Capital Income in an Intangible Economy

Matthieu Gomez* Émilien Gouin-Bonenfant†

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Abstract

Capital formation is increasingly driven by intangible investment, which critically relies on specialized labor (e.g., managers, researchers, etc). In this paper, we study the implication of this rising share of intangible investment for the macroeconomy. We consider a general neoclassical model with tangible and intangible investment, calibrated using data on the allocation of capital income in the corporate sector (i.e., tangible investments, intangible investments, and payouts). We emphasize that rising intangibles make the supply of capital more inelastic, both in the short-run and in the long-run, owing to the limited supply of specialized labor. Rising intangibles also change the incidence of capital taxation: whereas the tax burden entire falls on production workers in the neoclassical growth model, it is borne disproportionately by specialized workers and capital owners in intangible economies.

*Columbia University, mg3901@columbia.edu.
†Columbia University, eg3041@columbia.edu.

1 Introduction

Modern capital formation increasingly takes the form of intangible investments, in particular research and development (Corrado and Hulten, 2010). One key property of intangible investment, relative to tangible investment, is that it critically relies on talented, specialized workers (e.g., managers, researchers, etc). This paper examines the implication of this rising intangible share for the economy.

Overview of results. The starting point of the paper is that investment goods (i.e., machines, computers, plants) cannot be frictionlessly transformed into capital. Capital formation also requires specialized labor (i.e., researchers, entrepreneurs, financiers) and accumulated capital (i.e., building on past research, adjustment frictions). We represent this through a general aggregate investment functions:

$$\text{capital formation} = \text{existing capital}^{\theta} \cdot \left(\text{investment goods}^{\chi} \cdot \text{specialized labor}^{1-\chi}\right)^{1-\theta}. \tag{1.1}$$

This functional form provides a flexible two parameter investment function, that encompasses a wide range of existing models (with the neoclassical model as a special case). The fact that investment goods cannot be transformed into capital without friction, but instead also requires existing capital and specialized labor, has important implications on the aggregate and redistributive effects of economic shocks (e.g., taxes, technology or demand for savings).

The paper has three parts. First, we characterize the distributional implications of intangible capital (the presence of specialized labor in the investment function). We embed the equation for capital formation 1.1 in a standard neoclassical model and characterize how the investment technology affects the uses of capital income and the supply elasticity of capital. We emphasize the fact that investment in intangible economies (i.e., low χ) is less responsive to shocks (i.e., technology, discount rate, and/or tax shocks). The core idea is that intangible investments requires specialized labor, which is scarce, making investment less elastic, both in the short- and long-run. For instance, a given shock in savings is not fully absorbed by the corporate sector and instead results in higher prices, both for the value of existing capital but also for the wage of specialized workers.

We then calibrate the model using a combination of long-run facts about the distribution of income and short-run evidence on the investment-q elasticity. We rely on the following central accounting identity, that specifies how capital income is used:

$$\text{capital income} = \text{payout} + \text{tangible investments} + \text{intangible investments}. \tag{1.2}$$

The central goal of our accounting exercise is to allocate 100% of aggregate capital income into those three categories. Compared to the BEA's current methodology, we use a more comprehensive definition of intangible investment, seeking to capture *all* payments to nonproduction labor (i.e., to entrepreneurs, managers, researchers, etc.), including via noncash payments (i.e., via stock options, acquisitions, etc.).[1]

[1]Regarding the first point, Corrado et al. (2022) points that that the market value of leading US tech firms vastly exceeds their tangible assets and capitalized R&D expenditures (essentially what the BEA does) and recognizes that " [...] understanding modern firms and indeed modern economies requires broadening the concept of capital beyond tangible assets to include intangibles,

We document a large rise in the labor content of corporate investment: the intangible share increases from roughly 40 to 65%.

We then use the calibrated model to simulate a number of counterfactuals. We find that a shift towards intangibles of the magnitude that we have seen in the data implies a significant redistribution of aggregate income away from production labor towards investment labor (-6 pp. of GDP). In addition, it makes the economy less elastic: the long-run supply elasticity declines by roughly half. As a case study, we study how this shift towards intangibles affects the incidence of capital taxes. In the neoclassical growth model (henceforth NGM)—where investment is only goods—it is well known that capital tax cut have a large effect on investment, and, as a result, capital taxes fall entirely on production workers via higher wages. In intangible economies, however, tax cut have a weaker effect on investment, hence a lower effect on production worker wages. As a result, capitalists and investment labor absorb more of the shock via revaluation gains (for capitalists) and higher wages (for investment labor). In the calibrated model, capital tax shocks fall

In Section 2, we write down a standard neoclassical model of capital accumulation, augmented with the general investment function (1.1) and two upward-sloping labor supply curves (production and investment workers). We characterize the distribution of income as well as the supply elasticity of capital, both in the short- and long-run.

In terms of the long-run distribution of income, we show that the capital share of investment θ governs the amount of investment rents that accrue to owners of capital.[2] The tangibility of investment χ determines how the remainder is split between tangible- and intangible-investment. We benchmark the model predictions against important special cases: neoclassical growth model (Cass-Koopmans), q-theory Hayashi (1982), and sweat capital (Bhandari and McGrattan, 2021).

In terms of the supply elasticity of capital, we then emphasize the fact that when inelastically supplied labor is an input in capital formation, both the short- and long-run capital supply elasticities are finite. This stands in contrast to the neoclassical growth model, where both the short- and long-run elasticities are infinite (i.e., goods and be turned into capital).

In Section 3, we map the model to the data. We construct a dataset on the uses of capital income in the US nonfinancial corporate sector. We focus on public firms (Compustat-CRSP) over the 1972-2022 period, where we have full financial statements, which, crucially, include noncash payments (e.g., equity grants, stock options, acquisitions financed with stocks). The empirical counterpart of (1.2) is the cashflow identity, which tracks how much cash is earned (cashflows from operations), how much is investment (minus cashflows from investing), and how much is paid out (minus cashflows from financing). Building on the existing literature, we make three key adjustments to the accounting data. First, we treat nonproduction labor costs as intangible investment, rather than production costs (see, e.g., Corrado, Hulten and Sichel, 2005; Corrado, Hulten and Sichel, 2009; Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Koh, Santaeulàlia-Llopis and Zheng, 2020). The idea is that if an employee

and that research and development spending is not the only way to capture intangible capital.". Regarding the second point, Eisfeldt et al. (2023) shows that the BEA misses a large share of stock payments to key employees by US manufacturing firms.

[2] We define investment rents in the sense of Ricardo: based on the difference between the average and marginal product of labor.

dose not contribute to the production process, then they must contribute to capital formation, broadly defined. Second, we account for IPOs and acquisitions net of delistings and allocate the implied cash-flows to tangible and intangible investments. Third, we account for noncash payments, treating them as economically equivalent to cash payments. This is particularly important for nonproduction labor (researchers, managers) who earn a large share of their compensation in stocks (see, e.g., Eisfeldt, Falato and Xiaolan, 2023).

We obtain an annual dataset on the aggregate (and industry-level) uses of capital income, which maps exactly into the firm budget constraint (1.2) in the model. We emphasize that our accounting is fully consistent with national accounting, which now acknowledges that some form of labor expenses are indeed investment. However, we go further than the BEA and consider any nonproduction labor expenses as investment. Corrado, Haskel, Jona-Lasinio and Iommi (2022) discuss this topic in detail, highlighting the fact that the following expenses are not currently considered investment in the national accounts: "market research and branding, operating models, platforms, supply chains, distribution networks, employer-provided training, attributed designs (industrial), and financial product development".

Three long-run facts stand out in the data: (1) the tangible investment share declines, (2) the intangible investment share increases, and (3) the payout share is low on average and highly countercyclical.

We then use the constructed dataset to calibrate the key elasticities of the model. We use a combination of long-run moments (aggregate labor share, tangible investment yield, intangible investment yield) and short-run moments (i.e., industry-level investment-q elasticity).

Finally, in Section 4, we use the calibrated the model to quantify the effect of rising intangibles. We calibrate the key elasticities of the model by long-run moments (aggregate labor share, tangible investment yield, intangible investment yield) and a short-run moment (i.e., industry-level investment-q elasticity). Equipped with the calibrated model, we simulate the effect of a rise in intangibility of the magnitude we have seen in the data (i.e., a 25 pp. decline in the model parameter χ, which governs the tangibility of investment). We find (i) a large transformative effect on the long-run composition of GDP, (ii) an important flattening of the capital supply curve, (iii) a rise in the incidence of capital taxes for capitalists and investment labor.

Literature review. A growing literature documents the rise in intangible capital (see, e.g., Eisfeldt and Papanikolaou, 2013, Peters and Taylor, 2017, Eisfeldt, Falato and Xiaolan, 2023, and Corrado, Haskel, Jona-Lasinio and Iommi, 2022). Beyond being harder to measure, intangible capital may exhibit distinct economic properties relative to tangible capital. In Eisfeldt and Papanikolaou (2013), intangible capital is unique because it is embedded in key talents. In Crouzet, Eberly, Eisfeldt and Papanikolaou (2022), intangible capital tends to be non-rival—allowing it to be used simultaneously in different production streams—and is characterized by limited excludability, which prevents firms from capturing all the associated benefits or rents. In this paper, we shift our focus from focusing on the economic properties of intangible capital to its distinct inputs, namely, that it is created by specialized labor in finite supply.

The closest study to ours is Luttmer (2018), which studies an economy in which households supply both managerial and production labor—with managerial labor contributing to both production and investment. Like the present paper, Luttmer (2018) highlights that organizational capital is produced using a specialized input by discussing the implications of a fixed supply of managerial capital. Our paper contributes to this work by providing new micro-moments to calibrate the model and using the calibrated models to run counterfactuals on the transition toward intangible capital and the incidence of corporate taxes. Another closely related paper is Bhandari and McGrattan (2021), which highlights that a significant portion of small firms' value stems from organizational capital accumulated through the owner's effort. The paper further explores the implications of this finding for the taxation of non-corporate businesses.

Finally, our paper offers an alternative explanation for investment stagnation despite high Q. The existing literature focuses on markups and market power (e.g., Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019; Barkai, 2020; Ball and Mankiw, 2023; De Ridder, 2024). In contrast, we emphasize that capital formation requires specialized labor (i.e., researchers, financiers). Since this specialized labor is in limited supply, it constrains the extent to which investment goods can be transformed into productive capital. Put differently, in our paper, the "fixed factor" that limits the ability (or willingness) of firms to scale in not to be found on the production side of firms but in their investment technology.

2 Model

2.1 Production block

General model. We focus on a representative firm. Output Y_t can be produced from capital K_t and production labor $L_{Y,t}$ through the standard Cobb-Douglas production function:

$$Y_t = z_{Y,t} K_t^{\alpha} L_{Y,t}^{1-\alpha}. \tag{production}$$

Our key departure from the standard neoclassical growth model is that output cannot be frictionlessly transformed into capital. Consistent with evidence on capital formation in the modern corporate sector, building new productive units of capital G_t requires specialized labor $L_{H,t}$ (i.e., researchers, entrepreneurs, financiers) as well as accumulated capital (i.e., building on past research, adjustment frictions). We represent this through the following aggregate investment function:

$$G_t = z_{H,t} K_t^{\theta} \left(I_t^{\chi} L_{H,t}^{1-\chi} \right)^{1-\theta}, \tag{capital formation}$$

$$K_{t+1} = (1-\delta) K_t + G_t. \tag{capital accumulation}$$

First, investment requires both tangible inputs I and specialized labor L_H; the parameter $\chi \in (0,1)$ governs the tangibility of capital. Second, there are diminishing marginal returns in both tangible inputs and specialized labor (holding constant the existing stock of capital); the capital share in investment is governed by $\theta \in (0,1)$. The sequences $\{z_{Y,t}, z_{H,t}\}_{t=1}^{\infty}$ represent Hicks-neutral productivity in production and investment, TFP and IST for short.

