Asset Prices and Wealth Inequality*

Matthieu Gomez †

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Abstract

This paper uses recently available data on the top of the wealth distribution to study the relationship between asset prices and wealth inequality. I document three stylized facts: (1) the share of wealth invested in equity increases sharply in the right tail of the wealth distribution, (2) when stock market returns are high, wealth inequality increases and (3) higher wealth inequality predicts lower future stock returns. These facts correspond to the basic predictions of asset pricing models with heterogeneous agents. Quantitatively, however, standard models with heterogeneous agents cannot fully capture the joint dynamics of asset prices and the wealth distribution. Augmenting the model with additional sources of fluctuations in wealth inequality, namely in the form of time-varying investment opportunities for wealthy households, is crucial to match the observed fluctuations in wealth inequality and in asset prices.

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†Columbia University; mg3901@columbia.edu

1 Introduction

Recent empirical studies have uncovered important fluctuations in wealth inequality over the last century.[1] Volatile stock market returns are a potential candidate to account for these fluctuations. Conversely, a large theoretical literature in asset pricing examines the role of household heterogeneity in shaping asset prices, but seldom considers its implication for wealth inequality. In this paper, I leverage recently available data on wealth inequality to examine empirically and theoretically the interplay between asset prices and the wealth distribution.

I focus on the following mechanism. Risk-tolerant investors hold more risky assets, accumulate more wealth, and disproportionately end up at the top of the wealth distribution. As a consequence, in periods when stocks enjoy large realized returns, investors at the top of the wealth distribution gain more than the rest, i.e. wealth inequality increases. In turn, as a larger share of wealth falls into the hands of risk-tolerant households, the aggregate demand for risk increases, which lowers risk premia and pushes up asset prices, i.e. higher wealth inequality predicts lower future returns. I confirm empirically each step of this mechanism. Wealthy households own more equity: the average household invests 40% of its wealth in equity, while the households in the top 0.01% invest 75%. In line with these magnitudes, in response to a realized stock return of 10%, the wealth share of the top 0.01% increases by 3.5% (7.5% minus 4%). Consistent with the last step of the mechanism, a one standard deviation increase in the wealth share of the top 0.01% predicts lower future excess returns by 5 percentage points.

I then evaluate whether this mechanism can quantitatively account for asset prices and the wealth distribution in equilibrium. I use the reduced form evidence I documented earlier to estimate a state-of-the-art asset pricing model with heterogeneous agents. I find that the standard model cannot jointly match asset prices and the wealth distribution. Specifically, the model cannot generate the high volatility of asset prices without implying an excessive level of inequality compared to the data. To solve this tension, I propose a parsimonious deviation from the standard model. More precisely, I augment the model with fluctuations in the investment opportunities of the rich relative to the poor. These shocks amplify fluctuations in asset prices without changing the average level of inequality, thereby resolving the tension put forth earlier. Furthermore, these shocks can explain why inequality sometimes increases in time of low asset returns, like in the 2000s.

The paper proceeds in three stages. First, I present three new stylized facts on the relation-

[1] See, for instance, Kopczuk and Saez (2004), Piketty (2014), and Saez and Zucman (2016).

ship between asset prices and the wealth distribution. Using the Survey of Consumer Finances, I document a substantial heterogeneity in portfolio holdings within the right tail of the wealth distribution. While the share of wealth invested in equity is essentially flat over the majority of the wealth distribution, it increases sharply within the top percentiles. As noted above, the average household invests 40% of his wealth in equity while the households in the top 0.01% invest 75% of their wealth in equity. Importantly, the disproportional exposure of the households in the top percentiles matters quantitatively for asset prices because these households hold a large fraction of aggregate wealth. This variation is almost entirely driven by differences in the amount that stockholders invest, rather than by participation decisions. In the time series, this heterogeneity implies that realized stock returns generate changes in wealth inequality. I use top wealth shares series constructed from tax filings and from Forbes 400 to estimate the exposure of the top percentiles to stock market returns.[2] I find that this exposure is remarkably in line with the estimates from portfolio holdings: in response to a realized stock return of 10%, the average wealth increases by 4%, while the average wealth for the top 0.01% increases by 7.5%. Therefore, the wealth share of the top 0.01% increases by the difference of 3.5%.

The flip side of this relationship is that, in an economy where inequality is high, the share of wealth owned by risk-tolerant investors is high, and therefore, in equilibrium, risk premia are low. Thus, higher inequality should predict lower future returns. Indeed, in the data, the wealth share of the top 0.01% is a robust predictor of stock market returns. A one standard deviation increase in the wealth share of the top 0.01% predicts lower future excess returns by 5 percentage points over the following year. In particular, the decrease in risk premia at the end of the 20th century is concomitant with a large increase in wealth inequality. The predictive power of the top wealth share is robust to the inclusion of other predictors put forward in the literature.

Second, I examine whether those facts are quantitatively consistent with a state-of-the-art asset pricing model with heterogeneous agents. Specifically, I study a continuous-time, overlapping generations framework where agents differ with respect to their risk aversion and intertemporal elasticity of substitution, along the lines of Gârleanu and Panageas (2015). I study the dynamics of asset prices and of the wealth distribution in the model. The model can qualitatively generate my three stylized facts. Moreover, as in the data, the wealth distribution exhibits a Pareto tail, shaped by the growth rate of the wealth of top investors relative to the rest of the economy.

Yet, quantitatively, the model cannot jointly match asset prices and the wealth distribution. Specifically, the model cannot generate volatile asset prices without implying an excessive level

[2] I use the series of top wealth shares constructed by Kopczuk and Saez (2004) and Saez and Zucman (2016).

of inequality. This is because, to generate volatile asset prices, the model requires a high degree of heterogeneity. Intuitively, large variations in asset prices can come from large variations in the relative wealth shares of different agents or large differences in their demand for assets. Both require a high degree of preference heterogeneity. But this persistent heterogeneity in preferences gives rise to an excessive level of inequality in the long run: calibrations that fit asset prices generate wealth distributions close to Zipf's law, i.e. with a power law exponent close to 1, whereas the wealth distribution in the data exhibits a thinner tail, a power law exponent of 1.5. Importantly, this tension arises independently of the source of preference heterogeneity: it is present whether one considers heterogeneity in risk aversion, in intertemporal elasticity of substitution, or in subjective discount rates.

Third, I propose a parsimonious deviation from the standard model. Specifically, I consider the impact of low-frequency changes in the investment opportunities of the rich relative to the poor. These shocks create exogenous changes in wealth inequality, thereby increasing fluctuations in asset prices. However, the transitory nature of the shocks limits their impact on the long run level of inequality. After incorporating these shocks in my framework, I show that the augmented model can match quantitatively asset prices and wealth inequality. Furthermore, these shocks help explain otherwise puzzling periods in the data, like the increase of wealth inequality in the 2000s, a period of low asset returns. During an episode when the wealthy have predictably better investment opportunities — for instance because of the development of a new technology — there is a persistent rise in wealth inequality. At the same time, because of these good future prospects, wealthy households demand more assets today, which gives rise to low asset returns during the episode.

Overall, these results suggest a strong link between asset prices and wealth inequality. While a number of studies focus on the role of the risk-free rate of return in shaping the wealth distribution, I document a more important role for the rate of return on risky assets.[3] Moreover, I show that a simple closed-circuit view, where aggregate endowment shocks feed through heterogeneous preferences to asset prices, does not give the full picture. It is necessary to consider additional, more specific, shocks to wealth inequality to understand asset prices.

Related Literature. This paper lies at the intersection of several strands of literature in finance and macroeconomics. This paper relates to the large asset pricing literature of models with heterogeneous investors, in particular Dumas (1989), Wang (1996), Basak and Cuoco (1998), Gollier

[3] See, for instance, Piketty (2014) and Acemoglu and Robinson (2015).

