Pricing

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Fall 2017

Informational Efficiency

- We talked about allocative efficiency: is there a possible redistribution of assets that would make everyone weakly better off (perfect risk sharing)
- Now we are going to study informational efficiency: do prices reflect information?

Efficientcy

There are three concepts of price efficiency.

- Weak efficiency: asset prices convey all information relating to the past time-series of data

  $E[R_{t+1}|R_t] = E[R_{t+1}]$

  One cannot predict future returns from current returns

- Semi-strong efficiency: asset prices convey all public information

  $E[R_{t+1}|\text{Public Information}] = E[R_{t+1}]$

  One cannot predict future returns from current publicly available information

- Strong efficiency: asset prices reflect private information.

  $E[R_{t+1}|\text{Private Information}] = E[R_{t+1}]$

  The third form of efficiency cannot exist, indefinitely, unless of course information is not costly.

Trading Game

- One asset can give dividend 490 (with prob 1/3), 240 (with prob 1/3) or 50 (with prob 1/3)
- Each investor receives some information: either that the dividend is not 490, that it is not 240, or that it is not 50
- Let us consider the following strategy
  - If investor receives not 490: sell at 240
  - If investor receives not 240: do nothing
  - If investor receives not 50: buy 240

- This strategy is a Nash equilibrium. If everyone follows this strategy, there is no trading volume and perfect information aggregation.

No Trade Theorem

- Asset with cashflow X in zero supply. Denote p its price today
- Each investor $1 \leq i \leq N$ receives a different signal about the cashflow σ_i
- Knowing p and signal σ_i, each investor chooses an asset allocation $\theta_i(\sigma_i, p) \leq \overline{\theta}$ to maximize profit

$$\max E[(X - p)\theta_i(\sigma_i, p)|\sigma_i, p]$$

- An equilibrium is defined by price p and asset allocation so that total demand equals total supply:

$$\sum_i \theta_i(\sigma_i, p) = 0$$

No Trade Theorem

Theorem (No Trade Theorem)
If there is an equilibrium, $\theta_i = 0$ *is a solution*

Proof.

1. **Payoffs are at least zero**. Since $\theta_i = 0$ is a possible action, the investor expected payoff after learning their signal $E[(X-p)\theta_i(\sigma_i, p)|\sigma_i, p]$ is at least zero. Therefore, the investor expected payoff before learning the signal $E[(X-p)\theta_i(\sigma_i, p)]$ is also at least zero
2. **Sum of Expected Payoffs is zero**.The sum of expected payoffs across investors is

$$\sum_{1 \leq i \leq N} E[(X-p)\theta_i(\sigma_i, p)] = E[(X-p) \sum_{1 \leq i \leq N} \theta_i(\sigma_i, p)]$$
$$= E[(X-p) \times 0]$$
$$= 0$$

(i.e. it is a zero sum game). Therefore, the investor expected payoffs before learning the signal is zero:

$$E[(X-p)\theta_i(\sigma_i, p)] = 0$$

3. Since the investor expected payoff before learning the signal is zero ($E[(X-p)\theta_i(\sigma_i, p)] = 0$), and since the investor expected payoff after receiving the signal is at least zero ($E[(X-p)\theta_i(\sigma_i, p)|\sigma_i, p] \geq 0$) , it must be that expected payoff after learning the signal is zero

$$E[(X-p)\theta_i(\sigma_i, p)|\sigma_i, p] = 0$$

□

Kyle (1985)

Model

- Model to understand how information can be incorporated in prices
- Some informed traders learn about asset payoff
- To avoid no trade theorem, we assume there are also some noise traders

Model

- Asset at the end of period has a payoff $X \sim N(0, \sigma^2)$
- Informed trader learns a signal ν with precision λ, i.e.

$$X = \nu + \epsilon$$

  - $\nu \sim N(0, \lambda^2\sigma^2)$
  - $\epsilon \sim N(0, (1-\lambda^2)\sigma^2)$
  - $\nu \perp \epsilon$ (i.e. ν and ϵ are independent)