The representative firm takes as given the interest rate R_t, as well as wage rates for production and investment labor $(w_{Y,t}, w_{H,t})$. It chooses production labor $L_{Y,t}$, investment labor $L_{H,t}$, and tangible investment I_t to maximize the present value of future payouts:

$$V_0 = \max_{\{L_{Y,t}, L_{H,t}, I_t, K_{t+1}\}} \sum_{t=1}^{\infty} R_{0\to t}^{-1} D_t,$$
$$\text{s.t.} \qquad D_t = Y_t - I_t - w_{Y,t}L_{Y,t} - w_{H,t}L_{H,t} \qquad \text{(budget constraint)}$$

subject to the equations for production, capital formation, and capital accumulation above. We denote the cumulative return as $R_{0\to t} \equiv \prod_{s=1}^{t} R_s$. The corresponding Lagrangian is

$$\mathcal{L}_0 = \sum_{t\geq 1} R_{0\to t}^{-1}\Big(Y_t - I_t - w_{Y,t}L_{Y,t} - w_{H,t}L_{H,t}\Big) + \sum_{t\geq 1} R_{0\to t}^{-1} q_t \Big((1-\delta)K_t + G_t - K_{t+1}\Big),$$

where q_t can be interpreted as the shadow value of productive capital units. Solving the representative firm problem yields the following set of optimality conditions:

$$w_{Y,t}L_{Y,t} = (1-\alpha)Y_t, \qquad \text{(firm foc } L_{Y,t}\text{)}$$
$$w_{H,t}L_{H,t} = (1-\theta)(1-\chi)G_t q_t, \qquad \text{(firm foc } L_{H,t}\text{)}$$
$$I_t = (1-\theta)\chi G_t q_t, \qquad \text{(firm foc } I_t\text{)}$$
$$R_{t+1}q_t = \alpha\frac{Y_{t+1}}{K_{t+1}} + \left(1-\delta+\theta\frac{G_{t+1}}{K_{t+1}}\right) q_{t+1}, \qquad \text{(firm foc } K_{t+1}\text{)}$$

with the transversality conditions $\lim_{T\to\infty} R_{0\to t}^{-1} q_T K_{T+1} = 0$. While our production function is based on a representative firm, in Appendix A.4, we provide an explicit microfoundation of the aggregate production and capital formation functions (as well as how aggregate TFP and IST shocks map to primitives).

Special cases. Our specification of the investment function nests and generalizes several important models used in the literature.

Neoclassical growth model ($\theta = 0, \chi = 1$). Our model nests the NGM in the special case $\theta = 0, \chi = 1$. In this case the capital formation equation simplifies to $G_t = z_{H,t}I_t$, which implies that goods and capital can be transformed into each other using a (potentially time-varying) linear technology.

Q-theory ($\chi = 1$). Another important special case is where investment requires both goods and capital, but not specialized labor ($\chi = 1$). In this case, the capital accumulation becomes $G_t = z_{H,t}K_t^{\theta}I_t^{1-\theta}$. This type of investment function is similar to the traditional q-theory of investment (Uzawa, 1969, Hayashi, 1982) which models capital accumulation with the general adjustment cost function $G_t = K_t\psi(I_t/K_t)$. Hence, our theory corresponds to the special case in which $\psi(I_t/K_t) = (I_t/K_t)^{1-\theta}$. The key insight is that the capital share θ is directly related to the curvature of the adjustment cost function in the traditional q-theory of investment. This parameter governs the *short-run* fluctuations in investment, as, in this particular formulation , the elasticity of today's investment I_t to today's q_t is $1/\theta$.

Sweat capital ($\chi = 0$). Another important special case is where investment requires both capital and specialized labor, but not goods themselves ($\chi = 0$). In this case, the capital accumulation becomes $G_t = z_{H,t} K_t^{\theta} L_{H,t}^{1-\theta}$. This corresponds to the various other models where firm expansion requires organizational capital (e.g., Luttmer, 2011; Bhandari and McGrattan, 2021). A similar idea is at the core of several important models, such as the Melitz (2003) model (firm entry requires labor) or the Romer (1986) model (innovation requires labor).

2.2 Equilibrium

Closing the model. Since the focus of the paper is on the firm side, we only provide a stylized microfoundation of the household problem. A representative household supplies differentiated labor $\{L_{Y,t}, L_{H,t}\}_{t=1}^{\infty}$ as well as consumption and wealth $\{C_t, V_t\}_{t=1}^{\infty}$ as to maximize welfare. Taking as given prices $\{w_{Y,t}, w_{H,t}, R_t\}_{t=1}^{\infty}$ and intial wealth V_0, the household solves a standard problem:

$$\begin{aligned} U_0 = & \max_{\{C_t, L_{Y,t}, L_{H,t}, V_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \left\{ \frac{C_t^{1-\rho}}{1-\rho} - \frac{(1-\mu) L_{Y,t}^{1+\sigma} + \mu L_{H,t}^{1+\sigma}}{1+\sigma} \right\}. \qquad (2.1) \\ \text{s.t.} \quad & C_t + V_t = w_{Y,t} L_{Y,t} + w_{H,t} L_{H,t} + R_t V_{t-1}. \end{aligned}$$

The initial wealth V_0 is given. The optimality conditions are

$$C_t^{-\rho} = \beta R_{t+1} C_{t+1}^{-\rho}, \qquad (\text{worker foc } C_t)$$

$$w_{Y,t} = (1-\mu) L_{Y,t}^{\sigma}, \qquad (\text{worker foc } L_{Y,t})$$

$$w_{H,t} = \mu L_{H,t}^{\sigma}. \qquad (\text{worker foc } L_{H,t})$$

The first equation says is the standard consumption Euler equation. The other two equations generate labor supply curves, where the parameter $\mu \in (0,1)$ governs the relative supply of the two labor types, and $\sigma > 0$ governs the labor supply elasticity.[3]

Equilibrium. An equilibrium is an initial condition K_0, an allocation $\{L_{Y,t}, L_{H,t}, I_t, K_t\}_{t \geq 1}$, and prices $\{w_{H,t}, w_{L,t}, R_t\}_{t \geq 1}$ that solve the firm problem (firm foc $L_{Y,t}$, firm foc $L_{H,t}$, firm foc I_t, firm foc K_{t+1}) and household problem (worker foc C_t, worker foc $L_{Y,t}$, worker foc $L_{H,t}$).

Lemma 2.1 (Equilibrium). *The equilibrium $(L_{Y,t}, L_{H,t}, I_t)$, as a function of the state variables $(K_t, z_{Y,t}, z_{H,t})$*

[3] Combining both equations, we have that (μ, σ) pins down the relative wage bill in terms of relative labor and wages:

$$\frac{w_{Y,t} L_{Y,t}}{w_{H,t} L_{H,t}} = \frac{1-\mu}{\mu} \left(\frac{L_{Y,t}}{L_{H,t}} \right)^{1+\sigma} = \left(\frac{1-\mu}{\mu} \right)^{-\frac{1}{\sigma}} \left(\frac{w_{Y,t}}{w_{H,t}} \right)^{1+\frac{1}{\sigma}}. \qquad (2.2)$$

and the co-state variable q_t, is given by

$$L_{Y,t} = \overline{l_Y} \cdot \left(z_{Y,t} \cdot K_t^{\alpha}\right)^{\frac{1}{\sigma+\alpha}},$$
$$L_{H,t} = \overline{l_H} \cdot \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1}{\sigma+\theta-\sigma\chi(1-\theta)}},$$
$$I_t = \bar{\iota} \cdot \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1+\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}},$$

where $\overline{l_Y}, \overline{l_H}, \bar{\iota} > 0$ are defined in Appendix A.1. The backward looking equation for K_t (capital accumulation) and the forward looking equation for q_t (firm foc K_{t+1}) complete the equilibrium.

It is instructive to focus on the case $\chi = 1$ at first (all investment is intangible). In this case, investment labor is $L_{H,t} \propto \left(z_{H,t} \cdot q_t \cdot K_t^{\theta}\right)^{\frac{1}{\sigma+\theta}}$. As usual, investment labor is increasing in capital because L_H is complement with K in the investment process. But notice that it is also increasing in q, the value of capital. Investment labor can therefore be thought of as having a "long duration" marginal product of labor: work done by investment labor today yields higher output in the future. For production labor, this is not the case, and as a result $L_{Y,t}$ does not depend directly on q_t.

2.3 The demand and supply of capital

Relative to the neoclassical growth model, our model generates an inelastic supply of capital, both in the short-run and in the long-run. We first discuss the two equations determining the price and quantity of capital (q, K) in the long-run as the solution to a supply and demand and supply system.

Lemma 2.2 (Steady-state)**.** *The steady-state level of capital K, and its value q, are determined by:*

$$q = \overline{q_D} \cdot K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}}, \qquad \text{(Demand)}$$
$$q = \overline{q_S} \cdot K^{\frac{1-\chi}{\frac{1}{\sigma}+\chi}}, \qquad \text{(Supply)}$$

where $\overline{q_D}, \overline{q_S} > 0$ are defined in Appendix A.1.

Figure 1 shows a numerical example of how the long-run capital market equilibrium is determined. The demand curve is a valuation equation. It says that, in order to earn a return R, investors require a lower valuation q when the stock of capital is high. As usual, this is because the marginal product of capital is declining in K.

The supply curve is essentially a wage equation, which states that investment labor requires higher wages in order to increase its input in capital formation. The slope of this supply curve is

$$\mathrm{d}\log K = \frac{\frac{1}{\sigma}+\chi}{1-\chi} \cdot \mathrm{d}\log q \qquad \text{(Capital elasticity, long-run)}$$

In contrast with the neoclassical model, when investment requires labor (i.e., $\chi < 1$), the elasticity of capital is finite and governed by the labor supply elasticity $\frac{1}{\sigma}$. Different combinations of (σ, χ) span long-run capital elasticities from zero to infinity.

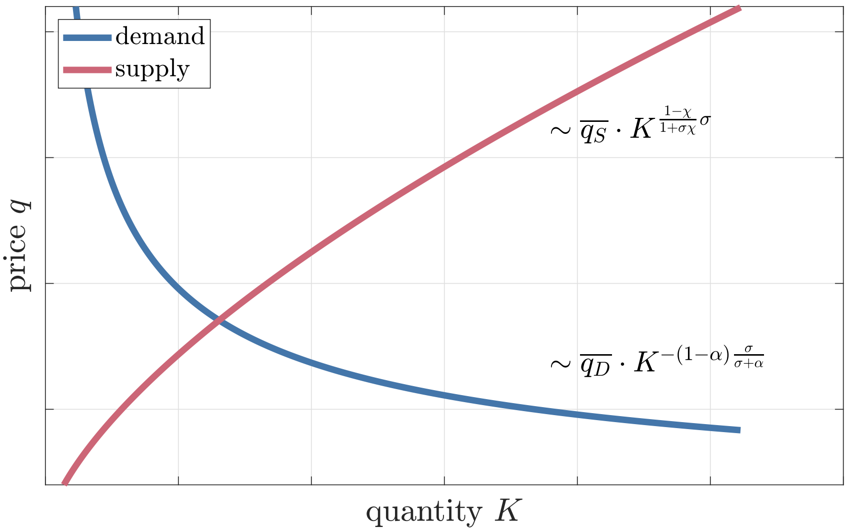


Figure 1: Long-run capital market equilibrium

The equilibrium value of capital q in steady-state is given by

$$q = \frac{1}{r + (1-\theta)\delta} \cdot \frac{\Pi}{K} \qquad \text{(Value of capital)}$$

It is the average return on capital Π/K, times a multiple that depends on the discount rate $r \equiv R - 1$. The quantity $(1-\theta)\delta$ can be seen as the rate at which a capitalists consuming its capital income every period would get diluted (see Appendix A.2).

Notice that neither the demand or supply elasticities in the long-run depend on θ. The parameter θ only matters for the short-run elasticity of capital. The following proposition derives the supply elasticity of capital to a perturbation in the path of the value of capital $\{\mathrm{d}q_t\}_{t=1}^{\infty}$:

Proposition 2.3 (Supply elasticity of capital)**.** *Conditional on a perturbation $\{\mathrm{d}q_t\}_{t=1}^{\infty}$ around the steady-state, the perturbation in the supply of capital is*

$$\mathrm{d}\log K_{T+1} = \frac{\frac{1}{\sigma} + \chi}{1-\chi} \cdot \delta \sum_{s=0}^{T} (1-\delta)^s \,\mathrm{d}\log q_{T-s} \tag{2.3}$$

$$\textit{where } \delta \equiv \delta \frac{(1+\sigma\chi)(1-\theta)}{\sigma + \theta - \sigma\chi(1-\theta)}.$$

While the capital share of investment θ did not matter for the long-run capital elasticity, it is the key determinant of the short-run capital elasticity. Capital being an input in production $\theta > 0$ makes capital less elastic in the short-run. This is exactly the logic behind the q-theory of investment, which emphasizes the presence of capital adjustment costs in the short-run. As in the long-run, capital supply is more elastic when tangibility χ is high. Finally, note that we recover the long-run elasticity as the limiting case of the short-run elasticity after setting $\mathrm{d}\log q_t = \mathrm{d}\log q$ for all t and taking the limit $t \to \infty$.

Table 1 compares the short- and long-run supply elasticity of capital in our model relative to different benchmark models. There are two key takeaways. First, in fully-tangible economies ($\chi = 1$), capital is infinitely elastic in the long-run (the capital supply curve is horizontal). This is because the relative price of capital in terms of goods is fixed over time. Adding a positive capital share of investment ($\theta > 0$) makes the short-run elasticity finite (see "q-theory"), but does not solve the problem of infinite elasticity in the long-run. Second, intangibility (i.e., $\chi < 0$) makes the supply elasticity of capital finite, both in the short- and long-run (see "sweat capital"). This is because labor supply is finite, which makes the response of capital formation to q constrained by the upward-sloping labor supply curve. In other words, investment booms in intangible economies raise the wages of investment labor, hence dampening the size of the boom.

