}^{I}\sum_{n=1}^{N_L} w_{n,t} L_{in,t} && \text{(budget constraint)} \\
& C_t = Z_C \prod_{i=1}^{I} Y_{iC,t}^{\alpha_{in}} && \text{(consumption good)} \\
& I_{n,t} = Z_{nI} \prod_{i=1}^{I} Y_{iI,t}^{\beta_{in}} && \text{(investment good)} \\
& Y_{i,t} = Z_{iY}\Big(\prod_{j=1}^{I} Y_{ij,t}^{\gamma_{ij}}\Big)\Big(\prod_{n=1}^{N_K} K_{in,t}^{\gamma_{inK}}\Big)\Big(\prod_{n=1}^{N_L} (G^t L_{in,t})^{\gamma_{inL}}\Big) && \text{(intermediary good)} \\
& K_{n,t} = \sum_{i=1}^{I} K_{in,t} && \text{(sectoral allocation)} \\
& K_{n,t+1} = (1-\delta_n)K_{n,t} + K_{n,t}^{\theta} I_{n,t}^{1-\theta} && \text{(capital accumulation)}
\end{aligned}
$$

We now show that this model is isomorphic to a Uzawa two-sector representation.

Proposition C.1 (Uzawa two-sector representation)**.** *The production side is isomorphic to*

$$
\begin{aligned}
C &= A_C\Big(\prod_n K_{nC}^{a_{nJ}}\Big)\Big(\prod_n L_{nC}^{a_{nL}}\Big), \\
I &= A_I\Big(\prod_n L_{nI}^{b_{nL}}\Big)\Big(\prod_n K_{nC}^{b_{nK}}\Big),
\end{aligned}
$$

where the production elasticities are given by

$$
\begin{aligned}
\alpha_{nL} &\equiv \Big(a'(I-\Lambda_Y)^{-1}\Lambda_L\Big)_n, \quad \alpha_{nK} \equiv \Big(a'(I-\Lambda_Y)^{-1}\Lambda_K\Big)_n, \\
\beta_{nL} &\equiv \Big(b'(I-\Lambda_Y)^{-1}\Lambda_L\Big)_n, \quad \beta_{nK} \equiv \Big(b'(I-\Lambda_Y)^{-1}\Lambda_K\Big)_n.
\end{aligned}
$$

Proof. The cost functions for Y, C, and L are:

$$
\begin{aligned}
\log p &= (I-\Lambda_Y)^{-1}\Lambda_L \log w_L + (I-\Lambda_Y)^{-1}\Lambda_K \cdot \log w_K \\
\log p_C &= \alpha'(I-\Lambda_Y)^{-1}\Lambda_L \cdot \log w_L + \alpha'(I-\Lambda_Y)^{-1}\Lambda_K \cdot \log w_K \\
\log q &= \beta'(I-\Lambda_Y)^{-1}\Lambda_L \cdot \log w_L + \beta'(I-\Lambda_Y)^{-1}\Lambda_K \cdot \log w_K
\end{aligned}
$$

Those imply Cobb-Douglas production functions with exponents given by the elements of $\alpha'(I-\Lambda_Y)^{-1}\Lambda_L$

and $\beta'(I-\Lambda_Y)^{-1}\Lambda_L$ (two $N \times 1$ vectors). CRS in production implies that $(\Lambda_L\ \Lambda_K)\iota = 1$. □

C.2 Equilibrium of general model

We focus on the case $N_K = 1$, $N_L = 2$ and we time subscripts. The equilibrium conditions are

$$\begin{aligned}
\alpha_{1L}C &= w_{1C}L_{1C}, \quad \alpha_{2L}C = w_{2C}L_{2C}, \quad \alpha_K C = w_K K_C && \text{(focs - consumption good)}\\
\beta_{1L}qI &= w_{1I}L_{1I}, \quad \beta_{2L}qI = w_{2I}L_{2I}, \quad \beta_K qI = w_K K_I && \text{(focs - capital good)}\\
L_{1I}/L_{1C} &= (\mu_1/(1-\mu_1))(w_{1I}/w_{1C})^{\phi}, \quad L_{2I}/L_{2C} = (\mu_2/(1-\mu_2))(w_{1I}/w_{1C})^{\phi} && \text{(focs - sorting)}\\
L_{1C}+L_{1I} &= L_1, \quad L_{2I}+L_{2C} = L_2, \quad K_C+K_I = K && \text{(market clearing)}\\
C^{-\gamma} &= \beta(1+r')C'^{-\gamma} && \text{(market clearing - financial market)}\\
K' &= (1-\delta)K + K^{\theta}I^{1-\theta} && \text{(capital accumulation)}\\
(1+r')q &= w_K + \left(1-\delta_n+\theta\frac{I'}{K'}\right)q' && \text{(arbitrage equation)}
\end{aligned}$$