(2001), Chan and Kogan (2002), Gomes and Michaelides (2008), Guvenen (2009), and Gârleanu and Panageas (2015). My contribution is to bring these models to the data using the recently available data on the wealth distribution. I argue that the data suggest the existence of additional shocks that redistribute investment opportunities across households. This ties my paper to a growing literature which examines the impact of re-distributive shocks on asset prices, either through technology shocks (Gârleanu et al. (2012), Kogan et al. (Forthcoming), Gârleanu and Panageas (2017)), fluctuating capital share (Lettau et al. (2016), Greenwald et al. (2014)), tax rates (Pastor and Veronesi (2016)) or idiosyncratic shocks (Constantinides and Duffie (1996), Storesletten et al. (2007), Schmidt (2016)).

The paper also relates to a growing literature in household finance which examines the heterogeneity in portfolio choice across the wealth distribution. Guiso et al. (1996), Carroll (2000), Wachter and Yogo (2010), Calvet and Sodini (2014) and Bach et al. (2015) have documented that the share of wealth invested in risky assets increases with wealth. I show that the increase is entirely accounted for by the top percentiles, and that this heterogeneity generates large fluctuations in wealth inequality over time. A number of papers have documented the heterogeneity in consumption exposure between stockholders and non-stockholders (Mankiw and Zeldes (1991), Brav et al. (2002), Malloy et al. (2009), Parker and Vissing-Jørgensen (2009)). This literature shows that the disproportional exposure of stockholders, together with the high volatility of asset prices, can explain the equity premium puzzle. My contribution is to focus on the heterogeneity *within* stockholders and examine whether the heterogeneity can generate a high volatility of asset returns in equilibrium to begin with.

This paper also contributes to the recent literature on wealth inequality. On the empirical side, I rely critically on the recent wealth shares constructed by Kopczuk and Saez (2004) and Saez and Zucman (2016). On the theoretical side, mechanisms generating the Pareto tail of the wealth distribution have been studied in Gabaix (1999) and Gabaix (2009). The relationship between asset prices and the wealth distribution in equilibrium models has been recently discussed in Benhabib et al. (2011), Achdou et al. (2016), Jones (2015) and Cao and Luo (2016). In particular, Eisfeldt et al. (2016) examine the joint equilibrium of asset prices and of the wealth distribution in an economy populated with investors that differ with respect to their level expertise. Relative to this literature, I explore the case of a stochastic economy, where households have different exposures to aggregate shocks, and therefore, where the wealth distribution is stochastic.

In a contemporaneous working paper, Toda and Walsh (2016) use the series on top income shares from Piketty and Saez (2003) to show that fluctuations in *income inequality* negatively predict future excess stock returns. In contrast, I show that fluctuations in *wealth inequality* negatively predicts future excess stock returns, using the series of top wealth shares from Kopczuk and Saez (2004). Fluctuations in income inequality are conceptually unrelated to fluctuations in wealth inequality.[4] Moreover, I examine this interplay between asset prices and wealth inequality within a quantitative model. I also emphasize the importance of other moments regarding the wealth distribution to estimate asset pricing models with heterogeneous agents: the exposure of top wealth shares to stock market returns and the Pareto tail of the wealth distribution.

Road Map The rest of my paper is organized as follows. In Section 2, I document three stylized facts consistent with heterogeneous agents models. In Section 3, I present a standard asset pricing model with heterogeneous agents to interpret these findings. In Section 4, I show that the standard model has difficulty matching asset prices and wealth moments. In Section 5, I propose a parsimonious deviation from the standard model. Section 6 concludes.

2 Three Facts on Asset Prices and Wealth Inequality

I now analyze data about the top of the wealth distribution to document three stylized facts predicted by heterogeneous agents models. In particular, I focus on the following mechanism. Risk-tolerant investors hold more risky assets and disproportionately end up at the top of the wealth distribution; thus, richer households own more risky assets. As a consequence, in periods when stocks enjoy large realized returns, investors at the top of the wealth distribution gain more than the rest; thus, inequality increases. In turn, as a larger share of wealth falls into the hands of risk-tolerant households, the aggregate demand for risk increases, which lowers risk premia and pushes up asset prices; thus, higher inequality predicts lower future returns.

After introducing the data, I document facts reflecting each step of this mechanism.

2.1 Data

In order to analyze the investments and the wealth dynamics of households across the wealth distribution, I combine different data sources.

Equity Investment. I measure the heterogeneity in investment decisions across households using the Survey of Consumer Finances (SCF). The survey is a repeated cross-section of about 4,000

[4] In particular, business cycles fluctuations in income inequality are driven by fluctuations in the aggregate level of realized capital gains, which partly reflects the amount of trades. See, for instance, Saez and Zucman (2016).

households per survey year, including a high-wealth sample. The survey is conducted every three years, from 1989 to 2013. The respondents provide information on their networth, including their investment in public and private equity. I define the equity share as the total investment in equity over networth. I define the set of entrepreneurs as the households with equity held in an actively managed business.[5]

It remains difficult for surveys to capture the very top households. In particular, by design, the Survey of Consumer Finance excludes the households who appear on Forbes Magazine's list of the 400 wealthiest Americans (Kennickell (2009), Saez and Zucman (2016)). Moreover, the SCF is only available every three years since 1989 and it cannot be used to measure business cycle fluctuations in the wealth distribution.

Wealth Share. I am interested in measuring changes in the wealth distribution and their relationship to stock returns. Therefore, I need yearly estimates of the wealth distribution that cover several business cycles. I use two datasets that jointly cover most of the last 100 years.

The first wealth series is the annual series of top wealth shares constructed by Kopczuk and Saez (2004). This series is constructed from estate tax returns, which report the wealth of deceased households. The wealth distribution of the deceased is used to capture the wealth distribution among the living using the mortality multiplier technique, which amounts to weighting each estate tax return by the inverse probability of death (depending on age and gender). The series is constructed using the whole universe of estate tax returns during the 1916-1945 period, and a stratified sample of micro-files for 1965, 1969, 1972, 1975 and 1982-2000.

I supplement the estate tax returns with the list of the wealthiest 400 Americans constructed by Forbes Magazine every year since 1982, which offers an unparalleled view on the right tail of the wealth distribution. The list is created by a dedicated staff of the magazine, based on a mix of public and private information.[6] The total wealth of individuals on the list accounts for approximately 1.5% of total aggregate wealth. By combining the lists over time, I am able to track the wealth of the same individuals over time.[7]

[5] The definition follows Moskowitz and Vissing-Jørgensen (2002).

[6] Forbes Magazine reports: "We pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don't pretend to know what is listed on each billionaire's private balance sheet, although some candidates do provide paperwork to that effect."

[7] This extends the construction of Capehart (2014) to the recent years. Recent empirical studies examining the Forbes 400 list also include Klass et al. (2006) and Kaplan and Rauh (2013).

One data series that has continuous coverage between 1917 and 2012 is Saez and Zucman (2016). The series is constructed from income tax returns. The series builds in smoothing over time to focus on low-frequency fluctuations in wealth. Therefore, it is not the most adequate to examine fluctuations in wealth at the business cycle frequency. Still, I show that my results hold qualitatively with this dataset in Appendix A.2.

Asset Prices. I measure stock returns from the value-weighted CRSP index and risk-free rates from the Treasury Bill rate after 1927. For the period before 1927, I obtain stock returns and risk-free rates from Shiller (2015).[8] I also use the set of predictors constructed in Welch and Goyal (2008), which includes, in particular, the price-dividend ratio.

2.2 Investment in Equity Across the Wealth Distribution

The basic building block of heterogeneous agents models is that there is a group of investors that disproportionately invests in equity. In contrast, if investment in equity were proportional to wealth, movements in stock prices would not generate movements in the wealth distribution and, conversely, movements in the wealth distribution would not generate movements in stock prices.

Figure 1a plots the average equity share within percentile bins across the wealth distribution. The average equity share of 0.4 masks a substantial heterogeneity across households. The equity share is essentially flat at 0.2 over the majority of the wealth distribution, but increases sharply within the top 1%. Figure 1b plots the equity share with respect to the *log* top percentiles, showing that the equity share is approximately linear in the log percentile at the top of the distribution. The figure suggests that the bulk of the heterogeneity is concentrated within the top percentiles. The top percentiles are likely to be important for asset prices because they own a large share of wealth: the vertical red line in Figure 1a shows that half of the total net worth is owned by the households in the top 3%.