Table 1: Comparing the supply elasticity of capital across benchmark models

Model	Constraint		Elasticity of capital	
	θ	χ	short-run $\left(\frac{\partial \log K_{t+1}}{\partial \log q_t}\right)$	long-run $\left(\frac{\partial \log K}{\partial \log q}\right)$
Neoclassical growth	$\theta = 0$	$\chi = 1$	$+\infty$	$+\infty$
Q-theory	–	$\chi = 1$	$\delta \frac{1-\theta}{\theta}$	$+\infty$
Sweat capital	–	$\chi = 0$	$\delta \frac{1-\theta}{\theta+\sigma}$	$\frac{1}{\sigma}$
General model	–	–	$\delta \frac{(1-\theta)(1+\sigma\chi)}{\theta+\sigma(1-\chi(1-\theta))}$	$\frac{1}{\sigma} \frac{1+\sigma\chi}{1-\chi}$

Notes. θ the investment capital share; χ is the tangibility of investment; δ is the depreciation rate; σ is the labor supply elasticity.

3 Measurement and calibration

We now use data on the U.S. corporate sector to calibrate our model, and in particular the two new parameters θ and χ. We first focus on measuring the uses of capital income in the data, which will be informative on the intangibility of capital χ. We then focus on measuring the elasticity of capital to shocks in the value of capital, which will be informative on the adjustment costs θ.

3.1 Measuring the uses of capital income

3.1.1 Model-implied distribution of capital income

We now describe the distribution of income in our economy. GDP in our economy is $Y + w_H L_H$, not Y. The reason is that $w_H L_H$ is not a production cost, it is a form of investment. This logic is consistent with the *System of National Accounts*.[4]

We define capital income as GDP minus labor costs; that is $\Pi_t \equiv Y_t - w_{Y,t} L_{Y,t}$. The firm budget constraint implies the following accounting identity, which accounts for the *uses* of capital income (i.e.,

[4]However, as discussed in the introduction, distinguishing between payments to production versus investment labor is difficult in practice and the BEA's methodology is known to have limitations in that regard.

how do firms spend their cash):

$$\underbrace{\Pi_t}_{\text{capital income}} = \underbrace{I_t}_{\text{tangible investments}} + \underbrace{w_{H,t}L_{H,t}}_{\text{intangible investments}} + \underbrace{D_t.}_{\text{payouts}} \quad . \tag{3.1}$$

It says that capital income has three uses: tangible investments, intangible investments, and payments to owners of the firm (henceforth "payouts").

Manipulating the first-order conditions of the firm implies that, in the long run, the distribution of capital income in the economy is given by:[5]

$$\frac{I}{\Pi} = \chi\frac{(1-\theta)\delta}{r+(1-\theta)\delta}; \qquad \frac{w_H L_H}{\Pi} = (1-\chi)\frac{(1-\theta)\delta}{r+(1-\theta)\delta}; \qquad \frac{D}{\Pi} = \frac{r}{r+(1-\theta)\delta}.$$

This equation expresses the share of tangible investment, the share of intangible investment, and the payout share in steady state in terms of only four parameters. For the payout share, what matters is the discount rate r and the dilution rate $\delta(1-\theta)$, which itself depends on depreciation rate δ and the capital share of investment θ. We use the term "dilution rate" for $\delta(1-\theta)$ because it corresponds to the extent to which a capitalist that consumes the entirety of capital income in a given period gets diluted over time.

The lower the discount rate r, the lower the share of capital income paid out the capitalists. Given a discount rate, a higher dilution rate $\delta(1-\theta)$ implies a lower current payout. This is because, in high dilution economies, capitalists must receive a higher current payout to guarantee a given long-run return (the payout stream has a low duration). As discussed earlier, the capital share of investment θ is key to determining how much capitalists get diluted, and hence the duration of their wealth.[6] What is not paid out is invested, in proportion to the tangibility of investment parameter χ.

3.1.2 Data

We use Compustat-CRSP merged data covering the period from 1972–2022. We use usual screens to focus on the nonfinancial corporate sector. Our goal is to construct industry-level series for: capital income Π, tangible investments I, intangible investments $w_H L_H$, and payments to capitalists D.

To measure the use of capital income in the data, we start from the statement of cashflows, as reported in Compustat. The key accounting identity is:

$$\text{cf from operations} + \text{cf from financing} + \text{cf from investing} = 0 \tag{3.2}$$

[5]Evaluating (firm foc I_t) and (firm foc $L_{H,t}$) in steady-state, we have

$$I = \chi(1-\theta)Gq = \chi\frac{(1-\theta)\delta}{r+(1-\theta)\delta}\Pi,$$
$$w_H L_H = (1-\chi)\frac{(1-\theta)\delta}{r+(1-\theta)\delta}\Pi,$$

where the second equalities use the formula for q (see Value of capital) as well as the steady-state condition $G/K = \delta$.

[6]The duration of a payout stream $\{D_t\}_{t=0}^{\infty}$ is defined as the value-weighted time to maturity $\sum_{t=1}^{\infty}\frac{R_{0\to t}^{-1}D_t}{\sum_{s=1}^{\infty}R^{-s}D_s}\cdot t$ and is typically used to measure the interest-rate sensitivity of an asset price. In the model, the steady-state duration of payouts is equal to $\frac{1}{r+(1-\theta)\delta}$. Hence, a higher dilution $(1-\theta)\delta$ implies lower duration.

The first term is the cashflows from operations, the second is the cashflows from financing activities, and the third is the cashflow for investing activities. We use the convention that changes in cash balances represent net payments to debt holders. Furthermore, we will later use the fact that cashflows from financing and investing can be decomposed into:

$$\text{cf from financing} = \text{cf from financing (equity)} + \text{cf from financing (debt)} \tag{3.3}$$

$$\text{cf from investing} = \text{cf from investing (capex)} + \text{cf from investing (acquisition)} \tag{3.4}$$

Equation (3.2) allows us to account for 100% of the cash that comes in due to profits (cashflows from operations), the net cash that comes out to pay owners (cashflows from financing activities), and the cash that comes out due to investment (cashflows from investment activities).

Notice that the cashflow identity is the financial accounting counterpart of the firm budget constraint (3.1) in the model. The mapping between model concepts and financial accounting terminology is summarized below:

$$\underbrace{\Pi_t}_{\substack{\text{capital income =}\\ \text{cashflows from operations}}} = \underbrace{(\Pi_t - w_{H,t}L_{H,t} - I_t)}_{\substack{\text{payments to capitalists =}\\ -\text{cashflows from financing}}} + \underbrace{(w_{H,t}L_{H,t} + I_t)}_{\substack{\text{total investment =}\\ -\text{cashflows from investment}}} \tag{3.5}$$

Building on the existing literature, we conduct some data imputations and adjustments, which we describe below.

<u>Adjustment #1: expensed payments to nonproduction labor.</u> One issue with accounting data is that the distinction between an expense (which are subtracted from sales to obtain cashflows from operation) and investment (which are not) can be arbitrary, especially in the case of intangibles.

The existing literature shows that this issue leads to an understatement of cashflows from operations and a corresponding understatement of (minus) cashflows from investment.[7] We follow the existing literature and correct cashflows from operation by adding 30% of SG&A expenses and 100% of R&D expenses. This is meant to account for the incorrect expensing of nonproduction labor expenses, which instead should be treated as a capital expenditure.

$$\text{cf adjustment (nonproduction labor)} \equiv 0.3 \cdot \text{SG\&A} + \text{R\&D}. \tag{3.6}$$

<u>Adjustment #2: net entry in sample.</u> When a private firm becomes public or goes public, it effectively represents a negative cashflow for passive capitalists, who re-balance their portfolio to always own the market. Similarly, when a firm exits or gets acquired, it represents a positive cashflow. We now describe how we measure the contribution of net firm entry to market capitalization growth.

First, we introduce a decomposition of the total market value growth (at the aggregate or within an industry) between time t and $t+1$ as the sum of three components due to stayers, entrants, and exits.

[7]See Peters and Taylor (2017) for evidence from firm-level data and Koh et al. (2020) from aggregate data.

Formally, we have that

$$g_{\text{total}} = g_{\text{stayer}} + g_{\text{entry}} + g_{\text{exit}}, \tag{3.7}$$

where the components are defined in this footnote.[8]

Using these definitions, we define the net cashflow due to net firm entry as

$$\text{cf adjustment (net entry)} \equiv -(g_{\text{entry}} + g_{\text{exit}}) \cdot \text{market capitalization.} \tag{3.8}$$

It corresponds to the net cash that a passive capitalists who owns the market receives due to net firm entry during a period.

Adjustment #3: payments in stocks. How should we record a payment when it is the form of stocks? First, we can decompose the growth in market value of a stayer into growth of price per share and the growth in number of shares. To match the timing of the payment to the timing of the stock issuance, we use the fully-diluted share count (which includes outstanding shares, as well as all possible sources of potential shares such as stock options and reserved shares).

Formally, we can decompose the growth of the market capitalization of stayers (i.e., firms who do not exit the sample) defined in (3.7) into a price and number of shares component:

$$g_{\text{stayer}} = g_{\text{price}} + g_{\text{shares}}, \tag{3.9}$$

where the components are defined in this footnote.[9]

Moreover, we observe the the contribution of the growth in the number of shares issued for cash $g_{\text{shares, cash}}$, and the remainder $g_{\text{shares}} - g_{\text{shares, cash}}$ is due to a combination of stock compensation and stock-financed acquisitions (i.e., "noncash payments"). Using the same logic as for firm entry (see equation 3.8), we define noncash payments as

$$\text{cf adjustment (noncash payments)} \equiv (g_{\text{shares}} - g_{\text{shares, cash}}) \cdot \text{market capitalization.} \tag{3.10}$$

The breakdown between stock compensation and acquisitions is only available after 2011. Therefore, we assume that acquisition is a constant share $\omega \in (0,1)$ of noncash payments (stock issuance not associated with cash). We compute ω at the industry year level after 2011, and use the average across years to split stock compensation before 2011.

[8]The formulas are:

$$g_{\text{total}} = \frac{\sum_{i\in\mathcal{F}_{t+1}} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}} - 1, \qquad g_{\text{stayer}} = \frac{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in(\mathcal{F}_t\cap\mathcal{F}_{t+1})} V_{i,t}} - 1,$$
$$g_{\text{entry}} = \frac{\sum_{i\in(\mathcal{F}_{t+1}\setminus\mathcal{F}_t)} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}}, \qquad g_{\text{exit}} = -\frac{(1+g_{\text{stayer}})\sum_{i\in(\mathcal{F}_t\setminus\mathcal{F}_{t+1})} V_{i,t+1}}{\sum_{i\in\mathcal{F}_t} V_{i,t}},$$

where $\mathcal{F}_t$ is the universe of firms at time t and V_i is the market capitalization of firm i.

[9]The definitions of g_{stayer}, g_{price}, and g_{shares} are

$$g_{\text{stayer}} = \frac{P_{i,t+1}N_{i,t+1}}{P_{i,t}N_{i,t}} - 1, \qquad g_{\text{price}} = \frac{P_{i,t+1}}{P_{i,t}} - 1, \qquad g_{\text{shares}} = \left(\frac{N_{i,t+1}}{N_{i,t}} - 1\right)(1+g_{\text{price}}),$$

where $P_{i,t}$ is the price per share and $N_{i,t}$ is the number of shares outstanding at firm i period t.

Adjustment #4: imputation of the tangible share of ambiguous investments.

How should one proceed to allocate the sum of cashflows due to mergers, acquisitions, and IPOs into tangible versus intangible investment? To do so, we use the "ambiguous income" approach introduced in Cooley et al. (1995). In our setup, the idea will be to assume that acquisitions have the same intangible content as other forms of investment. Denoting $\tilde{\chi} \in (0,1)$ the tangibility of investment in the rest of investment, we have that

$$\tilde{\chi} \equiv \frac{\text{cf from investing (capex)}}{\text{cf from investing (capex)} + \text{cf adjustments}\left(\text{nonproduction labor} + \psi \cdot (1-\omega)\cdot \text{noncash payments}\right)}, \tag{3.11}$$

where we discuss the role of ψ shortly. We compute $\tilde{\chi}$ at the industry year-level. Equipped with this last estimate, we are ready to construct our final variables.

