RBC with Asset Prices

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TFP Shocks

General Equilibrium

- Assume
  1. firm have “AK” technology.

$$aK_t dt$$

  2. Investment cost function $C_t(I_t, K_t) = c(i_t)aK_t$ where $i_t = I_t/K_t$
- Law of motion of capital is

$$\frac{dK_t}{K_t} = (i_t - \delta)dt + \sigma dZ_t$$

- Profits can be written

$$Y_t dt = (1 - i_t - c(i_t))A_t K_t dt$$

Returns

- Denote $q_t K_t$ the value of a firm with installed capital K_t

$$\frac{dq_t}{q_t} = \mu_q dt + \sigma_q dZ_t$$

- The return of a claim to the representative firm is:

$$dR_t = \underbrace{\frac{Y_t dt}{q_t K_t}}_{\text{Dividend Yield}} + \underbrace{E[\frac{d(q_t K_t)}{q_t K_t}]}_{\text{Capital Gain}} \tag{1}$$

$$= \frac{a(1 - i_t - c(i_t))}{q_t} dt + (i_t - \delta + \mu_q + \sigma_q \sigma)dt + (\sigma + \sigma_q)dZ_t \tag{2}$$

General Equilibrium

- Assume representative agent with Epstein-Zin preferences In this guess, guess constant price q^*, constant investment i^* (i.e. ff capital drops by 1%, investment, consumption, wealth all drop by 1%)
- The law of motion of profits is then

$$\begin{aligned}\frac{dY_t}{Y_t} &= a\frac{d(K_t)}{K_t} \\ &= (i^* - \delta)dt + \sigma dZ_t\end{aligned}$$

- Euler equation gives:

$$\kappa = \gamma\sigma$$

$$r = \rho + \frac{1}{\psi}(i^* - \delta) - \frac{1 + \frac{1}{\psi}}{2}\gamma\sigma^2$$

Using Equation (1), we obtain q^* in term of ι^*

$$q^* = \frac{1 - i^* - c(i^*)}{\rho - (1 - \frac{1}{\psi})(i^* - \delta - \frac{\gamma}{2}\sigma^2)}$$

General Equilibrium

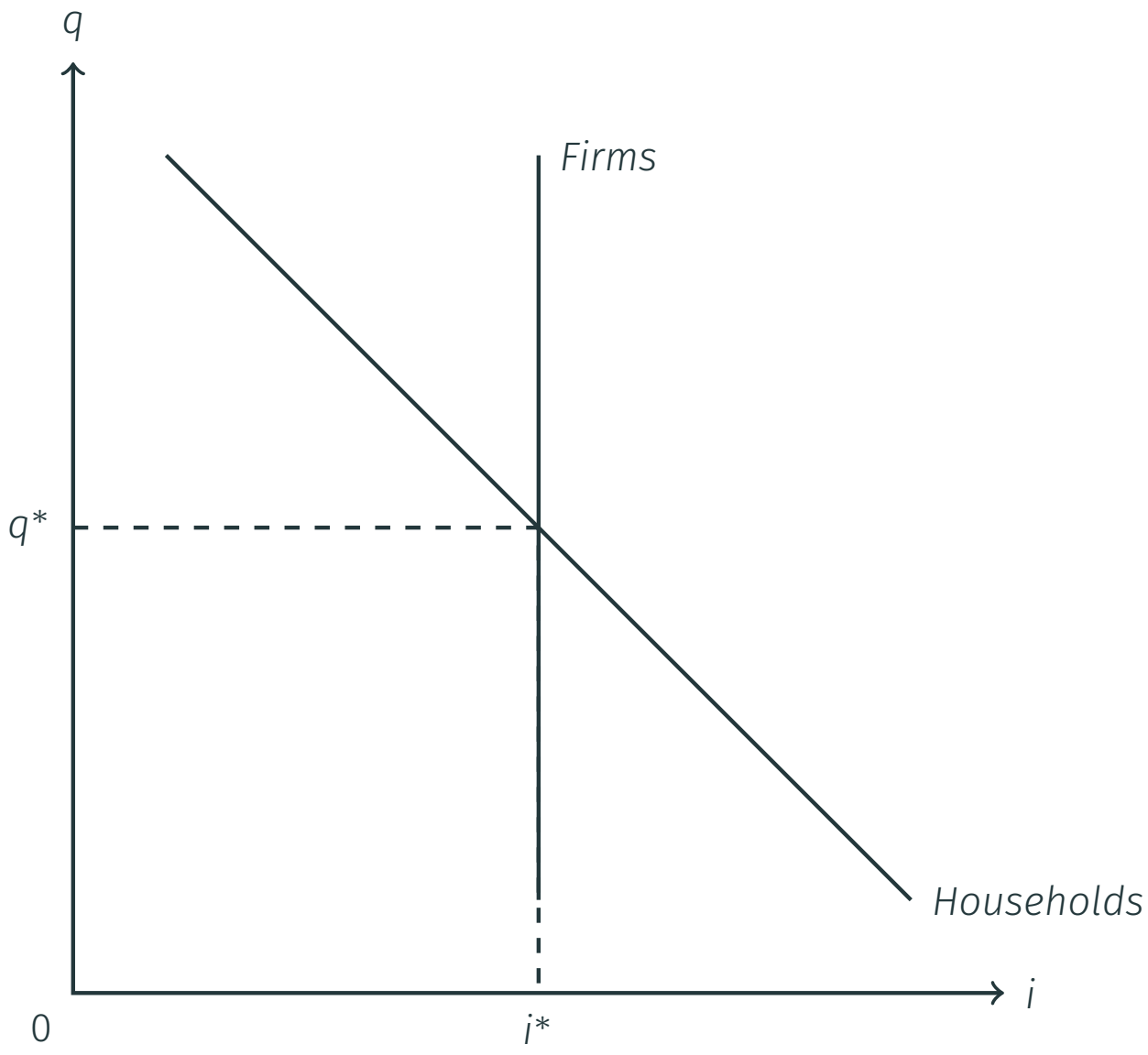
- We obtain two equation in two unknowns:

