Discount Rates

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The old view (1970)

1. Expected returns do not move much over time; stocks returns are unpredictable.
2. Prices move on news of cash-flow.

The modern view

1. Expected returns move a lot over time, stocks are predictable.
2. Prices move on news of discount rate changes.

Price-Dividend Ratio

Static Model: Gordon Growth Formula

- The gross simple return on a stock is given by:

$$1 + R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t}$$
$$\Rightarrow P_t = \frac{D_{t+1} + P_{t+1}}{1 + R_{t+1}}$$

- Iterating forward:

$$P_t = \sum_{k=1}^{K} \frac{D_{t+k}}{R_{t+1} \dots R_{t+k}} + \frac{P_{t+K}}{R_{t+1} \dots R_{t+K}}$$

- Letting $K \to \infty$ yields the dividend discount model (DDM) of stock prices:

$$P_t = \sum_{k=1}^{\infty} \frac{D_{t+k}}{R_{t+1} \dots R_{t+k}}$$

Static Model: Gordon growth formula

- Assume that returns and dividend growth are constant, i.e. $R_{t+k} = R$ an $\mathbb{E}_t[D_{t+k}] = (1+G)^{k-1}\mathbb{E}_t[D_{t+1}]$, with $G < R$. We have

$$P_t = E_t\left[\sum_{k=1}^{\infty} \frac{D_{t+k}}{(1+R)^k}\right] = \sum_{k=1}^{\infty} \frac{(1+G)^k D_t}{(1+R)^k} = D_t \frac{1+G}{R-G}$$

- We obtain Gordon Growth Formula

$$\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{R-G}$$

Static Model: Gordon growth formula

$$\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{R - G}$$

Some implications:

- If divivend growth G increases, prices decrease
- If expected returns R rise, prices decrease
- Small changes in expected returns (or in expected growth) have a large impact in prices if R close to G (i.e. P/D is high)

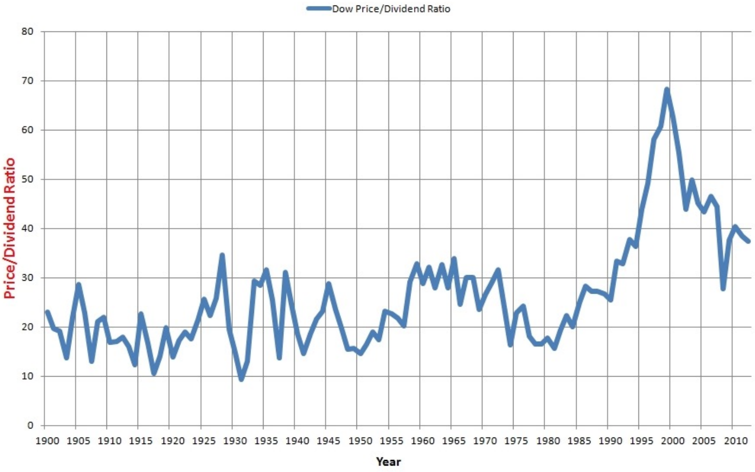
  $$\frac{d \ln P}{dR} = -P/D$$

  In particular, this means that growth stocks should be more volatile.
- Uncertainty about growth rates increases firm value. Intuitively

  $$\mathbb{E}\left[\frac{1}{R - G}\right] > \frac{1}{R - \mathbb{E}[G]}$$

  Pastor Veronesi *Technological Revolutions and Stock Prices*

Dow Price/Dividend Ratio History



Dynamic Model: Campbell-Shiller

- Denote DP_t the Dividend to Price ratio = D_t/P_t
- Definition of returns

$$\begin{aligned} 1 + R_{t+1} &= \frac{D_{t+1} + P_{t+1}}{P_t} \\ &= \frac{D_{t+1}}{D_t} \frac{DP_t}{DP_{t+1}} (1 + DP_{t+1}) \end{aligned}$$

- Assume again that returns and dividend growth are constant, we obtain Gordon Growth formula:

$$\log(1 + DP) = \log(1 + R) - log(1 + G)$$

- Now we are not assuming DP_t constant.

$$\log(1 + DP_{t+1}) = \log(1 + R_{t+1}) - \log(\frac{D_{t+1}}{D_t}) + \log \frac{DP_{t+1}}{DP_t}$$