Formulas Having defined the key variables, we now write down the final formulas:

$$\begin{aligned}
\text{tangible investments} = & - \text{cf from investing (capex)}, \\
& - \tilde{\chi} \cdot \text{cf from investing (acquisition)}, \\
& - \tilde{\chi} \cdot \text{cf adjustments}\left(\text{net entry} + \omega\cdot \text{noncash payments}\right), \\
\text{intangible investments} = & - \text{cf adjustments}\left(\text{nonproduction labor} + \psi \cdot (1-\omega)\cdot \text{noncash payments}\right), \\
& - (1-\tilde{\chi}) \cdot \text{cf from investing (acquisition)}, \\
& - (1-\tilde{\chi}) \cdot \text{cf adjustments}\left(\text{net entry} + \omega\cdot \text{noncash payments}\right), \\
\text{payments to capitalists} = & - \text{cf from financing} \\
& + \text{cf adjustments}\left(\text{net entry} + \text{noncash payments}\right).
\end{aligned}$$

Combining these formulas, we obtain an expression for capital income

$$\begin{aligned}
\text{capital income} =& \text{tangible investments} + \text{intangible investments} + \text{payments to capitalists} \\
=& \text{cf from operations} + \text{cf adjustments (nonproduction labor)} - (1-\psi)\cdot(1-\omega)\cdot \text{noncash payments},
\end{aligned}$$

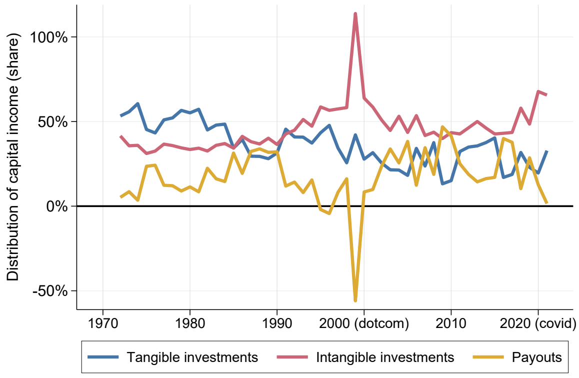
where the second equality uses the cashflow identity (3.2). The number $\psi(1-\omega)$ corresponds to the share of noncash payments that is unreported in R&D and SG&A. Following changes in accounting standards in 2006, we set $\psi = 0.5$ before 2006 and $\psi = 0$ afterwards.

A few remarks are in order. First, that payments to capitalists are invariant to the calibration $(\tilde{\chi}, \omega)$. This is because they only represent assumptions on how cashflows are distributed between tangible investments and intangible investments. Second, capital income is invariant not only to the calibration $(\tilde{\chi}, \omega)$, but also the adjustments for noncash payments and net entry. This is because noncash payments and net entry are purely redistributive flows (from capitalists to tangible investments and to innovators). The only adjustment that affects the level of capital income is the adjustment for expensed nonproduction labor. The idea is that we reclassify line items as capital expenditures, not expenses.

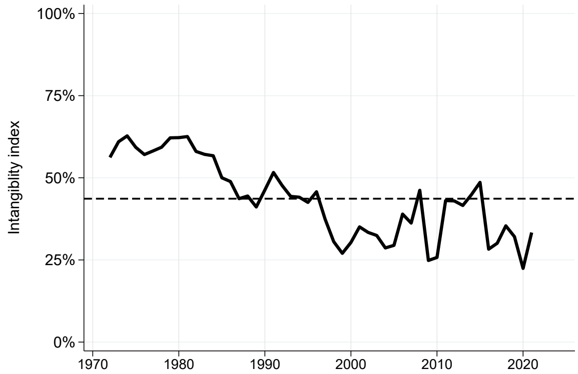
3.1.3 Results

Figure 2a reports the distribution of capital income over time. Three facts stand out: (1) the tangible investment share declines, (2) the intangible investment share increases, and (3) the payout share is low on average and highly countercyclical.

Those three facts are consistent with findings in the literature relating to the stagnation of capital expenditures post-GFC (Gutiérrez and Philippon, 2017; Crouzet and Eberly, 2019), the importance of payments to nonproduction labor (Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017; Eisfeldt et al., 2023), and the empirical evidence on the aggregate equity payout yield (Fama and French, 2005; Boudoukh et al., 2007; Fried and Wang, 2019). Zooming in on the tangibility of investment, Figure 2b shows a large decline in the tangibility of investment, from roughly 60% in the 1970s to 35% post-2000.



(a) Uses of capital income



(b) Tangibility of investment

Figure 2: Capital income in the corporate sector (1972–2022)

Table 2 reports the average distribution of capital income over the 1972–2022 period, including a detailed breakdown, and robustness checks. Tangible investments account for 42% of capital over the period, mostly due to direct capital expenditures, but also in part due to acquisition of existing businesses (who themselves are partial tangible). Intangible investments account for 44%. Cash and non-cash compensation to nonproduction labor accounts for most of it, but almost 20% comes from payouts associated with cash and noncash payout that arise in acquisitions.

Finally, payments to capitalists account for a mere 14% of capital income over the sample. While net cash payouts to equity holders account for 19% of capital income, they were offset by a 12% equity dilution (i.e., noncash payments). This sample average is skewed by the tech bubble of the 1990s, where payments to capitalists were negative for a few years (i.e., acquisitions and IPOs exceeded dividends and interest payouts, see Fried and Wang, 2019 for a discussion of this fact).

In the Appendix, we contrast our results with aggregate data from the National Accounts. Our definition of capital is meant to be (mostly) consistent with the national accounts definition. There are two key differences. First, our treatment of noncash payments is more comprehensive than what the BEA does. It is well understood that the BEA undercounts labor income by missing payments or underestimating their true value (see Zwick, 2022 for a detailed discussion on the topic). In contrast, our approach uses the actual growth in the number of (fully-diluted) shares times the market value

Table 2: Distribution of capital income (1972–2022 average).

Uses of capital income (%)	Baseline	Robustness $\tilde{\chi}=0$	$\tilde{\chi}=1$	$\omega=0$	$\omega=1$
Tangible investments	36	37	45	37	42
Tangible capital expenditures	37	37	37	37	37
Mergers, acquisitions, and IPOs	-1	0	8	0	5
$\tilde{\chi}\cdot$ cash acquisitions	3	0	8	3	3
$\tilde{\chi}(1-\omega)\cdot$ non-cash payments	2	0	5	0	5
$\tilde{\chi}\cdot$ net entry in public universe	-6	0	-6	-3	-3
Intangible investments	46	46	38	42	46
Intangible capital expenditure	38	38	38	39	35
$0.3\cdot$ selling, general, and admin. expenses	24	24	24	24	24
research and development expenses	12	12	12	12	12
$\psi\cdot\omega\cdot$ non-cash payments	2	2	2	4	0
Mergers, acquisitions, and IPOs	8	8	0	3	10
$(1-\tilde{\chi})\cdot$ cash acquisitions	5	8	0	5	5
$(1-\tilde{\chi})(1-\omega)\cdot$ non-cash payments	3	5	0	0	7
$(1-\tilde{\chi})\cdot$ net entry in public universe	1	-6	0	-2	-2
Payout to capitalists	18	18	18	18	18
Net cash equity payout	19	19	19	19	19
$-$ Non-cash payments	-13	-13	-13	-13	-13
$-$ Net entry in public universe	6	6	6	6	6
Net debt payout	6	6	6	6	6

at the time to impute the value of payments in stocks. Second, our definition of what an innovator potentially differs from the BEA. The BEA mostly capitalizes expenses on labor in the case of research and development (see Corrado et al., 2009). Instead, we opt for a more comprehensive definition of "nonproduction labor", which includes not only scientists, bu also key managers, entrepreneurs, and early financiers. Appendix Figure A1 plots capital income and enterprise value, both in our sample and in the the Integrated Macroeconomic Accounts. Appendix Figure A2 compares the time variation in the use of capital income in the national accounts versus Compustat.

3.2 Calibrating the model

First, we set two parameters externally. We set $\sigma = 2$, to match a labor supply elasticity of 0.5, a low value consistent with the macro evidence. The parameter β is chose to match a roughly 7% log return (unlevered), which is the average over our sample. In Appendix B.2, we show that, along a balanced growth path with growth π, the log return is constant at $\log R = -\log\beta + \rho\pi$, where ρ is the household's EIS. Assuming $\rho = 1$ and $\pi = 2\%$, we get $\beta = 0.95$. Second, we calibrate the parameter $\chi = 0.41$, which governs the importance of tangible inputs in capital formation using our evidence on the tangibility of investment (2b) .

Third, we calibrate the remaining parameters four parameters (α, θ, δ) internally by targeting two long-run moments (the total investment yield and the labor income share) and one short-run moment (the tax elasticity of investment). The total investment yield is the sum of tangible and intangible investments, as a share of enterprise value. In Appendix A.2, we discuss how this moment has the interpretation of a dilution rate: it measures how much a capitalists that would consume all the capital income would get diluted over time. The second moment is the labor share, which is defined as total payments to labor (to both production and investment labor) as a share of aggregate income.

Table 3: Internally calibrated parameters

Moment	Notation	Formula	Target
Total investment yield	$\frac{I+w_H L_H}{V}$	$(1-\theta)\delta$	13%
Capital share	$\frac{\Pi}{GDP}$	$\frac{\alpha}{1+(1-\chi)\alpha\frac{(1-\theta)\delta}{r+(1-\theta)\delta}}$	33%
Tax elasticity of investment	$\frac{\partial \log(I_0+w_{H,0}L_{H,0})}{\partial \log \tau_K}$	$\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}$	-4

Notes. α is the capital share in production; θ is the production share in investment; χ is the tangibility of investment; δ is the depreciation rate; r is the return; ϕ is the annual decay of the tax cut.

Note that, together, these two long-run moments can not separately identify $1-\theta$ from δ. The reason is that a high investment yield can be due to the fact that capitalists extract little rents from investment (θ is low) or that the quantity of investment is high (δ is high). Income shares alone can not distinguish between these two cases. Guided by the earlier intution that θ effectively governs the elasticity of capital supply in the short-run (as in the q-theory of investment), we use a short-run moment for our last empirical target.

We draw on evidence from Chodorow-Reich, Smith, Zidar and Zwick (2024) on the empirical response of US firms to the 2017 Tax Cuts and Jobs Act. We focus on their result that uses tax files for a sample of roughly $N = 7000$ (nonfinancial, non-passthrough domestic firms). The variable of interest is log investment change (post-TCJA versus pre-TCJA). The authors estimate that a 1% decline in the corporate income tax rate leads to a roughly 4% decline in investment (see their Table 3, columns 2 and 3). We interpret their estimates as a partial equilibrium short-run response in the model (i.e., holding prices w_Y, w_H, R constant).

In Appendix B.2, we map their regression coefficient to a simple formula that is the product of two terms. We show that a tax cut (with an annual decay rate of τ) implies a short run response of log total investment of $\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}$ (see Table 3). The first terms accounts for the fact that (i) capital payout is levered to the tax rate and (ii) the tax cut is not fully permanent. The second term is $\frac{1}{\theta}$, which governs the short-run elasticity of capital in partial equilibrium.

Using these short- and long-run targets, we obtain the following model calibration:

$$\alpha = 0.38, \qquad \theta = 0.3, \qquad \delta = 0.19, \qquad \chi = 0.41. \qquad \text{(preferred calibration)}$$

4 Counterfactuals

We now use our calibrated model to generate counterfactuals. We first look at the effect of a shift towards intangible capitals. We then discuss the effect of a change in corporate taxes.

4.1 Model experiment: Shift towards intangibles

We now conduct an experiment where we quantify the long-run effect of a decline in the tangibility of investment χ from 0.54 to 0.29 (i.e., a 25 pp. decline around the baseline calibration), in line with what we have seen in the data over the 1972-2022 period (see Figure 2b).

Table 4: Distributional effect (shift towards intangibles).