$$c'(i^*) = q^*$$

$$q^* = \frac{1 - i^* - c(i^*)}{\rho - (1 - \frac{1}{\psi})(i^* - \delta - \frac{\gamma}{2}\sigma^2)}$$

  - First equation says that investment increases in prices.
  - Second equation says that price decreases in investment (at least for $\psi \leq 1$)

Comparative Statics with Infinite Adjument Cost



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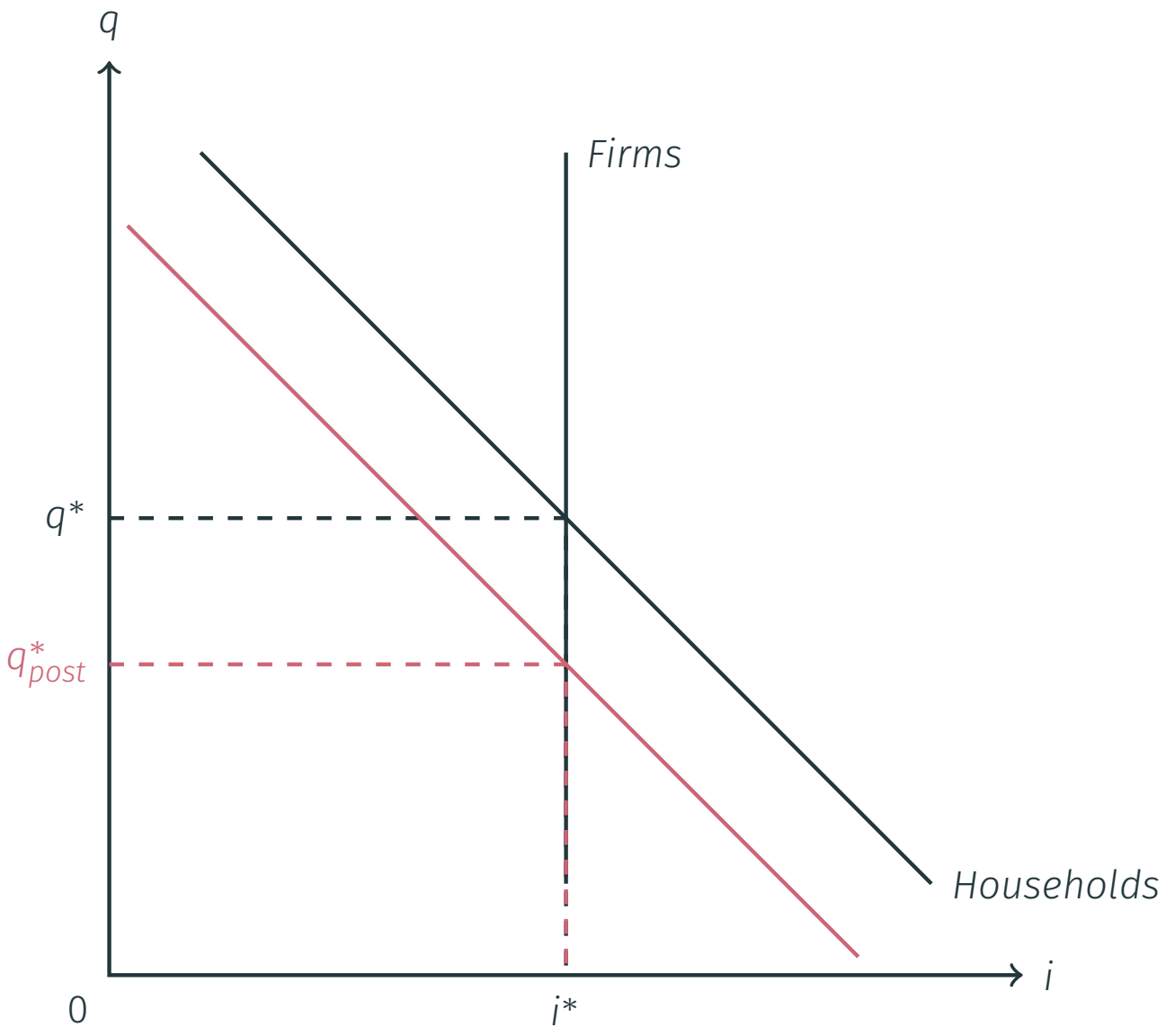
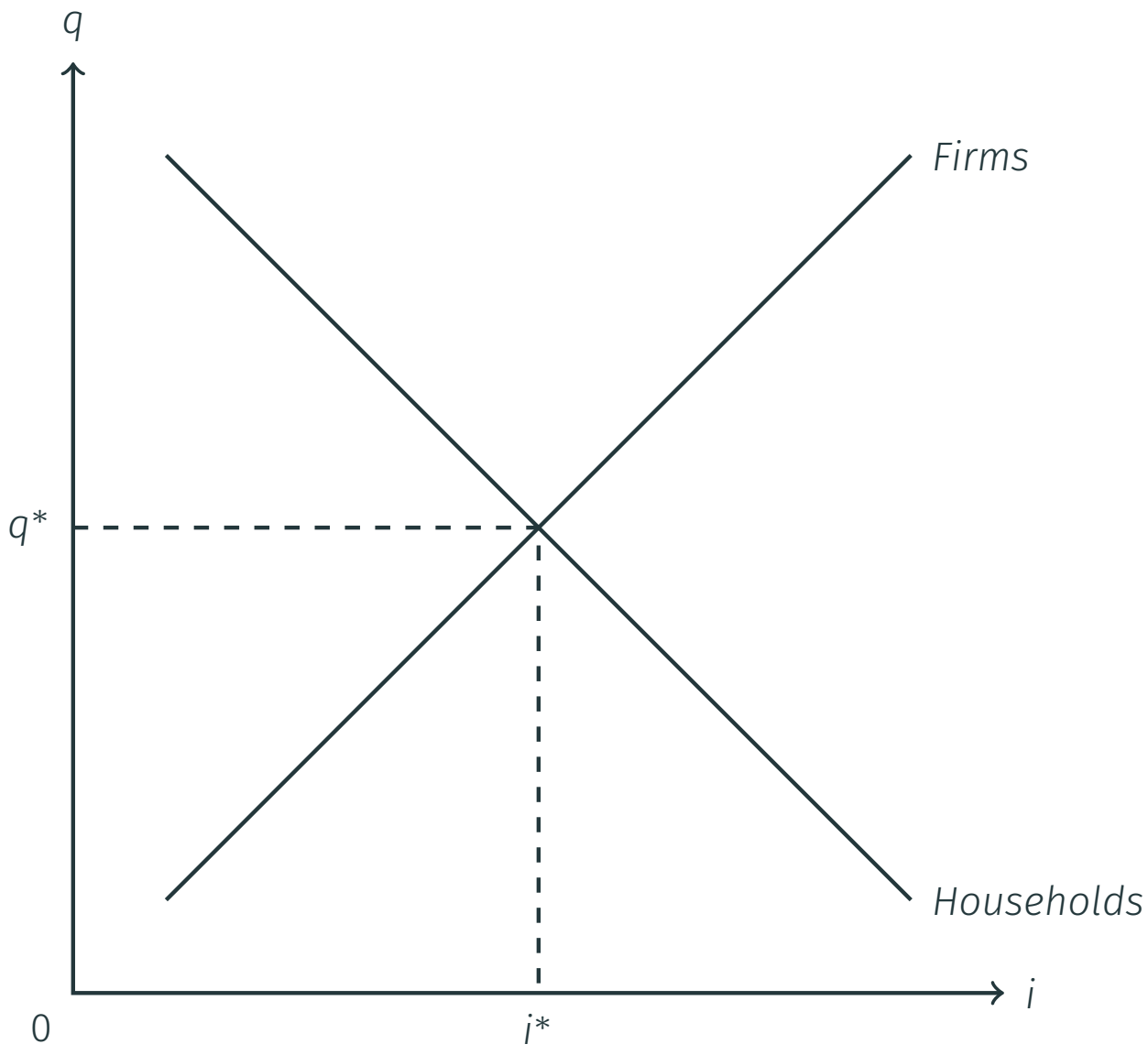