Panel A of Table 1 reports the corresponding average equity share in four groups of households: all households, households in the top 1%−0.1%, households in the top 0.1%−0.01%, and households in the top 0.01%. The average equity share for households in the top 0.01% is 0.75, while the average equity share for all households is 0.4; thus, wealthy households hold twice as much equity as the average household.

A stylized fact in the household finance literature is that stock market participation increases with wealth (Vissing-Jørgensen (2002b)). Therefore, the increase in the equity share within the

[8] Available at http://aida.wss.yale.edu/~shiller/data/chapt26.xlsx.

top percentiles could be driven by an increase in the proportion of stockholders (i.e. the extensive margin). However, Panel B of Table 1 shows that the percentage of stockholders is constant within the top percentiles (90%). The increase in the equity share is entirely driven by the increase *within* stockholders. The heterogeneity between stockholders and non-stockholders generates a lot of variations at the bottom of the wealth distribution, but these variations account for a small share of total wealth.

Investment in risky assets comes mainly in two forms: public equity and private equity. Panel A of Table 1 decomposes the increase in equity share across the top percentiles between the two types of equity. The decomposition reveals that the increase in the equity share is mostly driven by an increase in the share of wealth invested in private equity. Panel C of Table 1 shows that the proportion of entrepreneurs increases sharply in the top percentile: the proportion of households with an actively managed business is 78.5% in the Top 0.01%, compared to 10% in the general population.[9] The wealth of these entrepreneurs is mostly invested in their private business. A potential concern is that, if entrepreneurs cannot trade or sell their firms easily, the heterogeneity in private equity holdings may have no impact on stock market prices. However, Panel C of Table 1 shows that entrepreneurs hold large amounts of public equity (15%). Even with illiquid businesses, entrepreneurs can adjust their overall risky holdings at the margin.

These results show that households at the top of the wealth distribution disproportionately invest in equity. This suggest that they are disproportionately exposed to aggregate risk. An important caveat is that we do not observe the details of the equity positions, public or private, and they might have different characteristics, potentially correlated with wealth.[10] To get around this issue, I now turn to time series evidence on top wealth shares.

2.3 Wealth Exposure to the Stock Market Across the Wealth Distribution

Because households across the wealth distribution invest differently in equity, stock market returns generate fluctuations in the wealth distribution. Following a positive return, wealthy households gain more relative to other agents, and therefore wealth inequality increases. I now quantify this mechanism.

To do so, I use wealth series from Kopczuk and Saez (2004) and Forbes 400. I measure the

[9] Similarly, Hurst and Lusardi (2004), using the Panel Study of Income Dynamics (PSID), show that the propensity of entrepreneurship increases sharply with wealth in the top percentiles.

[10] For instance, Roussanov (2010) study preferences such that the exposure to idiosyncratic shocks increases with wealth, but the exposure to aggregate shocks decreases with wealth

exposure of top households to the stock market by regressing the growth of total wealth in a percentile group on excess stock market returns, i.e.,

$$\log \frac{W_{t+1}^{p\to p'}}{W_{t-1}^{p\to p'}} = \alpha + \beta(r_t^M - r_t^f) + \gamma r_t^f + \epsilon_t \tag{1}$$

where $W_t^{p\to p'}$ denotes the total wealth of households between the percentiles p and p' in year t, r_t^M is the stock market return, and r_t^f is the risk-free rate.[11]

The first four columns in Table 2 (Panel A) report the estimates for β, the wealth exposure to the stock market, for four groups of households: all households, households in the top $1 - 0.1\%$, households in the top $0.1\% - 0.01\%$, and households in the top 0.01%. The estimated exposure β increases monotonically with the top percentiles, from $\beta = 0.44$ for the average household, to $\beta = 0.75$ for the households in the top 0.01%. These estimates exactly match the equity shares estimated in Table 1, even though the time periods are different. This is what we expect if a dollar invested in equity has the same exposure as a dollar invested in the stock market.

The last two columns of Panel A in Table 2 report the wealth exposure of the Top 400 and of the Top 100 from Forbes. The estimates for the households in the extreme tail of the distribution are similar in magnitude to the estimates for the households in the top 0.01% from tax data.

Since top households are comparatively more exposed to the stock market, high stock market returns increase inequality. Panel B of Table 2 confirms this relationship by regressing top wealth *shares* on stock market returns. The estimate 0.31 corresponds to the difference of exposure between households at the top and the average household $(0.75 - 0.44)$.

Some forces might drive a distinction between the exposure of top wealth shares and the relative exposure of individuals in the top percentiles. This is because top percentiles do not necessary include the same individuals over time. In particular, some fluctuations in top wealth shares may be generated by fluctuations in the size of idiosyncratic shocks. Intuitively, when the variance of idiosyncratic shocks increases, top wealth shares increase through a composition effect. If changes in idiosyncratic variance are positively correlated with stock returns, this results in a positive bias.

I address this bias in two ways. First, I use the panel dimension of Forbes 400 to track the same individuals over time. Appendix Table A1 compares the exposure of the wealth of households in the Top 40 to the exposure of the individual households inside the Top 40. The estimates are similar (0.71 vs 0.74). Second, to examine the magnitude of the bias in estate tax returns, I control in

[11] The L.H.S. is the growth of $W^{p\to p'}$ between $t-1$ and $t+1$, to avoid overlapping time periods. This is because $W_t^{p\to p'}$ is the average of wealth owned by the group over the year, rather than the wealth at a given point in time. See also the notes in Table 2.

regression (1) for changes in idiosyncratic variance, as measured by the changes in the idiosyncratic variance of firm level stocks.[12] Appendix Table A2 shows that changes in idiosyncratic variance have a positive, non-significant effect on top wealth shares. In particular, the inclusion of this control does not impact the estimate for the exposure of top wealth shares to stock returns, β.

The reason why this composition effect turns out to be small empirically is that the wealth distribution is very unequal. Intuitively, because wealth is so concentrated, fluctuations due to entry and exit at the bottom of the percentile are small relative to fluctuations in the wealth of households inside the percentile. Formally, for a wealth distribution Pareto-distributed with power law exponent ζ, I show in Appendix B.3 that a rise in idiosyncratic variance $\Delta\sigma^2$ increases the growth of top wealth shares by $(\zeta - 1)\Delta\sigma^2/2$ in the following year. Because $\zeta \approx 1.5$ for the wealth distribution in the U.S., the formula says that a one standard deviation rise in the idiosyncratic variance, $\Delta\sigma^2 = 0.05$, increases top wealth shares by 0.5% in the following year. This is much smaller than the impact of a one standard deviation rise in stock prices, $r_t^M = 0.17$, which increases top wealth shares by 6%, as measured in Table 2. Therefore the potential bias due to changes in idiosyncratic volatility is small.[13]

2.4 Top Wealth Shares Predict Future Excess Returns

The previous evidence suggests that wealthy households are more willing to take on aggregate risk. As top wealth shares increase, wealth is rebalanced from risk-averse households to risk-tolerant households, and, therefore, the total demand for risk in the economy increases. In equilibrium, the compensation for holding risk decreases. Hence, higher top wealth shares should predict lower future excess returns.

I estimate the predictive power of top wealth shares by regressing excess stock returns on the wealth share of the top 0.01%, i.e.,

$$\sum_{1\leq h\leq H} r_{t+h}^M - r_{t+h}^f = \alpha + \beta_H \text{Log Wealth Share Top } 0.01\%_t + \epsilon_{Ht} \tag{2}$$

where h denotes the horizon.

The first line in Table 3 reports the results of the predictability regression at the one-year and three-year horizons. The estimates are statistically and economically significant. A one standard

[12]To the best of my knowledge, there is no time series on the idiosyncratic variance of the wealth growth of households.