Steady-state. We now characterize the steady-state of the model:

$$\begin{aligned}
\alpha_{1L}C &= w_{1C}L_{1C}, \quad \alpha_{2L}C = w_{2C}L_{2C}, \quad \alpha_K C = w_K K_C && \text{(focs - consumption good)}\\
\beta_{1L}qI &= w_{1I}L_{1I}, \quad \beta_{2L}qI = w_{2I}L_{2I}, \quad \beta_K qI = w_K K_I && \text{(focs - capital good)}\\
L_{1I}/L_{1C} &= (\mu_1/(1-\mu_1))(w_{1I}/w_{1C})^{\phi}, \quad L_{2I}/L_{2C} = (\mu_2/(1-\mu_2))(w_{1I}/w_{1C})^{\phi} && \text{(focs - sorting)}\\
L_{1C}+L_{1I} &= L_1, \quad L_{2I}+L_{2C} = L_2, \quad K_C+K_I = K && \text{(market clearing)}\\
\delta K &= I, \quad q = \frac{w_K}{r+(1-\theta)\delta} && \text{(steady-state capital + financial market)}
\end{aligned}$$

First, using (focs - capital good - 3) and (arbitrage equation + financial market), we have that the long-run share of capital allocated to the investment sector is

$$\frac{K_I}{K} = \frac{\delta}{r+\delta}\beta_K$$

Plugging into (focs - consumption good - 3 + focs - capital good - 3), we have that consumption share of GDP is

$$\frac{C}{qI} = \frac{\beta_K}{\alpha_K}\frac{1-s_K}{s_K}$$

Denote the share of an input in the investment sector by s, so that $s_K \equiv \frac{K_I}{K}$ and so on. We can express the wage premium as

$$\frac{w_{nI}}{w_{nC}} = \frac{\beta_L/\beta_K}{\alpha_L/\alpha_K}\frac{s_K}{1-s_K}\frac{s_{nL}}{1-s_{nL}}.$$

Substituting in (focs - sorting), we have that the labor allocation problem for type n is the solution to

$$\begin{aligned}
\frac{s_{nL}}{1-s_{nL}} &= \frac{\mu_n}{1-\mu_n}\left(\frac{\beta_L/\beta_K}{\alpha_L/\alpha_K}\frac{s_K}{1-s_K}\right)\left(\frac{s_{nL}}{1-s_{nL}}\right)^{\phi}\\
\implies \frac{s_{nL}}{1-s_{nL}} &= \left(\frac{\mu_n}{1-\mu_n}\right)^{\frac{1}{1+\phi}}\left(\frac{\beta_L/\beta_K}{\alpha_L/\alpha_K}\frac{s_K}{1-s_K}\right)^{\frac{\phi}{1+\phi}}
\end{aligned}$$

Equipped with (s_K, s_{1L}, s_{2L}), it is trivial to solve the rest of the steady-state allocation (q, K).

Capital supply elasticity. From now on, we drop the arbitrage equation and have q be exogeneous. This allows us to analyze the supply-side of the economy. In the short-run, K is fixed, so that there are only three unknowns $(s_K, s_{1L}, s_{2,L})$ and three equations:

$$\begin{aligned}
\frac{\alpha_{1L}C}{\beta_{1L}qI} &= \left(\frac{\mu_1}{1-\mu_1}\right)^{\frac{1}{\phi}}\left(\frac{L_{1C}}{L_{1I}}\right)^{1+\frac{1}{\phi}} \\
\frac{\alpha_{2L}C}{\beta_{2L}qI} &= \left(\frac{\mu_2}{1-\mu_2}\right)^{\frac{1}{\phi}}\left(\frac{L_{2C}}{L_{2I}}\right)^{1+\frac{1}{\phi}} \\
\frac{\alpha_K C}{\beta_K qI} &= \frac{K_C}{K_I}
\end{aligned}$$