Comparative Statics with Finite Adjusment Cost



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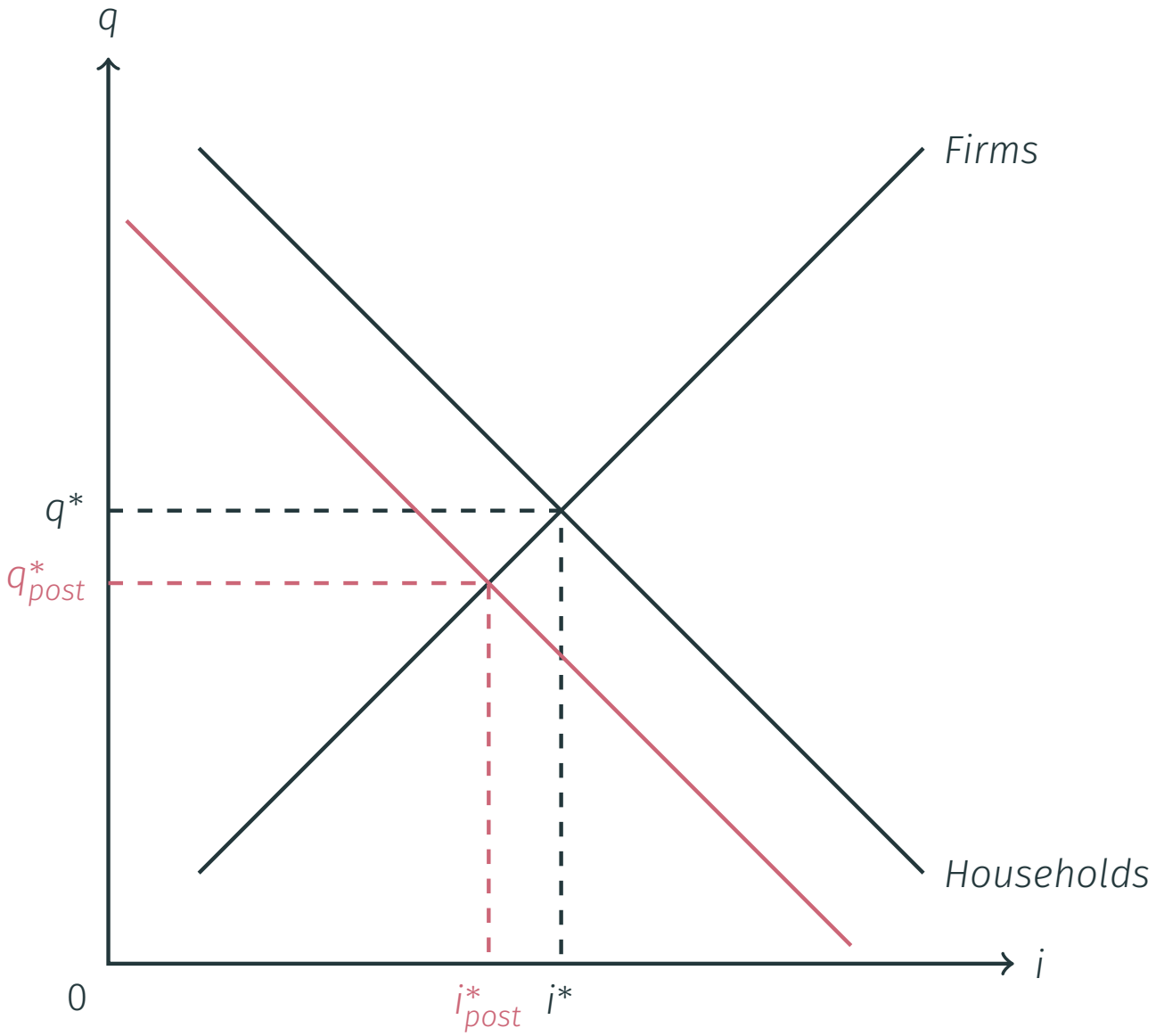

Real Impact of Long Run Risk on Economy

- These two equations allow to see GE impact of a change in σ, or γ...
  If $\psi \geq 1$, increase fundamental risk γ, σ or bad news in future growth leads to an increase in consumption, decrease in investment, and therefore decrease in long run output
- Note that these shocks cannot really explain comovement consumption and investment (one needs correlated shock to capital and shock to news)
- One potential is NK with sticky prices: suppose increase in risk, and central bank does not decrease short term interest rate enough. In this case consumption has to adjust by falling.