[13]This result is consistent with Gabaix et al. (2016). They show that changes in idiosyncratic variance generate slow transition dynamics.

deviation increase in the log of the wealth share of the top 0.01% is associated with a decrease of excess returns by 5 percentage points over the next year.

Figure 2 plots the wealth share of the top 0.01% along with a moving average of excess stock returns over the following eight years. Fluctuations in the wealth share of the top 0.01% do a particularly good job at tracking the low-frequency fluctuations in excess stock returns. Excess stock returns were low when inequality was high in the 1920s. Excess stock returns increased following the decrease in inequality in the 1930s, and decreased following the increase in inequality in the 1980s.

The fact that wealth inequality mostly captures the low-frequency fluctuations in excess returns is not surprising. This is because wealth inequality is persistent. Using the Dickey-Fuller generalized least squares (DF-GLS) test, one cannot reject that the series of the wealth share of the top 0.01% has a unit root. A natural concern is that, in this case, conventional t-statistics are misleading (Elliott and Stock (1994), Stambaugh (1999)). To address this concern, I rely on a test developed in Campbell and Yogo (2006), which is valid when the predictor variable has a largest root close to, or even larger than, one. The results of this test, reported in Table A5, show that the wealth share of the top 0.01% still significantly predicts returns, even though the evidence becomes thinner as one allows for explosive dynamics in the predictor.

Finally, I assess whether the information in the wealth share of the top 0.01% is subsumed by other predictors put forward in previous literature. I use the list of predictor variables constructed in Welch and Goyal (2008). For each regressor, I report the β_1, β_2 as well as the R^2 corresponding to the following bivariate predictive regression

$$\sum_{1 \leq h \leq H} r^M_{t+h} - r^f_{t+h} = \alpha + \beta_H \text{Log Wealth Share Top } 0.01\%_t + \gamma_H \text{Predictor}_t + \epsilon_{Ht} \tag{3}$$

Table 3 summarizes the results. The first column reports the coefficient on the wealth share of the top 0.01%, the second column reports the coefficient on the other predictor, and the last column reports the adjusted R^2 of the regression. While the first three columns report the results with $H = 1$, the last three columns report the results with $H = 3$. To facilitate the comparison between the different predictors, all regressors are normalized to have a standard deviation of one. The table shows that the predictive power of the wealth share of the top 0.01% is robust to the inclusion of other predictors. In particular, the estimate for β_1 remains stable across the different specifications.

I have shown that households at the top of the wealth distribution invest disproportionately in equity, that fluctuations in stock prices generate fluctuations in inequality, and, in turn, that the level of inequality determines future excess returns. Those three facts are at the heart of asset pricing models with heterogeneous agentss. I now examine the quantitative properties of these models.

3 An Asset Pricing Model with Heterogeneous Preferences

I consider a continuous-time pure-exchange economy. I present a model where overlapping generations of households differ in their preferences. The baseline model builds on Gârleanu and Panageas (2015). I derive the behavior of asset prices and characterize the properties of the weatlh distribution.

3.1 Structure

Endowment. I consider a continuous-time pure exchange economy. I assume that the aggregate endowment exhibits i.i.d. growth. Its law of motion is

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dZ_t$$

where Z_t is a standard Brownian motion.

Demographics. The specification of demographics follows Blanchard (1985). The economy is populated by a mass one of agents. Each agent faces a constant hazard rate of death $\delta > 0$ throughout his life. During a short time period dt, a mass δdt of the population dies and a new cohort of mass δdt is born, so that the total population stays constant.

Labor Income. An agent i born at time $s(i)$ is endowed with the labor income process $L_i = \{L_{it} : t \geq s(i)\}$, given by

$$L_{it} = \omega Y_t \times \xi_i \times G(t - s(i)) \tag{4}$$

The first term of this formula, ωY_t, corresponds to the fraction of the aggregate endowment distributed as labor income. The second term, ξ_i, is an individual specific level of income. I assume that it is i.i.d. across agents, with mean 1. The value of ξ_i is realized at birth. This component captures the heterogeneity in labor income within a generation.

The third term, $G(t - s)$, captures the life-cycle profile of earnings of households. The function G is normalized so that aggregate earnings equal ωY_t at each point in time, i.e.

$$\int_{-\infty}^{t} \delta e^{-\delta(t-s)} G(t-s) ds = 1$$

The rest of the endowment $(1 - \omega)Y_t$ is paid by claims to the representative firm.

Preferences. Agents have recursive preferences as defined by Duffie and Epstein (1992). They are the continuous-time versions of the recursive preferences of Epstein and Zin (1989). For an agent i with a consumption process $C_i = \{C_{it} : t \geq 0\}$, his utility $U_i = \{U_{it} : t \geq 0\}$ is defined recursively by:

$$U_{it} = E_t \int_t^{+\infty} f_i(C_{iu}, U_{iu}) du$$

$$f_i(C, U) = \frac{\rho + \delta}{1 - \frac{1}{\psi_i}} \left(\frac{C^{1-\frac{1}{\psi_i}}}{((1-\gamma_i)U)^{\frac{\gamma_i - \frac{1}{\psi_i}}{1-\gamma_i}}} - (1-\gamma_i)U \right)$$

These preferences are characterized by three parameters. The subjective discount factor is ρ, the coefficient of relative risk aversion is γ_i and the elasticity of intertemporal substitution (EIS) is ψ_i.

I assume there are two types of agents, labeled A and B, that differ with respect to their coefficient of relative risk aversion γ_i and their elasticity of intertemporal substitution ψ_i. I denote A the risk-tolerant agent, i.e. $\gamma_A > \gamma_B$. At every point in time a proportion π_A of newly born agents are of type A.

Markets. Households can trade two assets. First, they can trade claims to the representative firm. They can also trade instantaneous risk-free claims in zero net supply. The price of both of those claims is determined in equilibrium. Denote r_t the risk-free rate and dR_t the return of a dollar invested in the representative firm:

$$dR_t = \mu_{Rt} dt + \sigma_{Rt} dZ_t$$

Household Problem. Denote W_{it} the financial wealth of agent i at time t. As in Blanchard (1985), agents can access a market for annuities. There are life insurance companies that collect the agents' financial wealth when they die. In exchange, agents receive an income stream equal to δW_{it} per unit of time.

The problem of households is as follows. An household i born at time $s(i)$ chooses a consumption path $C_i = \{C_{it} : t \geq s\}$ and an amount of dollars invested in the representative firm $\theta_i = \{\theta_{it} : t \geq s\}$ to maximize his lifetime utility

$$V_{it} = \max_{C_i, \theta_i} U_{it}(C_i)$$

subject to the dynamic budget constraint

$$dW_{it} = (L_{it} - C_{it} + (r_t + \delta) W_{it} + \theta_{it}(\mu_{Rt} - r_t))dt + \theta_{it}\sigma_{Rt} dZ_t \text{ for all } t \geq s$$

I now make a useful change of variables. Because markets are dynamically complete, there is a unique stochastic discount factor η_t:

$$\frac{d\eta_t}{\eta_t} = -r_t dt - \kappa_t dZ_t$$

where r_t is the risk free interest rate and κ_t is the price of aggregate risk. The expected return of a dollar invested in the representative firm can be written:

$$\mu_{Rt} = r_t + \kappa_t \sigma_{Rt}$$

Households are not subject to liquidity constraints; hence, they can sell their future labor income stream and invest the proceeds in financial claims. Define N_{it}, the total wealth of household i as the sum of his financial wealth and his human capital, i.e. the present value of his labor income:

$$N_{it} = W_{it} + E_t[\int_{u=t}^{+\infty} e^{-\delta(u-t)} \frac{\eta_u}{\eta_t} L_{ts(i)}]$$

In particular, denote $\xi_i \phi_t Y_t$ the wealth of a newborn agent, i.e.,

$$\phi_t = E_t[\int_{t}^{+\infty} e^{-\delta(u-t)} \frac{\eta_u}{\eta_t} \omega \frac{Y_u}{Y_t} G(u-t)]$$

The household problem can now be reformulated as follows. Household i chooses a consumption rate $c_i = \{c_{it} = C_{it}/N_{it} : t \geq s(i)\}$ and a wealth exposure to aggregate shocks $\sigma_i = \{\sigma_{it} : t \geq s(i)\}$ such that for all $t \geq s(i)$

$$V_{it} = \max_{c_i, \sigma_i} U_{it}(c_i N_i) \tag{5}$$

$$\text{s.t. } \frac{dN_{it}}{N_{it}} = \mu_{it} dt + \sigma_{it} dZ_t$$

$$\text{with } \mu_{it} = r_t + \delta + \kappa_t \sigma_{it} - c_{it} \tag{6}$$

$$\text{and } N_{is(i)} = \xi_i \phi_{s(i)} Y_{s(i)} \tag{7}$$

Equilibrium. Informally, an equilibrium is characterized by a map from shock histories Z_t to prices and asset allocations such that, given prices, agents maximize their expected utilities and markets clear. Denote $\mathbb{I}_A = [0, \pi_A]$ the set of agents in group A and $\mathbb{I}_B = [1 - \pi_A, 1]$ the set of all agents in group B.