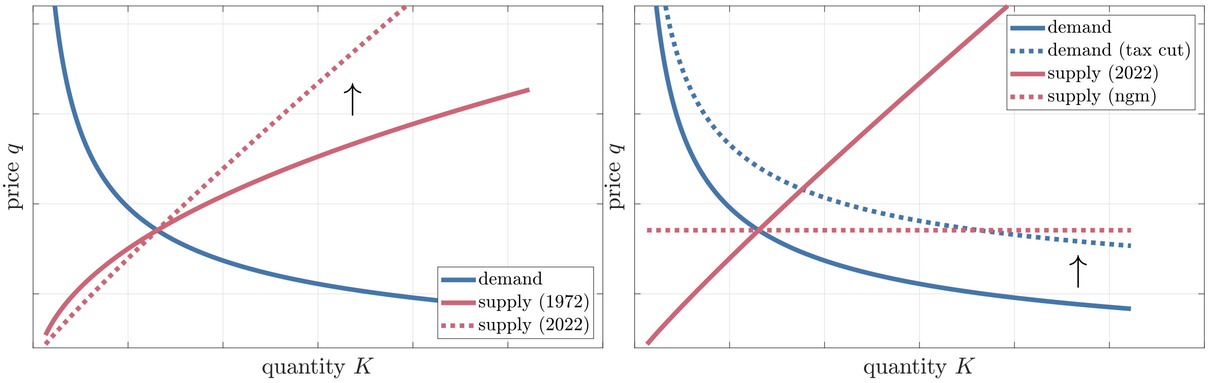
Variable	Symbol	Baseline (pp.)	1972→2022
Panel A – Expenditures			
Consumption	$Y - I$	76	-0
Tangible investments	I	10	-6
Intangible investments	$w_H L_H$	14	6
Panel B – Income			
Capital income	$Y - w_Y L_Y$	33	0
Labor income (production)	$w_Y L_Y$	53	-6
Labor income (investment)	$w_H L_H$	14	6

First, Table 4 reports the effect of rising intangibility on the composition of GDP (see equation A.1 for national accounting definitions). On the expenditure side (Panel A), there is a rise in intangible investments (6 pp. of GDP) offset by a decline in tangible investments (-6 pp. of GDP). On the income side (Panel B), we see decline in the production labor income share 53 pp. compensated by a rise in the investment labor income share (53 pp.). In the baseline calibration, labor supply is quite inelastic ($\sigma = 2$), which means that most of this reallocation of incomes between production and investment labor is due to changes in relative wages (i.e., $\mathrm{d}\log \frac{w_H L_H}{w_Y L_Y} = \frac{2}{3} \cdot \mathrm{d}\log \frac{w_H}{w_Y}$).

What this means is that we should observe rising wage polarization in the labor market, where demand for workers with skills that are important for capital formation should lead to higher relative wages. This is reminiscent of an influential literature on US wage inequality post-1980, which finds that occupations requiring "abstract" skills have seen a relative rise in wages relative to those requiring "routing" skills (see, e.g., Autor and Dorn, 2013).

Figure 3a shows the long-run effect of rising intangibles on the capital supply curve, which amounts to a counter-clockwise rotation of the supply curve.[10] Notice that capital supply becomes much less elastic, due to the growing importance of imperfectly-elastic labor in capital formation.

[10] Note that a change in χ affects the long-run equilibrium Y due to an endowment effect. For visual purposes, we construct Figure 3a by showing a rotation around the initial equilibrium, which amounts to consider a joint change in χ and z_H that ensure no change in the long-run (q, K).



(a) Model experiment, rise of intangibles

(b) model experiment, tax cut

Figure 3: Long-run capital market equilibrium (market equilibrium).

Table 5: Capital supply elasticity.

	Formula	Baseline	Q-theory $\chi = 1$	1972 $\chi = 0.54$	2022 $\chi = 0.29$
Short-run	$\frac{\partial \log K_{t+1}}{\partial \log q_t}$	0.14	0.43	0.17	0.11
Long-run	$\frac{\partial \log K}{\partial \log q}$	1.54	$+\infty$	2.23	1.1

Table 5 reports the model-implied elasticities. The baseline model has low elasticities compared to the case of full tangibility (i.e., setting $\chi = 1$ as in the standard q-theory of investment). In the short-run it is 0.14 versus 0.43, and in the long-run it is 1.54 versus $+\infty$. Simulating the transition from 1972 to 2022 (i.e., rise in intangibles), we obtain a decline in both the long-run elasticity by more than half. Next, we explore the implications of this decline in the elasticity of capital on the incidence of shocks.

4.2 Model experiment: Capital taxation

Consider an extension of the baseline model, where the only difference with the baseline model is in the firm budget constraint, which becomes:

$$D_t = (1 - \tau_{K,t})(Y_t - w_{Y,t} L_{Y,t}) - I_t - w_{H,t} L_{H,t}, \qquad \text{(budget constraint')}$$

where $\{\tau_{K,t}\}_{t=1}^{\infty}$ is a sequence of capital income tax rates, or more precisely taxe on payouts.

Figure 3b plots the effect of a permanent tax cut $\mathrm{d}\log(1 - \tau_K) > 0$ on the long-run capital market equilibrium. As usual in supply-demand systems, a positive demand shock will increase prices and quantities. When the supply curve is horizontal (as in the ngm), then the demand shock is entirely absorbed by quantities K. But in the baseline, long-run capital supply is imperectly elastic due to finite labor supply. As a result, a demand shock, in this case a tax cut, leads to a rise of both prices q_t and quantities K_t.

To set ideas, consider a permanent 1 pp. decline in τ_K. Using a standard comparative statics approach and the long-run demand and supply elasticities in the model, we obtain

$$\frac{\mathrm{d}\log K}{\mathrm{d}\log(1-\tau_K)}\cdot 1\% = \frac{1}{\text{demand elasticity} + \text{supply elasticity}} = 0.9\%,$$
$$\frac{\mathrm{d}\log q}{\mathrm{d}\log(1-\tau_K)}\cdot 1\% = \frac{\text{supply elasticity}}{\text{demand elasticity} + \text{supply elasticity}} = 0.6\%.$$

The shock is absorbed through both a higher valuation of capital q *and* a rise in capital formation K (see Appendix C.2 for derivations). As a benchmark, in the NGM, the response of capital would be roughly twice as high (1.9%) with no response of valuations.

More generally, a shift towards intangibles means that the market increasingly clears via higher valuations rather than actual capital formation. As we discuss next, this imperfect pass-through of demand shocks to quantities has important effects on the incidence of capital taxes (i.e., who wins and loses in terms of welfare). Intuitively, a jump q induced by a tax cut will benefit investment labor (via higher wages) and initial capitalists (via a revaluation gain).

Incidence of capital taxes. We first consider an arbitrary perturbation $\{\mathrm{d}\tau_{K,t}\}_{t=0}^{\infty}$ around the undistorted steady-state. Which factors of production win and/or lose in response to this shock? We follow Fagereng et al. (2024) and apply the envelope condition on the household value function (2.1), combined with the Euler condition worker foc C_t, to obtain the total welfare effect of the change in prices induced by the tax shock.

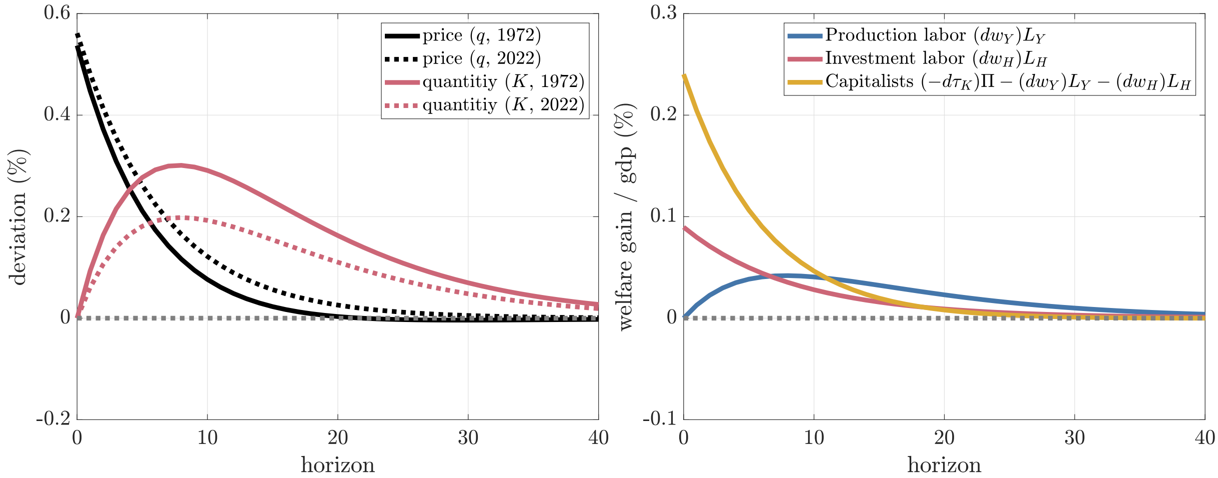
Proposition 4.1. *The equilibrium welfare effect, in units of $t = 0$ consumption, associated with the perturbation $\{\mathrm{d}\tau_{K,t}\}_{t=1}^{\infty}$ around the undistorted steady-state, is given by*

$$\textit{Welfare Gain} = \underbrace{\sum_{t=1}^{\infty} R^{-t}(\mathrm{d}w_{Y,t})L_Y}_{\textit{welfare gain (production labor)}} + \underbrace{\sum_{t=1}^{\infty} R^{-t}(\mathrm{d}w_{H,t})L_H}_{\textit{welfare gain (investment labor)}} + \underbrace{\mathrm{d}V_0 + \sum_{t=1}^{\infty} R^{-t}(\mathrm{d}R_t)V}_{\textit{welfare gain (capitalists)}}$$

Proposition (4.1) decomposes the welfare effect into the contribution of changes in prices for the three factors of production: production labor, investment labor, and capital. For labor, the welfare effect captures the fact that the tax shock affects the path of wages $\{\mathrm{d}w_{Y,t}, \mathrm{d}w_{H,t}\}_{t=1}^{\infty}$. For capitalists, there is an initial revaluation of wealth $\mathrm{d}V_0$ as well as the contribution of changes in forward returns $\{\mathrm{d}R_t\}_{t=1}^{\infty}$. The total (private sector) welfare effect is equal to the present value of the change in the tax rate times capital income (i.e., the mechanical effect of taxes on profits holding everything else constant): Welfare Gain $= \sum_{t=1}^{\infty} R_{0\to t}^{-1}(\mathrm{d}\tau_{K,t})\Pi_t$.

Experiment. We now simulate a 1% cut around the steady-state, which decays at an annual rate of $1-\phi = 0.9$. We consider the case of a small-open economy, where forward returns are constant (i.e., $\mathrm{d}R_t = 0$). For this exercise, this is equivalent to setting the EIS to $\rho = 0$ (i.e., $R_t = \beta^{-1}$). This simplifies the formula for the welfare effect for capitalists in Proposition (4.1), which becomes only the revaluation gain $\mathrm{d}V_0$ (i.e., the rise in their wealth due to the fact that future post-tax capital income has increased).

Figure 4a plots the response of capital accumulation and valuations over a 40 year period after the shock. We show the dynamics implied by the model using the 1972 and 2022 calibrations to describe how the shift towards capital affects the response of the economy. In both cases, the value of capital jumps on impact, but it takes several years for the stock of capital to peak. Notice, however, that in the intangible economy (2022 calibration), the stimulative effect of the tax cut is much lower, consistent with the earlier calculations regarding the supply elasticity of capital (see Table 5).



(a) Capital and valutations

(b) Welfare effects

Figure 4: Model experiment: Tax cut

How does the general equilibrium (GE) response of investment respond to the partial equilibrium (PE) target used for calibration? In PE, we target a high elasticity, but in a GE experiment where the tax applies to all of capital income (which represents 4% of GDP in the baseline), we expect a much weaker response due to rising wages. Indeed, we find a GE response that is roughly 4 times smaller:

$$\frac{\partial \log(I_0 + w_{H,0}L_{H,0})}{\partial \log(1-\tau_K)} = 4, \qquad \text{(Tax elasticity of investment, PE)}$$

$$\frac{\mathrm{d} \log(I_0 + w_{H,0}L_{H,0})}{\mathrm{d} \log(1-\tau_K)} = 0.95. \qquad \text{(Tax elasticity of investment, GE)}$$

We can further decompose the gap between PE and GE into the contribution of $\mathrm{d}w_Y$ and $\mathrm{d}w_H$

Welfare effects. The welfare effect for workers works through changes in their wage, which are themselves fully pinned down by the path of the state and co-state variables $\{q_t, K_t\}_{t=0}^{\infty}$ (see Lemma 2.1). A rise in capital K benefits both types of workers, via their complementarity with existing capital, but the value of capital q directly affects the marginal product of investment labor. Figure 4b plots the resulting path of (undiscounted) welfare effects for both types of labor, expressed as a share of steady-state GDP. While the tax shock appears to benefit both types of labor equally, it is worth pointing out that production labor accounts for roughly 4 times more labor income than investment labor in steady-state (see Table 4).

The welfare effect for capitalists can be decomposed as the present value of higher future payouts:

$$\mathrm{d}V_0 = \sum_{t=1}^{\infty} R^{-t}\Big((-\,\mathrm{d}\tau_{K,t})\Pi - (\mathrm{d}w_{Y,t})L_Y - (\mathrm{d}w_{H,t})L_H\Big). \qquad \text{(Revaluation gain)}$$

Notice that those higher payouts are very front-loaded, especially compared to the back-loaded increase in production worker wages (see Figure 4b). This is because wages rise slowly with capital accumulation, while the benefit for capitalists on post-tax capital income is immediate.

