Discount Rates

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The old view (1970)

1. Expected returns do not move much over time; stocks returns are unpredictable.
2. Prices move on news of cash-flow.

The modern view

1. Expected returns move a lot over time, stocks are predictable.
2. Prices move on news of discount rate changes.

Price-Dividend Ratio

Static Model: Gordon Growth Formula

- The gross simple return on a stock is given by:

$$1 + R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t}$$
$$\Rightarrow P_t = \frac{D_{t+1} + P_{t+1}}{1 + R_{t+1}}$$

- Iterating forward:

$$P_t = \sum_{k=1}^{K} \frac{D_{t+k}}{(1 + R_{t+1}) \ldots (1 + R_{t+k})} + \frac{P_{t+K}}{(1 + R_{t+1}) \ldots (1 + R_{t+K})}$$

- Letting $K \to \infty$ yields the dividend discount model (DDM) of stock prices:

$$P_t = \sum_{k=1}^{\infty} \frac{D_{t+k}}{(1 + R_{t+1}) \ldots (1 + R_{t+k})}$$

Static Model: Gordon growth formula

- Assume that returns and dividend growth are constant, i.e. $R_{t+k} = R$ an $\mathbb{E}_t[D_{t+k}] = (1+G)D_t$, with $G < R$. We have

$$\begin{aligned} P_t &= E_t\left[\sum_{k=1}^{\infty} \frac{D_{t+k}}{(1+R)^k}\right] \\ &= \sum_{k=1}^{\infty} \frac{(1+G)^k D_t}{(1+R)^k} \\ &= D_t \frac{1+G}{R-G} \end{aligned}$$

- We obtain Gordon Growth Formula

$$\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{R-G}$$

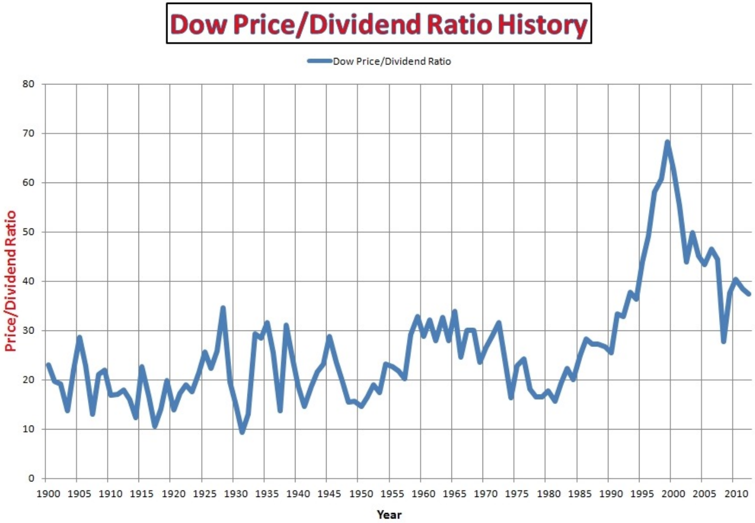
Static Model: Gordon growth formula

$$\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{R - G}$$

Some implications:

- If divivend growth G increases, prices decrease
- If expected returns R rise, prices decrease
- Small changes in expected returns (or in expected growth) have a large impact in prices if R close to G (i.e. P/D is high)

$$\frac{d \ln P}{dR} = -P/D$$


Dow Price/Dividend Ratio History
Dow Price/Dividend Ratio
80
70
60
50
40
30
20
10
0
Price/Dividend Ratio
1900 1905 1910 1915 1920 1925 1930 1935 1940 1945 1950 1955 1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010
Year

Dynamic Model: Campbell-Shiller

- Denote $PD_t = P_t/D_t$ the Price to Dividend ratio
- Definition of returns

$$
\begin{aligned}
1 + R_{t+1} &= \frac{D_{t+1} + P_{t+1}}{P_t} \\
&= \frac{D_{t+1}}{D_t} \frac{PD_{t+1}}{PD_t} (1 + \frac{1}{PD_{t+1}})
\end{aligned}
$$