Denote $p_t Y_t$ the total wealth in the economy. It is the sum of the firm valuation and the human capital of existing agents. Conjecture that p_t follows an Ito process:

$$\frac{dp_t}{p_t} = \mu_{pt} dt + \sigma_{pt} dZ_t$$

Definition 1. *An equilibrium is a set of stochastic processes for the interest rate* $r = \{r_t; t \geq 0\}$*, market price of risk* $\kappa = \{\kappa_t : t \geq 0\}$*, consumption and investment decisions* $c_i = \{c_{it} : t \geq 0\}, \sigma_i = \{\sigma_{it} : t \geq 0\}$ *such that*

1. (c_i, σ_i) *solve* (5) *given* (r, κ)

2. *Markets clear*

$$\int_{i \in \mathbb{I}_A} N_{it} c_{it} di + \int_{i \in \mathbb{I}_B} N_{it} c_{it} di = Y_t \text{ (Consumption)} \tag{8}$$

$$\int_{i \in \mathbb{I}_A} N_{it} \sigma_{it} di + \int_{i \in \mathbb{I}_B} N_{it} \sigma_{it} di = p_t Y_t (\sigma + \sigma_{pt}) \text{ (Risky asset)} \tag{9}$$

By Walras's law, the market for risk-free debt clears automatically.

3.2 Solving the Model

Solution Method. All households with the same preference parameters face the same trade-off, irrespective of their wealth or age. This is because their utility function is homogeneous and their death rate is constant over time. In particular, the consumption rate c_{it} and the wealth exposure σ_{it} are the same for all agents with the same preferences. For the purpose of determining prices, we can abstract from the distribution of wealth within each group: we only need to keep track of the share of aggregate wealth that belongs to the agent in group A:

$$x_t = \frac{\int_{i \in \mathbb{I}_A} N_{it} di}{\int_{i \in \mathbb{I}_A} N_{it} di + \int_{i \in \mathbb{I}_B} N_{it} di} \tag{10}$$

I restrict my attention to Markov equilibrium where all processes are functions of x_t only. We have four policy functions $c_{jt}, \sigma_{jt}, j \in \{A, B\}$, two value functions, two prices κ_t and r_t, and two valuations (firm value and human capital) to solve for. The four first order conditions, the two HJB equations, the two market clearing condition, and the no arbitrage condition for the firm value and for human capital are enough to derive the equilibrium. In the Appendix Appendix B.2, I reduce the system to a system of PDEs. I solve this system of PDEs using an implicit time-stepping scheme.[14]

I now turn to two particular parts of the equilibrium.

Evolution of Household Wealth. I first characterize the law of motion of households' wealth. This law of motion will determine the law of motion of the wealth distribution, which I observe

[14] More details about the solution method can be found at https://github.com/matthieugomez/EconPDEs.jl.

in the data. Denote p_{jt} the wealth-to-consumption ratio of an agent in group $j \in \{A, B\}$. In equilibrium, the process p_{jt} follows an Ito process

$$\frac{dp_{jt}}{p_{jt}} = \mu_{p_j t} dt + \sigma_{p_j t} dZ_t$$

The following proposition characterizes the law of motion of the wealth of households within each group:

Proposition 1 (Law of Motion for Households Wealth)**.** *The wealth of households in group $j \in \{A, B\}$ follows the law of motion*

$$\frac{dN_{jt}}{N_{jt}} = \mu_{jt} dt + \sigma_{jt} dZ_t$$

where μ_{jt} and σ_{jt} are given by

$$\sigma_{jt} = \frac{\kappa_t}{\gamma_j} + \underbrace{\frac{1-\gamma_j}{\gamma_j(\psi_j - 1)} \sigma_{p_j t}}_{H_{it}} \tag{11}$$

$$\mu_{jt} = \psi_j (r_t - \rho) + \frac{1+\psi_j}{2\gamma_j} \kappa_t^2 + \underbrace{\frac{1-\gamma_j}{\gamma_j(\psi_j - 1)} \kappa_t \sigma_{p_j t} + \frac{1-\gamma_j \psi_j}{2(\psi_j - 1)\gamma_j} \sigma_{p_j t}^2 + \mu_{p_j t}}_{\Phi_{it}} \tag{12}$$

The volatility of wealth, σ_{jt}, has two components: a myopic demand and a demand due to time-varying investment opportunities. The myopic demand equals the ratio of the market price of risk to the risk aversion. The lower the risk aversion, the higher the myopic demand. The hedging demand H_{it} captures deviations from the mean-variance portfolio due to variations in investment opportunities. In the calibrations explored below, this term will be positive because expected returns are countercyclical.

The drift of wealth, μ_{jt} has three components. The first term is the standard term due to intertemporal substitution, determined by the EIS ψ_i and the difference between the interest rate r_t and the subjective discount factor ρ: $\psi_j(r_t - \rho)$. The second term comes from risky assets. Agents with a lower risk aversion invest disproportionately in risky assets and therefore earn higher returns. Hence, their wealth grows at a faster rate than the rest of the economy. The third term Φ_{it} captures changes in investment opportunities.

In short, the proposition suggests that agents with a lower risk aversion invest more in risky assets and grow faster than the rest of the households. Therefore, we naturally obtain my first two stylized facts: agents at the top of the distribution buy more risky assets and households at the top are more exposed to aggregate shocks compared to the rest of the households.

In particular, because agents in group A choose a different wealth exposure compared to agents in group B, the share of wealth owned by agents in group A, x_t, is stochastic. The next proposition characterizes the law of motion of x_t:

Proposition 2. *The law of motion of x is*

$$\frac{dx_t}{x_t} = \mu_{xt}dt + \sigma_{xt}dZ_t$$

where μ_{xt} and σ_{xt} are given by

$$\sigma_{xt} = (1 - x_t)(\sigma_{At} - \sigma_{Bt}) \tag{13}$$

$$\mu_{xt} = (1 - x_t)(\mu_{At} - \mu_{Bt}) + (1 - x)\frac{\phi_t}{p_t}\delta(\frac{\pi_A}{x_t} - \frac{1 - \pi_A}{1 - x_t}) - (\sigma + \sigma_{pt})\sigma_{xt} \tag{14}$$

The volatility of x_t is directly related to the difference between the wealth volatility of the agents in group A and the wealth volatility of the agents in group B.

The drift of x_t is the sum of three terms. The first term is the difference between the wealth drift of the agents in group A and the wealth drift of the agents in group B. The second term corresponds to the birth of individuals in group A compared to the birth of individuals in group B. The third term corresponds to an Ito correction term.

Market price of risk. The third step of our basic mechanism is that, when more wealth falls into the hands of risk tolerant households, stock prices increase and future returns are lower. To gain some intuition on this relationship in the model, I now consider the determination of the equilibrium price of risk κ_t.