Table 6 reports the share of private welfare gains that accrue to each factor of production. In the baseline, 20% of the tax incidence falls on production workers, while 26% falls on investment labor and 54% on capitalists. Note that this stands in sharp contrast with the NGM, where 100% of the incidence falls on productions workers. This is because, when capital is fully elastic ($\theta = 0$), there is no revaluation effect since $q = 1$ at all times.

To understand the difference between the baseline and the NGM, it is useful to consider an intermediary model (i.e., the q-theory special case), where we impose ful tangiblity $\chi = 1$ while keeping a positive capital share $\theta = 0.3$ as in the baseline calibration. In that case, we a slightly lower capitalist share.

Table 6: Incidence of capital taxes (share of private sector welfare gains).

Factor	Baseline	NGM	Q-Theory	1972	2022
		$\theta = 0, \chi = 1$	$\chi = 1$	$\chi = 0.54$	$\chi = 0.29$
Production labor	20	100	50	26	17
Investment labor	26	0	0	22	30
Capitalists	54	0	50	52	53

We also run the same model experiment in the low- and high-intangible calibrations that correspond roughly to 1972 and 2022. Notice that the shift towards intangibles has led to a reallocation of the incidence of capital taxes, away from production labor and towards investment labor and capitalits.

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A Appendix for Section 2

A.1 Proofs

Proof of Lemma 2.1. We now provide equilibrium expressions for the choices of the firm $(L_{Y,t}, L_{H,t}, I_t)$ in terms of the state variables $(K_t, z_{Y,t}, z_{H,t})$ and the co-state variable q_t. We only need the the combined FOCs for workers and firms (i.e., firm foc $L_{Y,t}$; firm foc $L_{H,t}$; firm foc I_t; worker foc $L_{Y,t}$; worker foc $L_{H,t}$) summarized by three equations:

$$(1-\mu)L_{Y,t}^{\sigma} = (1-\alpha)\frac{Y_t}{L_{Y,t}}, \tag{eq1}$$

$$\mu L_{H,t}^{\sigma} = (1-\theta)(1-\chi)\frac{G_t}{L_{H,t}}q_t, \tag{eq2}$$

$$1 = (1-\theta)\chi\frac{G_t}{I_t}q_t. \tag{eq3}$$

Combining (eq1) and the definition of Y we obtain the solution for $L_{H,t}$

$$L_{Y,t} = \left(\frac{1-\alpha}{1-\mu}\right)^{\frac{1}{\sigma+\alpha}} (z_{Y,t}K_t^{\alpha})^{\frac{1}{\sigma+\alpha}},$$

Combining (eq2) and (eq3), we have that

$$I_t = \frac{\chi}{1-\chi}\mu L_{H,t}^{1+\sigma}$$

Plugging into the definition of G_t, we obtain

$$\begin{aligned} G_t &= z_{H,t}K_t^{\theta}\left(\left(\frac{\chi}{1-\chi}\mu L_{H,t}^{1+\sigma}\right)^{\chi} L_{H,t}^{1-\chi}\right)^{1-\theta} \\ &= \left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} z_{H,t}K_t^{\theta}L_{H,t}^{(1+\sigma\chi)(1-\theta)} \end{aligned}$$

Whenever labor supply is elastic $\sigma\chi > 0$, the capital formation elasticity of investment labor is amplified. Plugging this expression for capital formation into (eq2), we obtain

$$L_{H,t} = \left(\frac{1}{\mu}(1-\theta)(1-\chi)\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)}\right)^{\frac{1}{\sigma+\theta-\sigma\chi(1-\theta)}} \left(z_{H,t}q_tK_t^{\theta}\right)^{\frac{1}{\sigma+\theta-\sigma\chi(1-\theta)}}.$$

Finally, we have

$$I_t = \frac{\chi}{1-\chi}\mu\left(\frac{1}{\mu}(1-\theta)(1-\chi)\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)}\right)^{\frac{1+\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}} \left(z_{H,t}q_tK_t^{\theta}\right)^{\frac{1+\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}}$$

□

Proof of Lemma 2.2. To solve for the steady-state (q, k), we use two equations:

$$Rq = \alpha \frac{Y}{K} + \left(1 - (1-\theta)\frac{G}{K}\right) q, \tag{eq4}$$

$$K = (1-\delta)K + G. \tag{eq5}$$

Combining (eq4) and (eq5), we have that

$$q = \frac{\alpha \frac{Y}{K}}{r + (1-\theta)\delta},$$

which is a valuation equation: capital income discounted at rate $r + (1-\theta)\delta$, to account for dilution. Using the solution for L_Y from Lemma 2.1, we have that

$$Y = K^{\alpha}\left(\bar{l}_Y K^{\frac{\alpha}{\sigma+\alpha}}\right)^{1-\alpha} \implies Y/K = \bar{l}_Y^{1-\alpha} K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}}$$

Therefore, we have that q is a decreasing function of K (a demand curve):

$$q = \frac{\alpha \bar{l}_Y^{1-\alpha}}{r + (1-\theta)\delta} K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}}. \tag{Demand curve}$$

Second, we use the expression for G derived in the proof of Lemma 2.1, and the expression for L_H, we have that

$$\begin{aligned} G &= \left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} K^{\theta}\left(\overline{l_H} q K^{\theta}\right)^{\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}} \\ \implies G/K &= \left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} \overline{l_H}^{(1+\sigma\chi)(1-\theta)} q^{\frac{(1+\sigma\chi)(1-\theta)}{\sigma+\theta-\sigma\chi(1-\theta)}} K^{-(1-\theta)(1-\chi)\frac{\sigma}{\sigma+\theta-\sigma\chi(1-\theta)}} \end{aligned}$$

Imposing $G/K = \delta$, which is equivalent to (eq5), we obtain

$$\delta^{\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} = \left(\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} \overline{l_H}^{(1+\sigma\chi)(1-\theta)}\right)^{\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} q^{1+\sigma\chi} K^{-(1-\chi)\sigma}$$

Therefore, we have that q is an increasing function of K (a supply curve):

$$q = \delta^{\frac{1}{1+\sigma\chi}\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} \left(\left(\frac{\chi}{1-\chi}\mu\right)^{\chi(1-\theta)} \overline{l_H}^{(1+\sigma\chi)(1-\theta)}\right)^{-\frac{1}{1+\sigma\chi}\frac{\sigma+\theta-\sigma\chi(1-\theta)}{1-\theta}} K^{\frac{1-\chi}{1+\sigma\chi}\sigma}$$

□

Proof of Proposition 2.3. The long-run supply elasticity is given by Lemma 2.2, where we express the long-run capital supply curve as

$$q = \overline{q_S} K^{\frac{1-\chi}{1+\sigma\chi}\sigma}.$$

Taking log and differentiating, we obtain

$$\log K = \frac{\frac{1}{\sigma} + \chi}{1 - \chi} \, \mathrm{d} \log q,$$

which characterizes movements along the long-run labor supply curve.

For the short-run elasticity, consider a perturbation $\{\mathrm{d} \log q_t\}_{t=1}^{\infty}$. We can characterize the response of K_{t+s}, at any horizon $s \geq 0$, using the following recursion:

$$\begin{aligned}
\mathrm{d} \log K_0 &= 0 \\
\mathrm{d} \log K_{t+1} &= (1-\delta) \, \mathrm{d} \log K_t + \delta \, \mathrm{d} \log G_t \\
&= (1-\delta) \, \mathrm{d} \log K_t + \delta\theta \left(1 + \frac{(1+\sigma\chi)(1-\theta)}{\sigma + \theta - \sigma\chi(1-\theta)}\right) \mathrm{d} \log K_t + \delta \frac{(1+\sigma\chi)(1-\theta)}{\sigma + \theta - \sigma\chi(1-\theta)} \, \mathrm{d} \log q_t \\
&= \left(1 - \delta \frac{\sigma(1-\theta)(1-\chi)}{\sigma + \theta - \sigma\chi(1-\theta)}\right) \mathrm{d} \log K_t + \delta \frac{(1+\sigma\chi)(1-\theta)}{\sigma + \theta - \sigma\chi(1-\theta)} \, \mathrm{d} \log q_t
\end{aligned}$$

Solving forward, we have that

$$\mathrm{d} \log K_{T+1} = \delta \frac{(1+\sigma\chi)(1-\theta)}{\sigma + \theta - \sigma\chi(1-\theta)} \left(\sum_{s=0}^{T} \left(1 - \delta \frac{\sigma(1-\theta)(1-\chi)}{\sigma + \theta - \sigma\chi(1-\theta)}\right)^{s} \right) \mathrm{d} \log q_{T-s}$$

Setting $\mathrm{d} \log q_{T-s} = \mathrm{d} \log q$ and taking the limit, we obtain

$$\lim_{T \to \infty} \mathrm{d} \log K_{T+1} = \frac{\frac{1}{\sigma} + \chi}{1 - \chi} \cdot \mathrm{d} \log q,$$

which indeed is the long-run capital supply elasticity. □

A.2 Accounting and definitions

We now summarize the accounting concepts that we use in the paper.

Accounting for GDP. GDP is $Y + w_H L_H$, not Y. This is because gross output Y does not include the value-added produced by investment labor. Capital income is then defined as GDP minus labor costs: $\Pi_t \equiv Y_t - w_{Y,t} L_{Y,t}$. Applying the standard GDP identity (i.e., income equals expenditures), we obtain:

$$\underbrace{\overbrace{w_{Y,t} L_{Y,t} + w_{H,t} L_{H,t}}^{\text{labor income}} + \overbrace{\Pi_t}^{\text{capital income}}}_{\text{income}} = \underbrace{\overbrace{Y_t - I_t}^{\text{consumption}} + \overbrace{I_t + w_{H,t} L_{H,t}}^{\text{investment}}}_{\text{expenditures}}. \tag{A.1}$$

Accounting for rents. The firm extracts rents in the model, in the sense of Ricardo (i.e., workers are paid below their average product). We define production and investment rents as:

$$\text{production rents} \equiv L_{Y,t} \cdot \left(\frac{Y_t}{L_{Y,t}} - \frac{\partial Y_t}{\partial L_{Y,t}} \right) = \alpha Y_t$$
$$\text{investment rents} \equiv \left(I^{\chi} L_H^{1-\chi} \right) \cdot \left(\frac{G_t}{I^{\chi} L_H^{1-\chi}} - \frac{\partial G_t}{\partial (I^{\chi} L_H^{1-\chi})} \right) = \theta q_t G_t.$$

Production rents are the difference between the average and marginal product of production labor. The second equality uses firm optimization and says that production rents represent a share α of gross output, as in the NGM. We define investment rents similarly, but where the input to investment is the bundle of tangible and intangible investments. The idea is that, in general, capital formation also creates rents that accrue to existing owners of capital, again because investment labor is paid below its average product. Using firm optimization we have that a share θ of the value of capital formation accrues to existing capitalists.

Accounting for returns and dilution. The general definition of return is the payout yield plus the growth of firm value: $R_{t+1} = \frac{D_{t+1}}{V_t} + \frac{V_{t+1}}{V_t}$. Using the firm budget constraint, we can alternatively write returns as

$$R_{t+1} = \frac{\Pi_{t+1}}{V_t} + \left(1 - \frac{I_{t+1} + w_{H,t+1} L_{H,t+1}}{V_{t+1}} \right) \frac{V_{t+1}}{V_t}, \tag{A.2}$$

where the first term is now the capital income yield: the owner's share of current profits. The second term accounts for the "diluted" share of firm growth. The idea is that, in order to pay out all of capital income, a firm would need to give away a share $\frac{I + w_H L_H}{V}$ of its ownership to outside investors in order to finance its growth.

Solving V_0 forward with $R_{t+1} = R$, we obtain

$$V_0 \equiv \sum_{t \geq 1} R_{0 \to t}^{-1} D_t = \sum_{t \geq 1} R_{0 \to t}^{-1} \prod_{1 \leq s < t} \left(1 - \frac{I_s + w_{H,s} L_{H,s}}{V_s} \right) \Pi_t, \tag{A.3}$$

Equation A.3 contains an important insight: capitalists at $t = 0$ do not own all of future capital income. They own a share that decays over time, because they must invest to maintain the stock of capital.

A.3 Model extension with multiple capital stocks

Suppose that there are $n = 1, \ldots, N$ capital stocks, each of which has its own accumulation technology $(\theta_n, \chi_n, \delta_n)$. The different capital stocks are aggregated according to a CRS Cobb-Douglas aggregator:

$$G_{n,t} = z_{H,n,t} K_{n,t}^{\theta_n} \left(I_{n,t}^{\chi_n} L_{H,n,t}^{1-\chi_n} \right)^{1-\theta_n}, \qquad \text{(capital formation - type } n\text{)}$$
$$K_{n,t+1} = (1 - \delta_n) K_{n,t} + G_{n,t}, \qquad \text{(capital accumulation - type } n\text{)}$$
$$K_t = \prod_{n=1}^{N} K_{n,t}^{\varphi_n}. \qquad \text{(capital aggregator)}$$

All the other model equations remain unchanged, which means that the baseline model is nested with $N = 1$.