Let's look at first-order dynamics

$$\begin{pmatrix}
\frac{s_K}{1-s_K}\alpha_K + \beta_K & \frac{s_{1L}}{1-s_{1L}}\left(\alpha_{1L} - \frac{1+\phi}{\phi}\right) + \beta_{1L} & \frac{s_{2L}}{1-s_{2L}}\alpha_{2L} + \beta_{2L} \\
\frac{s_K}{1-s_K}\alpha_K + \beta_K & \frac{s_{1L}}{1-s_{1L}}\alpha_{1L} + \beta_{1L} & \frac{s_{2L}}{1-s_{2L}}\left(\alpha_{2L} - \frac{1+\phi}{\phi}\right) + \beta_{2L} \\
\frac{s_K}{1-s_K}(\alpha_K - 1) + \beta_K & \frac{s_{1L}}{1-s_{1L}}\alpha_{1L} + \beta_{1L} & \frac{s_{2L}}{1-s_{2L}}\alpha_{2L} + \beta_{2L}
\end{pmatrix}
\begin{pmatrix}
\frac{\mathrm{d}\log s_K}{\mathrm{d}\log q} \\
\frac{\mathrm{d}\log s_{1L}}{\mathrm{d}\log q} \\
\frac{\mathrm{d}\log s_{2L}}{\mathrm{d}\log q}
\end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

C.3 Calibration details

We first extend the concept of "long-run" to a balanced growth path. Denote $\pi_X \equiv \log(X_{t+1}/X_t)$ the growth rate associated to a variable X. Differentiating the production functions for output and investment gives:

$$\begin{aligned}
\pi_Y &= \pi_{A_Y} + \alpha\pi_K + (1-\alpha)\pi_{L_Y}, \\
\pi_H &= \pi_{A_H} + \theta\pi_K + (1-\theta)(1-\chi)\pi_I + (1-\theta)\chi\pi_{L_H}.
\end{aligned}$$

To obtain a balanced growth path with constant labor inputs, we assume that the productivity of production labor and investment labor grow at the same rate π. Expressed in terms of Hicks-neutral productivity, this is equivalent to assuming $\pi_{A_Y} = (1-\alpha)\pi$, $\pi_{A_H} = (1-\theta)\chi\pi$. In this case, all variables (K, Y, C, I, w_Y, w_H) grow at the same rate π. Using the optimality conditions, we obtain

$$\begin{aligned}
1 + r &= \beta^{-1}(1+\pi)^{\gamma} \\
\frac{H}{K} &= \delta + \pi \\
q &= \frac{1}{r - \pi + (1-\theta)(\delta+\pi)}\frac{\alpha Y}{K}.
\end{aligned}$$

Taking the log of the first equation implies that we can calibrate β as the difference between log returns minus $\gamma\pi$.

The total investment yield is:

$$\frac{I + w_H L_H}{V} = (1-\theta)\frac{qH}{V} = (1-\theta)\frac{H}{K} = (1-\theta)(\delta+\pi).$$

As explained in Appendix A.1, the total investment yield corresponds to capitalists' dilution rate; that is, the extent to which an investor drawing down total capital income every period from the representative firm would get diluted over time. Consistently with this idea, note that the equation for q given above can be interpreted as the value of a income flow initially equal to αY (capital income) and growing at rate $\pi - (1-\theta)(\delta+\pi)$, which corresponds to the growth rate of the economy minus the dilution rate of capitalists.

Manipulating the first-order conditions of the firm implies that, in the long run, the uses of capital income in the economy are given by:

$$\begin{aligned}\frac{I}{\Pi} &= (1-\chi)\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)}\\ \frac{w_H L_H}{\Pi} &= \chi\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)}\\ \frac{D}{\Pi} &= \frac{r-\pi}{r-\pi+(1-\theta)(\delta+\pi)}.\end{aligned}$$

This equation expresses the share of tangible investment, the share of intangible investment, and the payout share in steady state in terms of five parameters.