Because all agents within the same group choose the same exposure to aggregate shocks, the market clearing for risky assets (9) can be written:

$$x_t\sigma_{At} + (1 - x_t)\sigma_{Bt} = \sigma + \sigma_{pt} \tag{15}$$

This market clearing simply says that the volatility of aggregate wealth is the wealth-weighted average of the volatility of the wealth of individual agents.

Substituting out the optimal choice σ_{jt} for households in group $j \in \{A, B\}$ given by (11), we obtain the market price of risk κ_t:

$$\kappa_t = \Gamma_t(\sigma + \sigma_{pt} - H_t) \tag{16}$$

where Γ_t corresponds to the aggregate risk aversion and H_t corresponds the total hedging demand

$$\Gamma_t \equiv 1/(\frac{x_t}{\gamma_A} + \frac{1-x_t}{\gamma_B})$$
$$H_t = x_t H_{At} + (1-x_t) H_{Bt}$$

The market price of risk κ_t is the product of the aggregate risk aversion Γ_t times the total quantity of risk $\sigma + \sigma_{pt}$, minus the total demand for risk due to the hedging H_t.

The aggregate risk aversion Γ_t is a wealth-weighted harmonic mean of individual risk aversions. The higher the share of wealth owned by the agents in group A, x, the lower the aggregate risk aversion Γ_t. Therefore, ignoring for a moment the hedging demand, an increase in the fraction hold by x_t decreases the market price of risk κ_t. This corresponds to the predictive regression of Section 2.4.

3.3 The Wealth Distribution

The model has sharp implications on the wealth distribution. In this section, I analyze the distribution of relative wealth. Denote households' relative wealth n_{it}, i.e.

$$n_{it} = \frac{N_{it}}{p_t Y_t}$$

By Ito's lemma, the law of motion of the relative wealth n_{it} is

$$\frac{dn_{it}}{n_{it}} = \tilde{\mu}_{it} dt + \tilde{\sigma}_{it} dZ_t$$

where $\tilde{\mu}_{it}$ and $\tilde{\sigma}_{it}$ are given by

$$\tilde{\sigma}_{it} = \sigma_{it} - \sigma - \sigma_{pt}$$
$$\tilde{\mu}_{it} = \mu_{it} - \mu - \mu_{pt} - \sigma\sigma_{pt} - (\sigma + \sigma_{pt})\tilde{\sigma}_{it}$$

Law of Motion of Wealth Density. I first characterize the dynamics of the wealth density in the model. Denote ψ_t the wealth distribution of newborn agents.[15]

[15] It corresponds to the distribution of human capital relative to average wealth. Denote ψ the density for ξ_i in (4). The expression for ψ_t is:

$$\psi_t(n) = \frac{p_t}{\phi_t}\psi(\frac{p_t}{\phi_t} n)$$

Proposition 3 (Kolmogorov Forward Equation with Aggregate Shocks)**.** *Denote g_{jt} the density of relative wealth within each group of agent $j \in \{A, B\}$. The law of motion of g_{jt} is given by*

$$dg_{jt}(n) = -\partial_n \left((\tilde{\mu}_{jt} dt + \tilde{\sigma}_{jt} dZ_t) n g_{jt}(n) \right) + \frac{1}{2} \partial_n^2 (\tilde{\sigma}_{jt}^2 n^2 g_{jt}(n)) dt + \delta(\psi_t(n) - g_{jt}(n)) dt$$

Denote g_t the density of relative wealth. We have

$$g_t(n) = \pi_A g_{At}(n) + (1 - \pi_A) g_{Bt}(n)$$

Given g_{jt}, the wealth distribution within each group $j \in \{A, B\}$, and the evolution of individual wealth $(\tilde{\mu}_{jt} dt, \tilde{\sigma}_{jt} dZ_t)$, the Kolmogorov Forward equation gives the wealth distribution tomorrow $g_{j,t+dt}$. The drift and volatility of individual wealth $(\tilde{\mu}_{jt}, \tilde{\sigma}_{jt})$ jointly determine the law of motion of the wealth distribution. In particular, because households choose different exposures to aggregate shocks, their relative wealth is stochastic, (i.e. $\tilde{\sigma}_{jt} \neq 0$), and therefore the wealth distribution is stochastic.

Law of Motion of Top Wealth Shares. I now integrate the Kolmogorov Forward equation to obtain the law of motion of top wealth shares. While the Kolmogorov Forward equation gives the law of motion of the *wealth density*, I now obtain the law of motion of *top wealth shares*. This makes it easier to relate the model to the data, because I only observe the dynamics of top wealth shares over time.

Let α a number between 0 and 1. Denote q_t the $\alpha-$quantile, i.e.,

$$\alpha = \int_{q_t}^{+\infty} g_t(n) dn$$

and denote T_t the wealth share of the top α, i.e.,

$$T_t = \int_{q_t}^{+\infty} n g_t(n) dn$$

For instance, for $\alpha = 1\%$, q_t is the wealth of an agent exactly at the 1% percentile of the distribution and T_t is the wealth share of the top 1%.

The following proposition characterizes the dynamics of T_t:

Proposition 4 (Law of Motion of Top Wealth Shares)**.** *The law of motion of the top wealth share T_t is*

$$\frac{dT_t}{T_t} = \mu_{Tt} dt + \sigma_{Tt} dZ_t$$

where μ_{Tt} and σ_{Tt} are given by

$$\sigma_{Tt} = \int_{n=q_t}^{+\infty} (\sigma_{At}\pi_A g_{At}(n) + \sigma_{Bt}(1-\pi_A) g_{Bt}(n)) n dn$$

$$\begin{aligned}\mu_{Tt} = & \int_{n=q_t}^{+\infty} (\mu_{At}\pi_A g_{At}(n) + \mu_{Bt}(1-\pi_A) g_{Bt}(n)) n dn \\ & + \underbrace{\frac{1}{2}\frac{q_t^2 g_t(q_t)}{T_t}\left(\sum_{j\in\{A,B\}} \sigma_{jt}^2 \pi_j g_{jt}(q_t)/g_t(q_t) - \left(\sum_{j\in\{A,B\}} \sigma_{jt}\pi_j g_{jt}(q_t)/g_t(q_t)\right)^2\right)}_{\textit{Heterogeneous Exposure}} \\ & \underbrace{- \delta(1 - \frac{\alpha q_t}{T_t})}_{\textit{Death}} + \underbrace{\frac{\delta}{T_t}\int_{n=q_t}^{\infty} (n - q_t)\psi_t(n) dn}_{\textit{Birth}}\end{aligned}$$

The volatility of the top wealth share, σ_{Tt}, is the average, wealth-weighted, volatility of individuals in the top percentile.

The drift of the top wealth share, μ_{Tt}, is the sum of four terms. The first term corresponds to the average, wealth-weighted, drift of individuals at the top. The second term is due to the death of individuals at the top. The third term is due to the birth of individuals at the top. The last term is due to the heterogeneous exposure of households at the threshold.

I now give an heuristic derivation for the death term. During a short time period dt, a mass $\alpha\delta dt$ of households in the top percentile die, which decreases the total wealth in the top percentile by $T_t\delta dt$. Because the population size in the top percentile is held constant, an equal mass of households at the threshold enter the top percentile, with a wealth q_t. Therefore, the total change in top wealth share T_t due to death is $-\delta dt(T_t - \alpha q_t)$.

I now give an heuristic derivation for the birth term. During a short time period dt, a mass $\int_{q_t}^{+\infty} \psi_t(n) dndt$ of households are born in the top percentile, which increases the total wealth in the top percentile by $\int_{q_t}^{+\infty} n\psi_t(n) dndt$. Because the population size in the top percentile is held constant, an equal mass of households at the threshold exit the top percentile, with a wealth q_t. Therefore, the total change in top wealth share T_t due to birth is $\delta dt(\int_{q_t}^{+\infty}(n - q_t)\psi_t(n)dn)$.