- We get

$$
\log(1 + \frac{1}{PD_{t+1}}) = \log(1 + R_{t+1}) - \log(\frac{D_{t+1}}{D_t}) - \log \frac{PD_{t+1}}{PD_t}
$$

Dynamic Model: Campbell-Shiller

- Denote $pd_t = \log PD_t, r_{t+1} = \log(1 + R_{t+1}), d_{t+1} = \log(D_{t+1})$

$$\log(1 + e^{-pd_{t+1}}) = r_{t+1} - \Delta d_{t+1} - \Delta pd_{t+1}$$
$$\Rightarrow pd_t = \Delta d_{t+1} - r_{t+1} + pd_{t+1} - \log(1 + e^{-pd_{t+1}})$$

- Approximate the function $\log(1 + e^{-x})$ by linear function:

$$pd_t = \Delta d_{t+1} - r_{t+1} + +\kappa + \rho pd_{t+1}$$

where

$$\rho = \frac{1}{1 + D/P} \approx 1 - D/P \approx 0.96$$
$$\kappa = -\log(\rho) - (1 - \rho)\log\left(\frac{1}{\rho} - 1\right)$$

Dynamic Model: Campbell-Shiller

- Iterating forward:

$$
\begin{aligned}
pd_t &= \Delta d_{t+1} - r_{t+1} + \kappa + \rho(\Delta d_{t+2} - r_{t+2} + \kappa + \rho pd_{t+2}) \\
&= \sum_{j=0}^{K} \rho^j [\Delta d_{t+j+1} - r_{t+j+1}] + \sum_{j=0}^{K} \rho^j \kappa + \rho^K pd_{t+K} \\
&= \sum_{j=0}^{\infty} \rho^j [\Delta d_{t+j+1} - r_{t+j+1}] + \frac{\kappa}{1-\rho}
\end{aligned}
$$

Dynamic Model: Campbell-Shiller

- The Campbell Shiller is an accounting decomposition that holds in any state of the world ex-post
- In particular, it is true in expectation (rational or irrational)

$$pd_t = \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t[\Delta d_{t+j+1}] - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t[r_{t+j+1}] + \frac{\kappa}{1-\rho}$$

What Drives Variations in Price-Dividend Ratio?

- For $K < \infty$:

$$var(pd_t) = cov\left(pd_t, \sum_{j=1}^{K} \rho^{j-1} \Delta d_{t+j}\right) - cov\left(pd_t, \sum_{j=1}^{K} \rho^{j-1} r_{t+j}\right) + \rho^K cov(pd_t, pd_{t+K})$$

- Dividing by the variance of pd_t

$$1 = b_d^{(K)} - b_r^{(K)} + b_{dp}^{(K)}$$

where

- $b_d^{(K)}$ denotes the slope estimate of a regression of $\sum_{j=1}^{K} \rho^{j-1} \Delta d_{t+j}$ on pd_t
- $b_r^{(K)}$ denotes the slope estimate of a regression of $\sum_{j=1}^{K} \rho^{j-1} r_{t+j}$ on pd_t
- $b_{dp}^{(K)}$ denotes the slope estimate of a regression of $\rho^k dp_{t+k}$ on pd_t

What Drives Variations in Price-Dividend Ratio?

Table 1: Results (from Cochrane "The Dog That Did Not Bark")

	Dependent Variable ($k = 15$)		
	$\sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j}$	$\sum_{j=1}^{k} \rho^{j-1} r_{t+j}$	$\rho^k dp_{t+k}$
pd_t	-0.11	-1.01	0.11

- price-dividend does not predict future dividend growth
- An alternative way to state the same result is to say that price-dividend predicts future returns

HW

- Reproduce the findings using Schiller data on price dividend ratio, dividend growth, and returns
- Bonus points: Try to do the same decomposition for housing asset (note that you need a measure of housing rent)

What Drives Variations in Returns?