  Based on this signal ν, they choose to buy a quantity of assets x.
- Noise traders chooses to buy a quantity of assets $u \sim N(0, \sigma_u^2)$ (independent of ν and ϵ).
- A competitive market maker receives the total buying quantity $x + u$ and chooses the price p equal to their expected payoff after learning $x + u$

Nash Equilibrium

- Having observed ν, the informed trader chooses a quantity of asset x to maximize their profit

$$\max_x E[x(\nu + \epsilon - p)|\nu]$$

  In particular, the strategy x is a function of ν.

- Having observed $x + u$, the market marker sets to price to equal the expected payoff:

$$p = E[\nu + \epsilon | x + u]$$

  In particular, the strategy p is a function of $x + u$

- The informed trader has a tradeoff: an order very responsive to ν means more profit for a given price but also higher price

  Suppose informed trader strategy is to buy a lot when ν is high. Then, in equilibrium, if the market maker observes a high $x + u$, they deduce easily that ν must be high, and therefore they increases sharply their price. Knowing this, the informed trader would prefer to purchase a bit less when ν is high.

Solution

- Formally we are looking for a Nash equilibrium (market maker strategy $p(x+u)$ is a best response to informed trader strategy $x(\nu)$ and informed trader strategy $x(\nu)$ is a best response to market maker strategy $p(x+u)$)
- Let us assume that the market maker strategy p is linear in $x+u$, i.e. $p = a + b(x+u)$ and that the informed trader strategy x is linear in ν, i.e. $x = \alpha + \beta\nu$.

Informed Trader Best Strategy

- Taking as given the market maker strategy $p = a + b(x + u)$, the informed trader chooses x to maximize her profits

$$
\begin{aligned}
\max_x E[x(\nu + \epsilon - p)|\nu] &= \max_x x E[\nu + \epsilon - p|\nu] \\
&= \max_x x(\nu - E[p|\nu]) \\
&= \max_x x(\nu - E[a + b(x + u)|\nu]) \\
&= \max_x x(\nu - a - bx)
\end{aligned}
$$

- Taking FOC with respect to x, we get

$$x = \frac{\nu - a}{2b}$$

- By definition of α and β, this means

$$
\begin{aligned}
\alpha &= -\frac{a}{2b} \\
\beta &= \frac{1}{2b}
\end{aligned}
$$

Market maker Best Strategy

- Taking as given the informed trader strategy $x = \alpha + \beta\nu$, the best response of the market maker is

$$p = E[\nu + \epsilon|(x + u)]$$

- Parenthesis: for two normal random variables X and Y, we have

$$E[Y|X] = E[Y] + \frac{cov(X,Y)}{var(X)}(X - E[X])$$

Proof. Denote $\hat{Y} = E[Y] + \frac{cov(X,Y)}{var(X)}(X - E[X])$ and $\epsilon = Y - \hat{Y}$. We have $E[\epsilon] = 0$ and $E[X\epsilon] = 0$ therefore ϵ is independent of X. Therefore $E[Y|X] = E[\hat{Y} + \epsilon|X] = \hat{Y}$

- Applying this formula to $Y = \nu + \epsilon$ and $X = x + u$

$$p = E[\nu + \epsilon] + \frac{cov(\nu + \epsilon, x + u)}{Var(x + u)}(x + u - E[x + u]) \tag{1}$$

Using $x = \alpha + \beta\nu$, we get

$$E[x + u] = E[\alpha + \beta\nu + u] = \alpha$$

$$Var(x + u) = Var(\alpha + \beta\nu + u) = \beta^2 Var(\nu) + Var(u)$$

$$cov(\nu + \epsilon, x + u) = cov(\nu + \epsilon, \alpha + \beta\nu + u) = cov(\nu, \beta\nu) = \beta Var(\nu)$$