The fourth term depends on the variance of risk exposures across households at the quantile q_t. When a *negative* shock hits the economy, top wealth shares decrease a little bit *less* than the wealth of households inside the top percentile, because some households from group B enter the top. Conversely, when a *positive* shock hits the economy, top wealth shares increase a little bit *more* than the wealth of households inside the top percentile, because some households from group A enter the top. As seen empirically in Section 2, because wealth is so concentrated, the impact of these fluctuations due to entry and exit is small.

The point I emphasize is that death is a key stabilizing force for top wealth shares. Because, agents at the top grow faster than the rest of the economy (i.e. $\tilde{\mu}_{At} \geq 0$), a model without death would feature explosive dynamics for top wealth shares.[16]

Pareto Tail. While a full characterization of the entire wealth distribution is not feasible, one can characterize relatively simply its right tail. In particular, I give certain conditions under which the stationary wealth distribution has a Pareto tail.

Definition 2. *A distribution g_t has asymptotically a right Pareto tail if there exists $\zeta > 0$ such that moments of order higher than ζ are asymptotically infinite, i.e. for $\xi \geq 0$*

$$\lim_{t \to +\infty} \int_0^{\infty} n^{\xi} g_t(n) dn = +\infty \text{ iff } \xi \geq \zeta$$

ζ is called the power law exponent of the distribution

I obtain a characterization of the power law exponent of the distribution in term of the average wealth growth of households in the top relative to the rest of the economy:

Proposition 5 (Pareto Tail)**.** *Denote*

$$\zeta = \frac{\delta}{E^x[\tilde{\mu}_A(x) - \frac{1}{2}\tilde{\sigma}_A(x)^2]}$$

where E^x denotes the expectation with respect to the stationary distribution of the state variable x.

If the following conditions are satisfied:

1. *Agents in group A grow faster than the economy, i.e.*

$$E^x[\tilde{\mu}_A(x) - \frac{1}{2}\tilde{\sigma}_A(x)^2] > \max(0, E^x[\tilde{\mu}_B(x) - \frac{1}{2}\tilde{\sigma}_B(x)^2])$$

2. *The stationary distribution of human capital has a tail thinner than ζ*

Then the wealth distribution has a right Pareto tail with power law exponent ζ. Moreover, the fraction of agents that are of type B tends to zero in the right tail of the distribution.

The higher the wealth growth of households at the top, the lower the power law exponent of the wealth distribution.

[16] See Gomez (2018) for a closer examination of the wealth dynamics of top households in the last fourty years.

4 Estimating the Model on Asset Prices and on the Wealth Distribution

I now bring the model to the data. I find that the model qualitatively generates the three stylized facts documented in Section 2. However, to generate volatile asset prices, the model requires a wealth distribution with a tail much thicker than the data. There is a key tension between matching quantitatively asset prices and the wealth distribution.

4.1 Estimation Approach

Method. I estimate the parameters of the model by the simulated method of moments (SMM), which minimizes the distance between moments from real data and simulated data. I proceed as follows. I select a vector of moments m computed from the actual data. Given a candidate set of parameters Θ, I solve the model, and compute the moments $\hat{m}(\Theta)$. I search the set of parameters $\hat{\Theta}$ that minimizes the weighted deviation between the actual and simulated moments

$$\hat{\Theta} = \arg\min_{\Theta}\{(m - \hat{m}(\Theta))' W (m - \hat{m}(\Theta))\} \tag{17}$$

where the weight matrix W adjusts for the fact that some moments are more precisely estimated than others.[17] Details on the simulation method are given in Appendix C.

I use the following set of moments.

Asset Prices Moments. I use four asset prices moments, corresponding to the average and standard deviation of the risk-free rate and of stock market returns, following Gârleanu and Panageas (2015). The data for the average equity premium, the volatility of returns, and the average interest rate are from Shiller (2015). The volatility of the real risk-free rate is inferred from the yields of 5-year constant maturity TIPS, as reported by Gârleanu and Panageas (2015).

Wealth Moments. I consider three moments about the wealth distribution. The first two moments capture the joint dynamics of the wealth distribution and asset returns, which corresponds to the stylized facts of Section 2. The third moment captures the average shape of the wealth distribution. As explained in Section 3.3, I focus on the right tail of the distribution.

The first moment is the elasticity of top wealth shares to stock market returns. It is estimated as the slope coefficient obtained by regressing the share of wealth owned by the top 0.01% on stock

[17] I use as the weight matrix W the variance covariance of the moments in the baseline calibration of the model.

returns. The moment was estimated to be $\beta^{\text{Exposure}} = 0.35$ in Table 2. The moment will discipline the heterogeneity in risk aversion of top households γ_A

The first moment is the elasticity of top wealth shares to stock market returns. It is estimated as the slope coefficient obtained by regressing the share of wealth owned by the top 0.01% on stock returns. The moment was estimated to be $\beta^{\text{Exposure}} = 0.35$ in Table 2. The moment will discipline the heterogeneity in risk aversion of top households γ_A

The third moment is the Pareto tail of the wealth distribution. I use the slope coefficient in a regression of log percentile on log networth for the households within the top 0.01%. Figure 3 plots the log percentile as a function of the log net worth for the U.S. distribution, in the SCF and in Forbes 400 data. The linear slope is characteristic of a distribution with a Pareto tail. I measure a power law exponent of $\zeta = 1.5$, consistent with previous studies.[18] This moment will displicine the average wealth growth of top households relative to the economy.

Calibrated Parameters. The choice of calibrated parameters follows Gârleanu and Panageas (2015). The law of motion of the endowment process is $\mu = 2\%$ and $\sigma = 4.1\%$. The rate of death is $\delta = 2\%$. The share of endowment distributed as capital income is $1 - \omega = 8\%$. It corresponds to the share of total household income received as interest income or dividend income. The life cycle income of households $G(u)$ is a sum of two exponentials approximating the hump shaped pattern of earnings observed in the data:

$$G(u) = B_1 e^{-\delta_1 u} + B_2 e^{-\delta_2 u}$$

with $B_1 = 30.72, B_2 = -30.29$.

Following the approach of Barro (2006), I report the stock market returns for a firm with a debt-equity ratio equal to the historically observed debt-equity ratio for the U.S. non financial corporate sector.[19]

Estimated Parameters. The model has 7 remaining parameters. 3 parameters correspond to the preference parameters of each households in group A $(\rho_A, \gamma_A, \psi_A)$ and 3 parameters correspond to the preference parameters in group B $(\rho_B, \gamma_B, \psi_B))$. The remaining parameter is the population share of the agents in group A, π_A.

[18] See, for instance, Klass et al. (2006).

[19] As Barro (2006), I choose a debt-equity ratio equal to $\lambda \approx 0.5$.

4.2 Estimation Results

For each estimation, I report the parameters and the moments in Table 4. I plot the equilibrium functions in Figure 4.

Baseline I first examine whether a model estimated exclusively on asset prices generates the relationship between asset returns and the wealth distribution measured in Section 2. To do so, I first report in Column (1) of Table 4 the original calibration of the model by Gârleanu and Panageas (2015), which exclusively targets asset price moments. Qualitatively, the model generates the two stylized facts described in Section 2.

First, households in the top percentile are disproportionately invest in the stock market. Therefore, following large stock returns, top wealth shares increase. The model predicts that the exposure of top wealth shares β^{exposure} to the stock market is 0.50, while it is slightly lower, 0.35, in the data.

Second, when a larger share of wealth falls into the hands of risk-tolerant households, the aggregate demand for risk increases, which lowers risk premia and pushes up asset prices. Thus, a higher top wealth share predicts lower future excess returns. The model yields a predictive coefficient $\beta^{\text{Predictability}}$ equal to -0.03 while it is -0.05 in the data, well into the standard error in Table 3. Therefore, the calibrated model appears to be consistent with the mechanism outlined in Section 2.

Does the model also generate a realistic level of inequality? Figure 3 compares the log-log relationship between top percentiles and financial wealth in the simulated model and in the data. The red curve represents the wealth distribution in the model. The curve is approximately linear. Therefore, the wealth distribution in the model has approximately a Pareto tail, as discussed in Section 3. However, the distribution has a much thicker tail than in the data. I estimate ζ equal to 1.0 in the model compared to ζ equal to 1.5 in the data. The model overestimates the level of inequality.