- Remember

$$pd_t = \Delta d_{t+1} - r_{t+1} + \kappa + \rho pd_{t+1}$$

In particular, this is true in expectation:

$$pd_t = E_t[\Delta d_{t+1}] - E_t[r_{t+1}] + \kappa + \rho E_t[pd_{t+1}]$$

- Therefore

$$r_{t+1} - E_t[r_{t+1}] = \Delta d_{t+1} - E_t \Delta d_{t+1} + \rho(pd_{t+1} - E_t pd_{t+1})$$

- Taking variance of the expression

$$\underbrace{Var_t(r_{t+1})}_{\approx 16\%^2} = \underbrace{Var_t(\Delta d_{t+1})}_{\approx 9\%^2} + \rho^2 \underbrace{Var_t(dp_{t+1})}_{\approx 13\%^2} + \rho \underbrace{cov_t(\Delta d_{t+1}, pd_{t+1})}_{\approx 0\%}$$

  - The variance of returns is much higher than the variance of dividend growth. This is the same as saying that the price-dividend moves
  - Approximately, variance of returns is 50% due to variance of current dividend growth and 50% due to future price dividend.

What Drives Variations in Returns?

- Using Campbell Shiller decomposition, we can also rewrite the surprise in the price dividend ratio as surprise in future expected dividend growth and surprise in future expected returns:

$$r_{t+1} - \mathbb{E}_t[r_{t+1}] = \Delta d_{t+1} - E_t \Delta d_{t+1} + (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j \Delta d_{t+1+j} - (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}$$

- Therefore, neglecting covariance terms which are small

$$\underbrace{Var_t(r_{t+1})}_{\approx 16\%^2} = \underbrace{Var_t(\Delta d_{t+1})}_{\approx 9\%^2} + \underbrace{Var_t(\mathbb{E}_{t+1}[\sum_{j=1}^{\infty} \rho^j \Delta d_{t+1+j}])}_{\approx 0\%^2} + \underbrace{Var_t(\mathbb{E}_{t+1}[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}])}_{\approx 13\%^2}$$

Approximately, variance returns is 50% due to variance of current dividend growth, 0% due to news about future expected dividend growth, and 50% due to news about future expected returns

What Drives Variations in Expected Returns?

- Suppose expected returns follow an AR(1) process:

$$r_{t+1} = r + x_t + u_{t+1} \quad (1)$$
$$x_{t+1} = \phi x_t + \xi_{t+1} \quad (2)$$

- The discount rate component is

$$\begin{aligned}(\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} &= (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j x_{t+1+j} \\ &= (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j \phi^{j-1} \xi_{t+1} \\ &= \frac{\rho \xi_{t+1}}{1 - \rho\phi}\end{aligned}$$

- Therefore

$$Var_t(E_{t+1}[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}]) = \frac{\rho^2}{(1-\rho\phi)^2} Var(\xi_{t+1})$$

$\Longrightarrow$ Small but persistent changes in discount rates can generate large fluctuations in asset returns

Why Aren't Returns Correlated?

- What is the correlation of returns?

$$\begin{aligned} Cov(r_{t+1}, r_{t+2}) &= E[r_{t+1}r_{t+2}] - E[r_{t+1}]E[r_{t+2}] \\ &= E[r_{t+1}E_{t+1}[r_{t+2}]] - E[r_{t+1}E_{t+1}[r_{t+2}]] \\ &= Cov(r_{t+1}, E_{t+1}[r_{t+2}]) \\ &= \underbrace{cov(E_t[r_{t+1}], E_{t+1}[r_{t+2}])}_{>0} + \underbrace{cov(r_{t+1} - E_t[r_{t+1}], E_{t+1}[r_{t+2}])}_{<0} \end{aligned}$$