A.4 A microfoundation

We now provide a microfoundation of the aggregate functions Y, G, and q. Suppose that there are $i = 1, \ldots, I$ price-taking firms who differ in their (deterministic) productivity process $\{z_{Y,i,t}, z_{H,i,t}\}_{t\geq 1}$, but otherwise all solve the same firm problem as in the baseline model above. We show that the the model aggregates to the representative firm problem where, except that aggregate technology now depends on the joint distribution of firm-level technology and capital:

$$z_{Y,t} = \left(\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}}\right)^{\alpha}, \quad z_{H,t} = \left(\sum_{i=1}^{I} \frac{q_{i,t}V_{i,t}}{q_t V_t} \cdot z_{H,i,t}^{\frac{1}{\theta}}\right)^{\theta}.$$

Production function Y. We follow Baqaee and Farhi (2018) and define the aggregate production function as the efficient level of production given (i) the distribution of productivity and capital and (ii) a given amount of labor L_Y.

$$Y \equiv \max_{\{L_{Y,i,t}\}} \sum_{i=1}^{I} Y_{i,t} \quad \text{s.t.} \quad \sum_{i=1}^{I} L_{Y,i,t} = L_{Y,t}$$

Taking the first-order condition and solving for $L_{Y,i,t}$ we obtain

$$L_{Y,i,t} = \left(\frac{1-\alpha}{\lambda_{Y,t}}\right)^{\frac{1}{\alpha}} K_{i,t} z_{Y,i,t}^{\frac{1}{\alpha}},$$

where $\lambda_{Y,t}$ is the marginal value of production labor. Aggregating on both sides and using the labor constraint, we obtain

$$L_{Y,t} = \left(\frac{1-\alpha}{\lambda_{Y,t}}\right)^{\frac{1}{\alpha}} \sum_{i=1}^{I} K_{i,t} z_{Y,i,t}^{\frac{1}{\alpha}} \tag{A.4}$$

Combining both equations, we have that

$$L_{Y,i,t} = \frac{\frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}}}{\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} \cdot z_{Y,i,t}^{\frac{1}{\alpha}}} L_{Y,t}. \tag{A.5}$$

The aggregate production function can therefore be calculated as

$$
\begin{aligned}
Y_t &= \sum_{i=1}^{I} z_{Y,i,t} K_{i,t}^{\alpha} L_{Y,i,t}^{1-\alpha} \\
&= \sum_{i=1}^{I} z_{Y,i,t} (K_{i,t}/L_{Y,i,t})^{\alpha} \cdot L_{Y,i,t} \\
&= \left(\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} z_{Y,i,t}^{\frac{1}{\alpha}} \right)^{\alpha} (K_t/L_{Y,t})^{\alpha} \sum_{i=1}^{I} L_{Y,i,t} \\
&= \left(\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} z_{Y,i,t}^{\frac{1}{\alpha}} \right)^{\alpha} K_t^{\alpha} L_{Y,t}^{1-\alpha} \qquad \text{(A.6)}
\end{aligned}
$$

The key is the third equation, where we use both (A.4) and A.5. Basically, we use the fact that the MPL is equalized across firms (to λ_Y). Aggregate TFP is thus a capital-weighted harmonic mean of individual TFP.

Capital formation function G. The aggregate capital formation function is the optimal allocation resources given the distribution of capital and productivity:

$$
\max_{\{L_{H,i,t}, I_{i,t}\}} \sum_{i=1}^{I} q_{i,t} G_{i,t} \quad \text{s.t.} \quad \sum_{i=1}^{I} L_{H,i,t} = L_{H,t}, \quad \sum_{i=1}^{I} I_{i,t} = I_t.
$$

Following the same steps as for the aggregate production function, the solution is

$$
\begin{aligned}
L_{H,i,t} &= \frac{\frac{q_{i,t}K_{i,t}}{q_t K_t} z_{H,i,t}^{\frac{1}{\theta}}}{\sum_{i=1}^{I} \frac{q_{i,t}K_{i,t}}{q_t K_t} z_{H,i,t}^{\frac{1}{\theta}}} L_{H,t} \\
I_{i,t} &= \frac{\frac{q_{i,t}K_{i,t}}{q_t K_t} z_{H,i,t}^{\frac{1}{\theta}}}{\sum_{i=1}^{I} \frac{q_{i,t}K_{i,t}}{q_t K_t} z_{H,i,t}^{\frac{1}{\theta}}} I_t \\
G_t &= \left(\sum_{i=1}^{I} \frac{q_{i,t}K_{i,t}}{q_t K_t} z_{H,i,t}^{\frac{1}{\theta}} \right)^{\theta} K_t^{\theta} \left(I_t^{\chi} L_{Y,t}^{1-\chi} \right)^{1-\theta}
\end{aligned}
$$

Aggregate IST is a value-weighted harmonic mean of individual IST.

Valuations q. Individual firm valuation is given by

$$
R q_{i,t} = \alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + \left(1 - \delta + \theta \frac{G_{i,t+1}}{K_{i,t+1}} \right) q_{i,t+1}
$$

Using the equilibrium allocation, we have that

$$\frac{Y_{i,t+1}}{K_{i,t+1}} = \frac{z_{Y,i,t}^{\frac{1}{\alpha}}}{\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} z_{Y,i,t}^{\frac{1}{\alpha}}} \frac{Y_t}{K_t}$$
$$\frac{G_{i,t+1}}{K_{i,t+1}} = \frac{z_{H,i,t}^{\frac{1}{\theta}}}{\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} z_{H,i,t}^{\frac{1}{\theta}}} \frac{G_t}{K_t}$$

Therefore, individual firm valuations can be expressed in terms of aggregates $\{Y_t, G_t, K_t\}$ as well as relative productivity

$$Rq_{i,t} = \left(\frac{z_{Y,i,t}}{z_{Y,t}}\right)^{\frac{1}{\alpha}} \alpha \frac{Y_{t+1}}{K_{t+1}} + \left(1 - \delta + \left(\frac{z_{H,i,t}}{z_{H,t}}\right)^{\frac{1}{\theta}} \theta \frac{G_{t+1}}{K_{t+1}}\right) q_{i,t+1}. \tag{A.7}$$

Assuming that $\lim_{T\to\infty} R_{0\to t}^{-1} q_{i,T} = 0$, we can easily show that $\sum_{i=1}^{I} \frac{K_{i,t}}{K_t} q_{i,t} = q_t$.

A.5 Equivalence to multi-sector model

We now show that the production technology (see Section 2.1) can be re-written as a multi-sector economy that produces that has three factors of production (K, L_Y, L_H) and three sectors that produce, respectively, a final good consumption C and a capital good G. We start with three equations from the model:

$$C = Z_Y K^{\alpha} L_Y^{1-\alpha},$$
$$I = Z_Y K^{\alpha} L_Y^{1-\alpha},$$
$$G = Z_H K^{\theta} I^{\chi(1-\theta)} L_H^{(1-\chi)(1-\theta)}.$$

We can instead express the system as

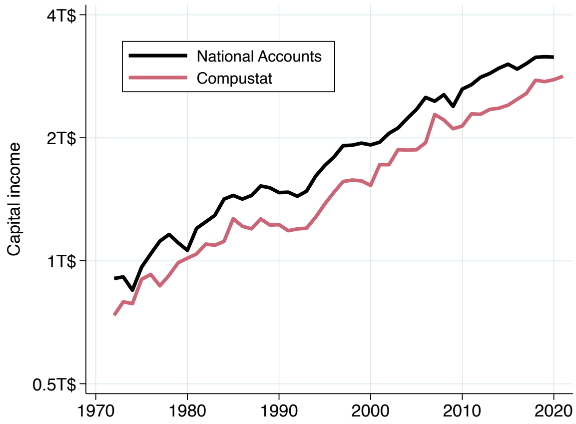
$$C = Z_Y K^{\alpha} L_Y^{1-\alpha},$$
$$G = Z_H Z_Y^{\chi(1-\theta)} K^{\theta+\alpha\chi(1-\theta)} L_Y^{(1-\alpha)\chi(1-\theta)} L_H^{(1-\chi)(1-\theta)}.$$

Notice that the capital good production function is constant return to scale in (K, L_Y, L_H). A decline in tangibility χ implies a greater importance of investment labor L_H at the expense of production labor and capital (K, L_Y).

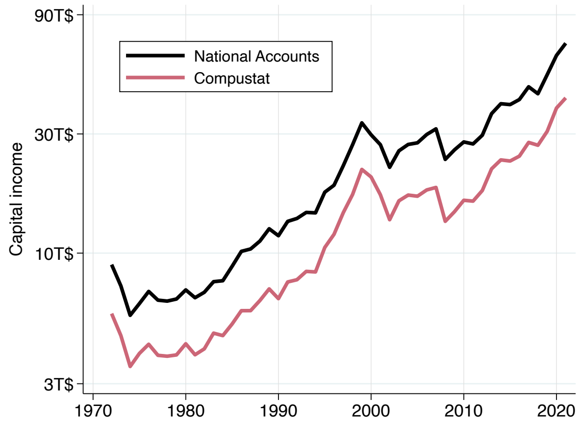
B Appendix for Section 3

B.1 Comparison with National Accounts

Appendix Figure A1 plots capital income and enterprise value, both in our sample and in the the Integrated Macroeconomic Accounts (Table S.5.a). Note that our sample accounts for roughly two-thirds of nonfinancial corporate sector gross capital income over the period, with a slight upward trend.



(a) Capital income



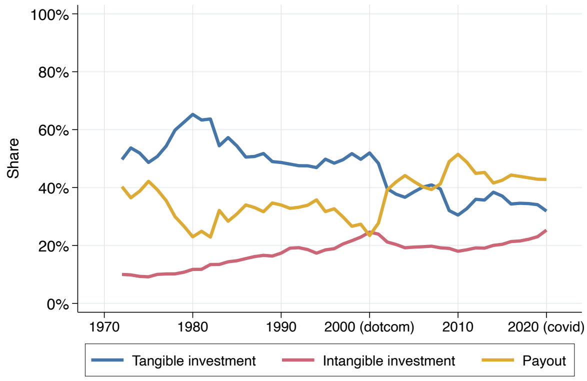
(b) Enterprise value

Figure A1: Sample validation against National accounts data (1970–2022).

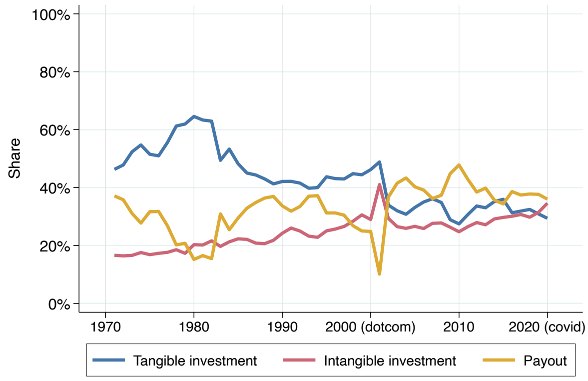
Table A1: Industry-level investment-q elasticity

Instrument horizon	Point estimate	95% Confidence Interval
On impact	1.41	[0.90 – 1.93]
One-year ahead	1.84	[0.00 – 3.67]

Figure A2 reports the use of capital income for entire U.S. economy using data from the national accounts (BEA). While Panel A2a reports the data aggregated across all industries, Panel A2b reweights the BEA data within industry cells to mimic the distribution of capital in Compustat, showing that part of the difference with the distribution of capital income in Compustat (as reported in Figure 2a) comes from the difference in industry composition across the two samples. Still, the figure due to the difference between public firms and private firms, as well as differences in the definition of intangible investment in our paper relative to the BEA classification.



(a) Economy-wide



(b) Reweighting industries to match public firms

Figure A2: Distribution of capital income (BEA)

B.2 Model equations for calibration

To map the model to the data, we first extend the concept of "long-run" to a balanced growth path (BGP).