Market maker Best Strategy

- Plugging these expressions into equation 1, we get

$$p = \frac{\beta\lambda^2\sigma^2}{\beta^2\lambda^2\sigma^2 + \sigma_u^2}(x + u - \alpha)$$

- By definition of a and b, this means

$$a = -\frac{\beta\lambda^2\sigma^2}{\beta^2\lambda^2\sigma^2 + \sigma_u^2}\alpha$$
$$b = \frac{\beta\lambda^2\sigma^2}{\beta^2\lambda^2\sigma^2 + \sigma_u^2}$$

Solution

- The investor best response gives α, β in term of a, b. The market maker best response gives a, b in term of α, β. That is we have four equations and four unknowns, which we can solve.

$$a = \frac{\beta\lambda^2\sigma^2}{\beta^2\lambda^2\sigma^2 + \sigma_u^2}\frac{a}{2b}$$
$$b = \frac{\lambda^2\sigma^2/(2b)}{\lambda^2\sigma^2/(4b^2) + \sigma_u^2}$$

which gives

$$a = 0$$
$$b = \frac{\lambda\sigma}{2\sigma_u}$$

- That is, best responses are given by

$$p = \frac{\lambda\sigma}{2\sigma_u}(x + u)$$
$$x = \frac{\sigma_u}{\lambda\sigma}\nu$$

- Define the liquidity of the market as the inverse of the derivative of p with respect to $x + u$

$$\frac{1}{\partial p / \partial x} = \frac{2\sigma_u}{\lambda\sigma}$$

  Markets is liquid means that one supplementary buy order does not increase price by much

- Define the agressiveness of informated traderas the derivative of x wrt ν

$$\frac{\partial x}{\partial \nu} = \frac{\sigma_u}{\lambda\sigma}$$

  Informed traders trade aggressively means that they purchase a lot more when the signal is more positive

- As $\sigma_u / \lambda\sigma \to 0$,
  1. Informed traders trade only a bit on their information: order flow very small
  2. Market marker increases sharply their price: price is very elastic to any order flow

  $\Rightarrow$ This economy tends to the "No Trade Theorem" situation

Price Informativeness

- Think of a succession of these games between market marker and informed trader (for each game, there is one price and one signal)
- Define price informativeness as the ratio variance price / variance cash flow across these different economies

$$\frac{Var(p)}{Var(\nu + \epsilon)}$$

  It is the R^2 in a regression of realized payoff $\nu + \epsilon$ on price p
- The variance of the price across these games is

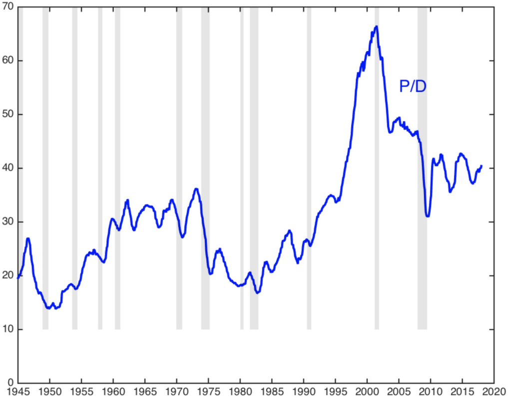
$$\begin{aligned} Var(p) &= Var(\frac{\lambda\sigma}{2\sigma_u}(x+u)) \\ &= \frac{\lambda^2\sigma^2}{4\sigma_u^2}(Var(x) + Var(u)) \\ &= \frac{\lambda^2\sigma^2}{4\sigma_u^2}(\frac{\sigma_u^2}{\lambda^2\sigma^2}\lambda^2\sigma^2 + \sigma_u^2) \\ &= \frac{\lambda^2\sigma^2}{2} \end{aligned}$$

- Therefore price informativeness is $\lambda^2/2$

Price Informativeness

- Price informativeness is between 0 and 1/2. There is a limit on price informativeness. Even if $\lambda \to 1$, the agent does not want to trade all his information. Markets are not strongly efficient
- High λ increases informativeness but also decreases liquidity. There is a tension between two social roles of financial market: liquidity and price informativeness