- Using the statistical model given by 1 and 2, and, assuming discount rate shocks and dividend shocks are uncorrelated,

$$\begin{aligned} Cov(r_{t+1}, r_{t+2}) &= cov(x_t, x_{t+1}) - cov(\frac{\rho\xi_{t+1}}{1-\rho\phi}, \xi_{t+1}) \\ &= \phi var(x_t) - \frac{\rho}{1-\rho\phi} var(\xi_{t+1}) \\ &= (\frac{\phi}{1-\phi^2} - \frac{\rho}{1-\rho\phi}) Var\xi_{t+1} \end{aligned}$$

For ϕ close to $\rho \approx 0.96$, correlation of returns is close to zero

Campbell Shiller at the Firm level

- We can define the an ROE E_{t+1} as

$$E_{t+1} = \frac{B_{t+1} + D_{t+1}}{B_t}$$

- Similarly to Campbell-Schiller, Vuolteenaho (2002) develops a variance decomposition based on ROE and Book/Market similar in spirit to Campbell's (1991) variance decomposition:

$$r_{t+1} - E_t[r_{t+1}] = \sum_{j=0}^{\infty}(E_{t+1} - E_t)(e_{t+1+j} - r_{f,t+1+j}) - \sum_{j=0}^{\infty}(E_{t+1} - E_t)[r_{t+1+j}]$$

  - ROE-based decomposition is better suited for individual securities.
  - Firm level stock returns are driven largely by cash-flow news.

Market Efficiency

- Two interpretations about predictability of returns
    1. Fama: Rational changes in expected returns required by households to hold risky assets
    2. Shiller: Sentiments (irrational expectations)
- One can ask households about returns expectations. Overall, one find that expectation of future returns by households does not predict future returns. This suggests sentiment is important.
- Issues
    - How well do people understand the question?
    - Surveys generally do not include wealthiest households, that have large impact on asset prices

Continuous Time

- Denote R_t the cumulative dollar amount of investing in the asset. The law of motion is given by:

$$\frac{dR_t}{R_t} = \frac{D_t dt + dP_t}{P_t}$$

- This can be written as

$$0 = D_t dt + E[dP_t] - P_t E[\frac{dR_t}{R_t}]$$
$$\Rightarrow P_t = E_t[\int_t^{+\infty} e^{\int_t^s - \frac{E[dR_u]}{R_u}} D_s ds]$$

- Denoging $V = P/D$.

$$\frac{dR_t}{R_t} = \frac{1}{V}dt + \frac{d(VD)}{VD}$$

Denoting μ_V and σ_V the the geometric drift and volatility of V. Denoting μ_D and σ_D the the geometric drift and volatility of V

$$\frac{dR_t}{R_t} = (\frac{1}{V} + \mu_{Vt} + \mu_{Dt} + \sigma_{Vt}\sigma_{Dt})dt + (\sigma_{Vt} + \sigma_{Dt})dZ_t$$

- Gordon Growth Formula: If $\mu_D, \sigma_D, E[dR_t/R_t]$ are constant, we get

$$V = \frac{1}{E[\frac{dR_t}{R_t}] - \mu_D}$$

Campbell-Shiller Decomposition

- Log-linear approximation of price dividend ratio around its mean:

$$\frac{1}{V} \approx \alpha(1 - \log \alpha) - \alpha \log(V)$$

with $\alpha = e^{-E[\log V]}$

- The equation

$$\frac{dR_t}{R_t} = \frac{D_t dt + dP_t}{P_t}$$

becomes

$$d \log R = \alpha(1 - \log \alpha) - \alpha \log(V) dt + d \log(V) + d \log(D)$$

- This can be written as

$$\log(V) = E[\int_t^{+\infty} e^{-\alpha(s-t)}(\kappa + d \log R_s - d \log D_s)]$$
$$= 1 - \log \alpha + E[\int_t^{+\infty} e^{-\alpha(s-t)}(d \log D_s - d \log R_s)]$$