Definition of BGP. The growth rates of variables along a BGP are

$$\begin{aligned}
\pi_Y &= \pi_{Z_Y} + \alpha\pi_K + (1-\alpha)\pi_{L_Y}, \\
\pi_G &= \pi_{Z_H} + \theta\pi_K + (1-\theta)\chi\pi_I + (1-\theta)(1-\chi)\pi_{L_H},
\end{aligned}$$

where $\pi_X \equiv \log X_{t+1} - \log X_t$ for a variable X. A BGP is an equilibrium where $\pi_{L_Y} = \pi_{L_H} = 0$ and $\pi_{Z_Y} = (1-\alpha)\pi$, $\pi_{Z_H} = (1-\theta)(1-\chi)\pi$. In this case, we have that the variables (K, Y, C, I, w_Y, w_H) grow at rate π. Using the optimality conditions, we obtain

$$\begin{aligned}
R &= \beta^{-1}(1+\pi)^{\rho} \\
\frac{G}{K} &= \delta + \pi \\
q &= \frac{1}{r + (1-\theta) - \theta\pi}\frac{\Pi}{K}
\end{aligned}$$

Moments for calibration. First, using (worker foc C_t), the log return is

$$\log R = -\log\beta + \rho\pi$$

To calibrate β, we can use log returns minus $\rho\pi$.

Second, combining (firm foc I_t) and (firm foc $L_{H,t}$), the long-run total investment yield is

$$\begin{aligned}
I + w_H L_H &= (1-\theta)qG, \\
&= (1-\theta)V\frac{G}{K,} \\
&= (1-\theta)V(\delta+\pi).
\end{aligned}$$

The total investment yield is therefore $\frac{I+w_H L_H}{V} = (1-\theta)(\delta+\pi)$: dilution is higher in a BGP with positive growth than in a steady-state. This is because the capital formation rate G/K is optimally higher.

Third, the capital share of income is

$$\begin{aligned}
\frac{\Pi}{Y + w_H L_H} &= \frac{\alpha Y}{Y + (1-\chi)(1-\theta)qG}, \\
&= \frac{\alpha}{1 + (1-\chi)(1-\theta)\frac{qG}{Y}} \\
&= \frac{\alpha}{1 + (1-\chi)\alpha\frac{(1-\theta)\delta}{r+(1-\theta)\delta}}
\end{aligned}$$

which is the same in both a steady-state and a BGP with positive growth. The last equation uses the fact that $qG = \frac{\frac{G}{K}}{r+(1-\theta)\delta}\alpha Y \implies qG/Y = \alpha\frac{\delta}{r+(1-\theta)\delta}$

The last moment is the short-run, partial equilibrium tax elasticity. Consider a capital income tax shock $\mathrm{d}\tau_{K,t} = (1-\phi)^t \mathrm{d}\tau_K$ at $t = 0$ around the undistorted steady-state. Using (firm foc I_t) and (firm foc $L_{H,t}$):

$$I_t + w_{H,t}L_{H,t} = (1-\theta)q_t G_t.$$

The log total derivative at $t = 0$ is

$$\begin{aligned}
\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) &= \mathrm{d}\log q_0 + \mathrm{d}\log G_0 \\
&= \left(1 + \frac{(1-\theta)(1+\sigma\chi)}{\theta+\sigma(1-\chi(1-\theta))}\right)\mathrm{d}\log q_0,
\end{aligned}$$

where the second equality uses the short-run capital elasticity formula (see Table 1). In partial equilibrium, wages are are taken as given, which corresponds to perfectly elastic supply $\sigma = 0$. In that case, we obtain

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = \frac{1}{\theta}\mathrm{d}\log q_0,$$

Using the envelope theorem on the firm value function, and using the fact that $\mathrm{d}w_{Y,t} = \mathrm{d}w_{H,t}$, we obtain

$$\mathrm{d}\log q_0 = -\frac{r}{r+\phi}\frac{r+(1-\theta)\delta}{r}\mathrm{d}\tau_K$$

The first term accounts for the duration of the tax change, where ϕ represents an annual rate of decay. The second term accounts for the ratio of the tax base (capital income) to the capital payout. Putting together, we obtain

$$\mathrm{d}\log(I_0 + w_{H,0}L_{H,0}) = -\frac{r+(1-\theta)\delta}{r+\phi}\frac{1}{\theta}\mathrm{d}\tau_K$$

B.3 Uses of capital income in the short-run

In the main paper, we have characterized the use of capital income in the short-run (i.e., conditional on a perturbation $\{\mathrm{d}z_{Y,t}, \mathrm{d}z_{H,t}\}_{t=1}^{\infty}$).

Proposition B.1 (Budget shares). *The distribution of capital income, in present value terms, is given by:*

$$\frac{\sum_{t\geq 1} R_{0\to t}^{-1}\,\mathrm{d}I_t}{\sum_{t\geq 1} R_{0\to t}^{-1}\,\mathrm{d}\Pi_t} = \frac{I}{\Pi}\cdot\mathcal{M}, \qquad \text{(tangible share, short-run)}$$

$$\frac{\sum_{t\geq 1} R_{0\to t}^{-1}\,\mathrm{d}(w_{H,t}L_{H,t})}{\sum_{t\geq 1} R_{0\to t}^{-1}\,\mathrm{d}\Pi_t} = \frac{w_H L_H}{\Pi}\cdot\mathcal{M}, \qquad \text{(intangible share, short-run)}$$

$$\frac{\sum_{t\geq 1} R_{0\to t}^{-1}\,\mathrm{d}D_t}{\sum_{t\geq 1} R_{0\to t}^{-1}\,\mathrm{d}\Pi_t} = 1 - \frac{I+w_H L_H}{\Pi}\cdot\mathcal{M} \qquad \text{(payout share, short-run)}$$

where the object $\mathcal{M}$ is a sufficient statistic that captures the relative effect of the perturbation of the value of capital *formation versus production*

$$\mathcal{M} = \frac{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}\log q_t G_t}{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}\log Y_t}$$

In the short-run, a sufficient statistic $\mathcal{M}$—the relative effect of the shock on the present value of capital formation versus output—summarizes the short-run response of budget shares *conditional on a shock*. The logic is straightforward: while the economy has constant income shares of production Y and investment qG individually (both have Cobb-Douglas technologies), technology shocks can differentially affect Y and qG, leading to short-run deviations in the distribution of capital income. In particular, shocks that increase the value of capital formation qG more than output Y lead to a lower payout share, at the expense of a higher total investment share. In contrast, shocks that mostly increase output Y increase the payout share. Finally, while we motive the exercise by considering two possible technology shocks, the same logic would apply to any other shocks (discount rate shock, tax shock, etc

Proof of Proposition B.1. We will use two facts. First, (firm foc $L_{Y,t}$) implies that the production labor income moves one-for-one with output and capital income:

$$\mathrm{d}\log(w_{Y,t}L_{Y,t}) = \mathrm{d}\log Y_t = \mathrm{d}\log \Pi_t.$$

Second, (firm foc $L_{H,t}$) implies that the production labor income moves one-for-one with investment and the value of capital formation:

$$\mathrm{d}\log(w_{H,t}L_{H,t}) = \mathrm{d}\log I_t = \mathrm{d}\log(q_t G_t).$$

We immediately get

$$\frac{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}I_t}{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}\Pi_t} = \frac{I}{\Pi}\cdot\mathcal{M}\frac{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}(w_{H,t}L_{H,t})}{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}\Pi_t} = \frac{w_H L_H}{\Pi}\cdot\mathcal{M},$$

where

$$\mathcal{M} = \frac{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}\log q_t G_t}{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}\log Y_t}$$

Residually, we have that

$$\frac{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}D_t}{\sum_{t\geq 1} R_{0\to t}^{-1} \,\mathrm{d}(\Pi_t)} = 1 - \frac{I + w_H L_H}{\Pi}\mathcal{M}$$

□

C Appendix for Section 4

C.1 Proofs

Proof of Proposition 4.1. The Lagrangian associated with the household problem (2.1) is

$$\mathcal{L}_0 = \sum_{t=1}^{\infty} \beta^t \left(\frac{C_t^{1-\rho}}{1-\rho} - \frac{(1-\mu)L_{Y,t}^{1+\sigma} + \mu L_{H,t}^{1+\sigma}}{1+\sigma} \right) + \sum_{t=1}^{\infty} \beta^t \lambda_t \Big(C_t + V_t - w_{Y,t}L_{Y,t} - w_{H,t}L_{H,t} - R_t V_{t-1} \Big).$$

Applying the envelope theorem, we have

$$\begin{aligned}
\mathrm{d}U_0 = \mathrm{d}\mathcal{L}_0 &= \sum_{t=1}^{\infty} \beta^t \lambda_t \Big((\mathrm{d}w_{Y,t-1})L_{Y,t-1} + (\mathrm{d}w_{H,t-1})L_{H,t-1} + (\mathrm{d}R_t)V_{t-1} \Big) + R_1\,\mathrm{d}V_0, \\
&= C_1^{-\rho} \sum_{t=1}^{\infty} \frac{\beta^t C_t^{-\rho}}{C_1^{-\rho}} \Big((\mathrm{d}w_{Y,t-1})L_{Y,t-1} + (\mathrm{d}w_{H,t-1})L_{H,t-1} + (\mathrm{d}R_t)V_{t-1} \Big) + R_1\,\mathrm{d}V_0, \\
&= C_1^{-\rho} \sum_{t=1}^{\infty} R_{0\to t}^{-1} \Big((\mathrm{d}w_{Y,t-1})L_{Y,t-1} + (\mathrm{d}w_{H,t-1})L_{H,t-1} + (\mathrm{d}R_t)V_{t-1} \Big) + R_1\,\mathrm{d}V_0.
\end{aligned}$$

Using the Euler equation (worker foc C_t), we can re-write it as

$$\frac{\mathrm{d}U_0}{C_1^{-\rho}} = \sum_{t=1}^{\infty} R_{0\to t}^{-1} \Big((\mathrm{d}w_{Y,t-1})L_{Y,t-1} + (\mathrm{d}w_{H,t-1})L_{H,t-1} + (\mathrm{d}R_t)V_{t-1} \Big) + R_1\,\mathrm{d}V_0.$$

In the case where $\rho = 0$ (small open economy), we have that $R_t = \beta^{-1}$ which means that $\mathrm{d}R_t = 0$. $\square$

C.2 Analytics of permanent tax shock

Consider the effect of a permanent shock $\mathrm{d}\log(1-\tau_K)$ in the long-run. Lemma 2.2 states that the equilibrium is determined by

$$(1-\tau_K)\overline{q_D}K^{-(1-\alpha)\frac{\sigma}{\sigma+\alpha}} = \overline{q_S}K^{\frac{1-\chi}{1+\sigma\chi}\sigma}$$

Taking a log total derivative on both sides, and using the fact that $\mathrm{d}\log\overline{q_S} = 0$ and $\mathrm{d}\log\overline{q_D} = -\frac{1}{r+(1-\theta)\delta} \cdot \mathrm{d}\log R$, we have that

$$\mathrm{d}\log K = \frac{1}{(1-\alpha)\frac{\sigma}{\sigma+\alpha} + \frac{1-\chi}{1+\sigma\chi}\sigma} \cdot \mathrm{d}\log(1-\tau_K)$$

The first term is one over the sum of supply and demand elasticities. Using the capital supply curve—which remains unchanged— we obtain the change in q

$$\mathrm{d}\log q = \frac{\frac{1-\chi}{1+\sigma\chi}\sigma}{(1-\alpha)\frac{\sigma}{\sigma+\alpha} + \frac{1-\chi}{1+\sigma\chi}\sigma} \cdot \mathrm{d}\log(1-\tau_K).$$

The first-term is a GE adjustment: the equilibrium change in $\frac{\mathrm{d}\log q}{\mathrm{d}R}$ is not the duration $\frac{\partial \log q}{\partial R}$.

C.3 Facts about q and K

First, we have that $V_t = q_t K_{t+1}$. Using (firm foc K_{t+1}), we can show that $V_t = q_t K_{t+1}$.

$$\begin{aligned} Rq_t &= \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta + \theta \frac{G_{t+1}}{K_t}) q_{t+1} \\ &= \alpha \frac{Y_{t+1}}{K_{t+1}} - (1-\theta) q_{t+1} \frac{G_{t+1}}{K_{t+1}} + \frac{K_{t+2}}{K_{t+1}} q_{t+1} \\ Rq_t K_{t+1} &= \alpha Y_{t+1} - (1-\theta) q_{t+1} G_{t+1} + q_{t+1} K_{t+2} \end{aligned}$$

Second, we can infer $\{g_{q,t}\}_{t=0}^{\infty}$ from the data. Using the law of motion of capital, we obtain

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + G_t \\ q_t K_{t+1} &= (1-\delta) q_t K_t + q_t G_t \\ q_t K_{t+1} &= (1-\delta)(1+g_{q,t}) q_{t-1} K_t + q_t G_t \\ V_t &= (1-\delta)(1+g_{q,t}) V_{t-1} + \frac{1}{1-\theta} \Big(I_t + w_{H,t} L_{H,t} \Big) \end{aligned}$$

where the last equation uses firm optimization. Given data on $\{V_t, I_t + w_{H,t} L_{H,t}\}_{-\infty}^{\infty}$ we can recover $\{g_{q,t}\}_{t=0}^{\infty}$.