Denote $dp_t = \log DP_t, r_{t+1} = \log(1 + R_{t+1}), d_{t+1} = \log(D_{t+1})$

$$\log(1 + e^{dp_{t+1}}) = r_{t+1} - \Delta d_{t+1} + \Delta dp_{t+1}$$

- Approximate the function $\log(1 + e^x)$ by linear function:

$$-dp_t = -r_{t+1} + \Delta d_{t+1} + \kappa - \rho dp_{t+1}$$

where

$$\rho = \frac{1}{1 + D/P} \approx 1 - D/P \approx 0.96$$

$$\kappa = -\log(\rho) - (1 - \rho) \log\left(\frac{1}{\rho} - 1\right)$$

- Iterating forward:

$$\begin{aligned}
pd_t &= \Delta d_{t+1} - r_{t+1} + \kappa + \rho(p_{t+1} - d_{t+1}) \\
&= \Delta d_{t+1} - r_{t+1} + \kappa + \rho[\Delta d_{t+2} - r_{t+2} + \kappa + \rho(p_{t+2} - d_{t+2})] \\
&= \sum_{j=0}^{K} \rho^j [\Delta d_{t+j+1} - r_{t+j+1}] + \sum_{j=0}^{K} \rho^j \kappa + \rho^K pd_{t+K} \\
&= \sum_{j=0}^{\infty} \rho^j [\Delta d_{t+j+1} - r_{t+j+1}] + \frac{\kappa}{1-\rho}
\end{aligned}$$

Dynamic Model: Campbell-Shiller

- The Campbell Shiller is an accounting decomposition that holds in any state of the world ex-post
- In particular, it is true in expectation (rational or irrational)

$$pd_t = \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t[\Delta d_{t+j+1}] - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t[r_{t+j+1}] + \frac{\kappa}{1-\rho}$$

What Drives Variations in Price-Dividend Ratio?

- For $K < \infty$:

$$var(pd_t) = cov\left(pd_t, \sum_{j=1}^{K} \rho^{j-1} \Delta d_{t+j}\right) - cov\left(pd_t, \sum_{j=1}^{K} \rho^{j-1} r_{t+j}\right) + \rho^K cov(pd_t, pd_{t+K})$$

- Dividing by the variance of pd_t

$$1 = b_d^{(K)} - b_r^{(K)} + b_{dp}^{(K)}$$

where

  - $b_d^{(K)}$ denotes the slope estimate of a regression of $\sum_{j=1}^{K} \rho^{j-1} \Delta d_{t+j}$ on pd_t
  - $b_r^{(K)}$ denotes the slope estimate of a regression of $\sum_{j=1}^{K} \rho^{j-1} r_{t+j}$ on pd_t
  - $b_{dp}^{(K)}$ denotes the slope estimate of a regression of $\rho^k dp_{t+k}$ on pd_t

What Drives Variations in Price-Dividend Ratio?

Table 1: Results (from Cochrane "The Dog That Did Not Bark")

	Dependent Variable ($k = 15$)		
	$\sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j}$	$\sum_{j=1}^{k} \rho^{j-1} r_{t+j}$	$\rho^k dp_{t+k}$
pd_t	-0.11	-1.01	0.11

- price-dividend does not predict future dividend growth
- An alternative way to state the same result is to say that price-dividend predicts future returns

Variance of Returns

What Drives Variations in Returns?

- Remember

$$-dp_t = -r_{t+1} + \Delta d_{t+1} + \kappa - \rho dp_{t+1}$$

In particular, this is true in expectation:

$$-dp_t = -E_t[r_{t+1}] + E_t[\Delta d_{t+1}] + \kappa - \rho E_t[dp_{t+1}]$$

- Therefore

$$r_{t+1} - E_t[r_{t+1}] = \Delta d_{t+1} - E_t \Delta d_{t+1} - \rho(dp_{t+1} - E_t dp_{t+1})$$

- Taking variance of the expression

$$\underbrace{Var_t(r_{t+1})}_{\approx 16\%^2} = \underbrace{Var_t(\Delta d_{t+1})}_{\approx 9\%^2} + \rho^2 \underbrace{Var_t(dp_{t+1})}_{\approx 13\%^2} - \rho \underbrace{cov_t(\Delta d_{t+1}, dp_{t+1})}_{\approx 0\%}$$

  - The variance of returns is much higher than the variance of dividend growth. This is the same as saying that the price-dividend moves
  - Approximately, variance of returns is 50% due to variance of current dividend growth and 50% due to future price dividend.

What Drives Variations in Returns?