Price Dividend



70
60
50
40
30
20
10
0
1945 1950 1955 1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010 2015 2020
P/D

- A present value model can be used to study the effects of alternative models of expected returns on prices and realized returns.
- The gross simple return on a stock is given by:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

$$\Rightarrow P_t = \frac{D_{t+1} + P_{t+1}}{R_{t+1}}$$

- Iterating forward:

$$P_t = \sum_{k=1}^{K} \frac{D_{t+k}}{\prod_{l=1}^{k} R_l} + \frac{P_{t+K}}{\prod_{l=1}^{K} R_l}$$

- Letting $K \rightarrow \infty$ and assuming that $\lim_{K \rightarrow +\infty} \frac{P_{t+K}}{\prod_{l=1}^{K} R_l} \rightarrow 0$

$$P_t = \sum_{k=1}^{+\infty} \frac{D_{t+k}}{\prod_{l=1}^{k} R_l}$$

Gordon growth formula

- Now assume that $R_k = R$ (constant return). and $\mathbb{E}_t[D_{t+k}] = G^k D_t$ (constant dividend growth), with $G < R$. We get

$$
\begin{aligned}
P_t &= E_t[\sum_{k=1}^{\infty} \frac{D_{t+k}}{R^k}] \\
&= \sum_{k=1}^{\infty} \frac{E_t[D_{t+k}]}{R^k} \\
&= \sum_{k=1}^{\infty} \frac{G^k D_t}{R^k} \\
&= D_t \frac{G}{R} \sum_{k=0}^{+\infty} \frac{G^k}{R^k} \\
&= D_t \frac{G}{R} \frac{1}{1 - \frac{G}{R}} \\
&= D_t \frac{G}{R - G}
\end{aligned}
$$

- We obtain Gordon Growth Formula

$$
\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{R - G}
$$

Gordon growth formula

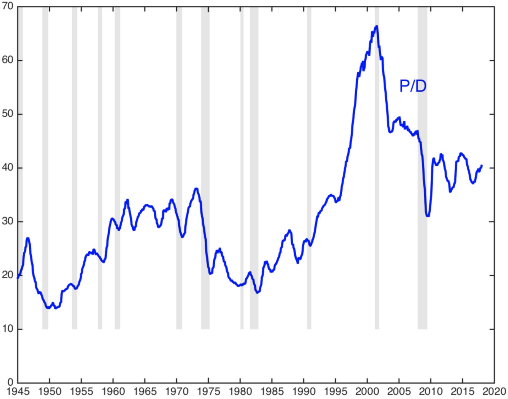
$$\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{R - G}$$

- Price dividend decreases in R-G
- Quantitatively, we have

$$\frac{\partial \ln(P/E_t[D_{t+1}])}{\partial R - G} = -\frac{P}{E_t[D_{t+1}]}$$
$$= -40$$

Therefore an increase of $R - G$ by 1% percentage point means that price-dividend ratio decreases by -40%.
$\Longrightarrow$ Small changes in expected growth rate or returns have large impacts on price

Test of Informational Efficiency



P/D
70
60
50
40
30
20
10
0
1945
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020

- To see if variations in P/D reflect variations in G, we can run the following regression

$$\frac{D_{t+k}}{D_t} = \alpha + \beta \log(\frac{P_t}{D_t}) + \epsilon_t$$

- We obtain $\beta \approx 0$ and R^2 small $\Rightarrow$ changes in prices don't really predict changes in future dividend growth

Shiller Graph

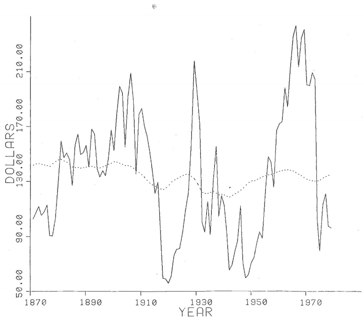


Figure 1 Detrended real Standard & Poor Composite Stock Price Index (solid line, p) and ex-post rational price (dotted line, p*), first of the year, 1871-1979. The variable p* is the present value of actual subsequent detrended real dividends, subject to an assumption about dividends after 1978. The variable p_t is from data set 1, described in appendix, and p_t^* is defined for this data set using $p_t^* = \bar{\gamma}(p_{t+1}^* + d_t)$ with p_{1979}^* set at the average value of p_t over the sample.