The capital share of income is

$$\frac{\Pi}{Y+w_H L_H} = \frac{\alpha Y}{Y+\chi(1-\theta)qH} = \frac{\alpha}{1+\chi(1-\theta)\frac{qH}{Y}} = \frac{\alpha}{1+\chi\alpha\frac{(1-\theta)(\delta+\pi)}{r-\pi+(1-\theta)(\delta+\pi)}},$$

The last moment is the short-run, partial equilibrium tax elasticity. Consider a capital income tax shock $\mathrm{d}\tau_{K,t} = (1-\phi)^t\,\mathrm{d}\tau_K$ at $t=0$ around the undistorted steady-state. Using (firm foc I_t) and (firm foc $L_{H,t}$):

$$I_t + w_{H,t}L_{H,t} = (1-\theta)q_t H_t.$$

The log total derivative at $t=0$ is

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = \mathrm{d}\log q_0 + \mathrm{d}\log H_0 = \left(1+\frac{1-\theta}{\theta}\right)\mathrm{d}\log q_0,$$

where the second equality is obtained using the fact that, in particular equilibrium, wages are taken as given. In that case, we obtain

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = \frac{1}{\theta}\,\mathrm{d}\log q_0,$$

Using the envelope theorem on the firm value function, and using the fact that $\mathrm{d}w_{Y,t} = \mathrm{d}w_{H,t} = 0$, we obtain

$$\mathrm{d}\log q_0 = -\frac{r}{r+\phi}\frac{r+(1-\theta)\delta}{r}\,\mathrm{d}\tau_K$$

The first term accounts for the duration of the tax change, where ϕ represents an annual rate of decay. The second term accounts for the ratio of the tax base (capital income) to the capital payout. Putting together, we obtain

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = -\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}\,\mathrm{d}\tau_K.$$

C.4 Proofs

Proof of Proposition 4.1. The Lagrangian associated with the household problem is

$$\mathcal{L}_0 = \sum_{t=1}^{\infty} \beta^t U(C_t, L_t) - \sum_{t=1}^{\infty} \lambda_t \Bigg(C_t + V_t - w_{Y,t} L_{Y,t} - w_{H,t} L_{H,t} - (1 + r_t) V_{t-1} \Bigg)$$
$$\text{where } U(C, L) \equiv \frac{1}{1-\gamma} \left(C_t - \frac{L_t^{1+\frac{1}{\sigma}}}{1 + \frac{1}{\sigma}} \right)^{1-\gamma}.$$

Applying the envelope theorem gives:

$$\begin{aligned}
\mathrm{d}U_0 &= \frac{\partial \mathcal{L}_0}{\partial V_0} \mathrm{d}V_0 + \sum_{t=1}^{\infty} \left(\frac{\partial \mathcal{L}_0}{\partial w_{Y,t}} \mathrm{d}w_{Y,t} + \frac{\partial \mathcal{L}_0}{\partial w_{Y,t}} \mathrm{d}w_{H,t} + \frac{\partial \mathcal{L}_0}{\partial r_t} \mathrm{d}r_t \right) \\
&= \lambda_1 (1 + r_1) \mathrm{d}V_0 + \sum_{t=1}^{\infty} \lambda_t \left(L_{Y,t} \mathrm{d}w_{Y,t-1} + L_{H,t} \mathrm{d}w_{H,t-1} + V_{t-1} \mathrm{d}r_t \right) \\
&= \beta \partial_C U(C_1, L_1)(1 + r_1) \mathrm{d}V_0 + \sum_{t=1}^{\infty} \beta^t \partial_C U(C_t, L_t) \left(L_{Y,t} \mathrm{d}w_{Y,t-1} + L_{H,t} \mathrm{d}w_{H,t-1} + V_{t-1} \mathrm{d}r_t \right), \\
&= \beta \partial_C U(C_1, L_1) \left((1 + r_1) \mathrm{d}V_0 + \sum_{t=1}^{\infty} R_{0 \to t}^{-1} \left(L_{Y,t} \mathrm{d}w_{Y,t-1} + L_{H,t} \mathrm{d}w_{H,t-1} + V_{t-1} \mathrm{d}r_t \right) \right),
\end{aligned}$$

where the second and third equalities use the household's focs with respect to C_t and V_t, respectively. In the case where $\gamma = 0$ (small open economy), we have that $1 + r_t = \beta^{-1}$ which means that $\mathrm{d}r_t = 0$. □