- Using Campbell Shiller decomposition, we can also rewrite the surprise in the price dividend ratio as surprise in future expected dividend growth and surprise in future expected returns:

$$r_{t+1} - \mathbb{E}_t[r_{t+1}] = \Delta d_{t+1} - E_t \Delta d_{t+1} + (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j \Delta d_{t+1+j} - (\mathbb{E}_{t+1} - \mathbb{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}$$

- Therefore, neglecting covariance terms which are small

$$\underbrace{Var_t(r_{t+1})}_{\approx 16\%^2} = \underbrace{Var_t(\Delta d_{t+1})}_{\approx 9\%^2} + \underbrace{Var_t(\mathbb{E}_{t+1}[\sum_{j=1}^{\infty} \rho^j \Delta d_{t+1+j}])}_{\approx 0\%^2} + \underbrace{Var_t(\mathbb{E}_{t+1}[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}])}_{\approx 13\%^2}$$

Approximately, variance returns is 50% due to variance of current dividend growth, 0% due to news about future expected dividend growth, and 50% due to news about future expected returns

What Drives Variations in Expected Returns?

- Suppose expected returns follow an AR(1) process:

$$r_{t+1} = r + x_t + u_{t+1} \tag{1}$$

$$x_{t+1} = \phi x t + \xi_{t+1} \tag{2}$$

- The discount rate component is

$$\begin{aligned}(\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{j=1}^{\infty}\rho^j r_{t+1+j} &= (\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{j=1}^{\infty}\rho^j x_{t+1+j} \\ &= (\mathbb{E}_{t+1} - \mathbb{E}_t)\sum_{j=1}^{\infty}\rho^j \phi^{j-1}\xi_{t+1} \\ &= \frac{\rho\xi_{t+1}}{1-\rho\phi}\end{aligned}$$

- Therefore

$$Var_t(E_{t+1}[\sum_{j=1}^{\infty}\rho^j r_{t+1+j}]) = \frac{\rho^2}{(1-\rho\phi)^2}Var(\xi_{t+1})$$

$\Longrightarrow$ Small but persistent changes in discount rates can generate large fluctuations in asset returns

Are Returns Correlated?

- What is the correlation of returns?

$$
\begin{aligned}
Cov(r_{t+1}, r_{t+2}) &= E[r_{t+1}r_{t+2}] - E[r_{t+1}]E[r_{t+2}] \\
&= E[r_{t+1}E_{t+1}[r_{t+2}]] - E[r_{t+1}E_{t+1}[r_{t+2}]] \\
&= Cov(r_{t+1}, E_{t+1}[r_{t+2}]) \\
&= \underbrace{cov(E_t[r_{t+1}], E_{t+1}[r_{t+2}])}_{>0} + \underbrace{cov(r_{t+1} - E_t[r_{t+1}], E_{t+1}[r_{t+2}])}_{<0}
\end{aligned}
$$

- Using the statistical model given by 1 and 2, and, assuming discount rate shocks and dividend shocks are uncorrelated,

$$
\begin{aligned}
Cov(r_{t+1}, r_{t+2}) &= cov(x_t, x_{t+1}) - cov(\frac{\rho\xi_{t+1}}{1-\rho\phi}, \xi_{t+1}) \\
&= \phi var(x_t) - \frac{\rho}{1-\rho\phi} var(\xi_{t+1}) \\
&= (\frac{\phi}{1-\phi^2} - \frac{\rho}{1-\rho\phi}) Var\xi_{t+1}
\end{aligned}
$$

For ϕ close to $\rho \approx 0.96$, correlation of returns is close to zero

Campbell Shiller at the Firm level

- We can define the an ROE E_{t+1} as

$$E_{t+1} = \frac{B_{t+1} + D_{t+1}}{B_t}$$

- Similarly to Campbell-Schiller, Vuolteenaho (2002) develops a variance decomposition based on ROE and Book/Market similar in spirit to Campbell's (1991) variance decomposition:

$$r_{t+1} - E_t[r_{t+1}] = \sum_{j=0}^{\infty}(E_{t+1} - E_t)(e_{t+1+j} - r_{f,t+1+j}) - \sum_{j=0}^{\infty}(E_{t+1} - E_t)[r_{t+1+j}]$$

  - ROE-based decomposition is better suited for individual securities.
  - Firm level stock returns are driven largely by cash-flow news.

Market Efficiency

Efficient markets

- Two interpretations about predictability of returns
  1. Fama: Rational changes in expected returns required by households to hold risky assets
  2. Shiller: Sentiments (irrational expectations)

Can we Test Efficiency Directly

- One can ask households about returns expectations. Overall, one find that expectation of future returns by households does not predict future returns. This suggests sentiment is important.
- Issues
  - How well do people understand the question?
  - Surveys generally do not include wealthiest households, that have large impact on asset prices