Figure 1: P/D vs P/D^* using exp post data on dividends and constant rate R $(P/D)^* = \sum_{k=1}^{+\infty} \frac{D_{t+k}}{R^k}$

- Maybe price dividend changes because of false expectations about future dividend growth G
- Maybe prices change because of changes in expected returns R (i.e. sometime

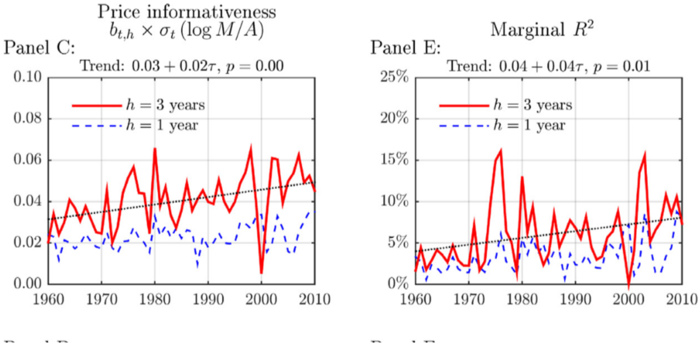
Cross Section

- In a given year, are firms with higher price dividends end up having higher dividend growth?
- In every year t, do the cross-sectional regression

$$\frac{D_{i,t+h}}{D_{i,t}} = \alpha_t + \beta_t \log(\frac{P_{it}}{D_{it}}) + \epsilon_{it}$$

- In contrast with time series regressions, we obtain $\beta > 0$ and $R^2 \approx 8\%$ increasing over time

Cross Section



- Firms with high market prices tend to have higher future earnings compared to firms with low market prices.
- Prediction power higher at the longer horizons $h = 3$
- Prediction show clear upward trend, especially at $h = 3$. The extent to which market prices can be used to distinguish firms that will deliver high earnings in the future from those that will not has increased over the past five decades.

Why do we Care about Informational Efficiency?

Why do we Care about Informational Efficiency?

- Remember investment model with firm i profit given by

$$v(\mu_i, k_i) = \mu_i k_i - \gamma \frac{k_i^2}{2}$$

with average productivity $E[\mu_i] = 1$ and interest rate normalized to 0

- Now assume that the CEO of the firm does not observe productivity μ_i but only a signal $E[\mu_i | I_m]$.
- FOC with respect to k_i gives

$$k_i = \frac{E[\mu_i | I_m]}{\gamma}$$

Why do we Care about Informational Efficiency?

- Remember that total productivity can be written as the sum of unweighted productivity $E[\mu_i] = 1$ and covariance term betwen productivity and capital

$$
\begin{aligned}
cov(k_i, \mu_i) &= E[\mu_i \frac{E[\mu_i|I_m]}{\gamma}] - \frac{E[\mu_i]E[E[\mu_i|I_m]]}{\gamma} \\
&= \frac{E[E[\mu_i|I_m]^2] - E[E[\mu_i|I_m]]^2}{\gamma} \\
&= \frac{Var(E[\mu_i|I_m])}{\gamma}
\end{aligned}
$$

- The higher $Var(E[\mu_i|I_m])$, the higher the allocation term.
  - If I_m has no information, then $Var(E[\mu_i|I_m]) = Var(E[\mu_i]) = 0$
  - If I_m allows to know μ_i, then $Var(E[\mu_i|I_m]) = Var(\mu_i)$