Re-estimating the Model In Column (2), I therefore re-estimate the model, targeting jointly asset returns and the wealth moments. The model now generates a realistic Pareto tail, with a power exponent of 1.4. The exposure of top wealth shares is also lower, equal to 0.38, as in the data. However, the model now misses asset prices. The model completely underestimates the volatility of returns and the equity premium. The volatility of returns in the model, 10.5%, is roughly half of the volatility of returns in the data, 18.2%. Similarly, the equity premium in the model, 2.8%, is roughly half of the equity premium in the data, 5.2%. These two failures suggest a tension between matching the volatility of asset returns and the level of wealth inequality in the data.

To emphasize the conflict between these two moments, Column (3) re-estimates the model, adding exclusively the Pareto tail of the distribution to asset price moments. Similarly to the previous results, the model broadly fits the other distributional moment and the properties of the risk-free rate. However, it still underestimates the volatility of returns, 11.1%, and the equity premium, 2.3%.

Tension Between Matching the Volatility of Returns and the Tail of the Distribution. The previous results suggest a trade-off between the volatility of stock returns and the Pareto tail of the distribution. To better understand this trade-off, I examine more precisely the mechanism generating volatile stock returns in the model. Applying Ito's lemma, the volatility of returns can be written

$$\sigma_R = \sigma + \frac{\partial \log P/D}{\partial \log x}\sigma_x \tag{18}$$

To generate the high volatility of returns, the model must have either large fluctuations in the share of wealth owned by agents A, σ_x, or a large elasticity of the price-dividend ratio to the share of wealth owned by agent A, $\frac{\partial \log P/D}{\partial \log x}$. Heterogeneity in preferences may increase the volatility of stock returns by impacting these two terms.

Column (4) re-estimates the model using asset price moments together with the exposure of top wealth shares to stock returns. The latter moment limits the heterogeneity in risk aversion between the two groups.[20] Still, the model can generate volatile stock returns, with σ_R equal to 17.6%. Because the supplementary moment decreases the volatility of x, σ_x, relative to the baseline calibration, the model, in order to generate volatile returns, must feature a high elasticity of the price-dividend ratio to x, $\frac{\partial \log P/D}{\partial \log x}$. The high elasticity of the price-dividend ratio is obtained by a high heterogeneity in EIS: the estimation yields $\psi_A/\psi_B = 50$. The agents in group A have a much higher propensity to save compared to the households in group B. Because of these large differences in saving decisions, the tail of the distribution is much thicker than the data, with a power law exponent of 1.2.

In contrast, Column (5) re-estimates the model by imposing a lower bound on the EIS $\psi_B \geq 0.2$.[21] This constraint limits the heterogeneity in EIS between the two groups. Still, the model can generate volatile returns, with σ_R equal to 17.4%. Because the new constraint decreases $\frac{\partial \log P/D}{\partial \log x}$

[20] If risk aversion differs too much between the two groups, the rich and the poor differ twoo much in their risk exposure (Equation (11)), and therefore top wealth shares move too much with stock returns (Equation (13)).

[21] This lower bound corresponds to the empirical results of Vissing-Jørgensen (2002a).

relative to the baseline calibration, the model, in order to generate volatile returns, must feature a high volatility of x, σ_x. The high volatility σ_x is obtained by a high heterogeneity in risk aversion: the estimation yields $\gamma_B/\gamma_A = 50$. This heterogeneity generates large differences in the demand for risky assets between A and B, yielding a counter-factually high exposure of top wealth shares to stock returns (1.9, compared to 0.35 in the data). Moreover, the agents in group A earn much higher returns on their wealth compared to the agents in group B. Because of these large differences in investment returns, the tail of the distribution is much thicker than the data, with a power law exponent of 1.0.

The last two estimations show that a high degree of preference heterogeneity, whether it comes from differences in EIS or from differences in risk aversion, generates too much wealth inequality compared to what is found in the data. Intuitively, large differences in preferences imply substantial and permanent differences between the wealth growth of the agents in the economy. In the long-run, these differences in growth rates contribute to a thicker right tail of the wealth distribution, as shown in Proposition 5. To generate volatile stock returns, the model requires a degree of preference heterogeneity so large that the wealth distribution has a power law exponent very close to one. This corresponds to the thickest tail accommodated by the model, Zipf's law.

5 The Role of Heterogeneity in Investment Opportunities

The previous section shows that there is tension in the standard heterogeneous agents model between asset prices and the wealth distribution. The high degree of heterogeneity necessary to generate large fluctuations in asset prices implies more inequality than there is in the data.

I now suggest a parsimonious deviation from the standard model to resolve this tension. I consider the impact of low-frequency changes in the investment opportunities of the rich relative to the poor. These shocks create low-frequency fluctuations in wealth inequality, thereby increasing fluctuations in asset prices. At the same time, since these shocks average to zero, they do not increase the average wealth growth of households at the top. Therefore, these shocks help resolve the tension present in the standard model.

Section 5.1 presents the augmented model. Section 5.2 estimates the model on asset prices and the wealth distribution. Section 5.3 examines additional predictions of the model.

5.1 Augmented Model

I now consider a parsimonious departure from the heterogeneous agents model presented in Section 4.

In particular, I examine a process ν_t, which generates differences between the financial returns available to the agents in group A and the financial returns available to the agents in group B. I introduce these shocks in a way that does not affect the aggregate endowment, i.e. they are purely redistributive. Specifically, I assume that the financial return available to the agents in group $j \in \{A, B\}$ is increased by a group specific term ν_{jt}, i.e. that the budget constraint of an agent in group $j \in \{A, B\}$ becomes

$$\mu_{jt} = r_t + \delta + \nu_{jt} + \kappa\sigma_{jt} - c_{jt} \tag{19}$$

ν_{At} and ν_{Bt} are chosen so that the difference between ν_{At} and ν_{Bt} equals ν_t ($\nu_{At} - \nu_{Bt} = \nu_t$) and so that the total return on wealth in the economy is left unchanged ($x_t\nu_{At} + (1 - x_t)\nu_{Bt} = 0$), that is

$$\nu_{At} = (1 - x_t)\nu_t$$
$$\nu_{Bt} = -x_t\nu_t$$

While I do not take a stand on the origin of these differential investment opportunities ν_t, the literature suggests some potential origins. These fluctuations could be generated by changes in technology (Gârleanu et al. (2012), Kogan et al. (Forthcoming)), changes in financial frictions (Kiyotaki and Moore (1997)), or changes in taxes (Piketty and Zucman (2015), Pastor and Veronesi (2016)). For instance, the following tax policy would exactly generate the specification in my model: the government levies a wealth tax $-\nu_t$ on agents in group A and redistribute the proceeds to all agents in proportion to their wealth.

I assume that ν_t is a mean reverting process which fluctuates around zero. More specifically, its law of motion is

$$d\nu_t = -\kappa_\nu\nu_t dt + \sigma_\nu dZ_t \tag{20}$$

where κ_ν is the mean reversion parameter and σ_ν is the exposure to aggregate shocks.[22] When $\sigma_\nu = 0$, the model reverts to the baseline model in Section 3.

[22]In particular I assume that the same shocks drive the aggregate endowment Y_t and ν_t in the economy. This assumption simplifies the presentation. The model can also match asset prices and the wealth distribution when fluctuations in ν_t are uncorrelated with aggregate endowment shocks.

For the agents in group $j \in \{A, B\}$, fluctuations in ν_{jt} have the same effects as fluctuations in the risk-free rate. Hence, the law of motion of their wealth is is the same as in baseline model, after substituting $r_t - \nu_{jt}$ for the risk-free rate.

Proposition 6 (Law of Motion for Households Wealth with Fluctuating ν_t)**.** *Denote p_{jt} the wealth consumption ratio of each agent, i.e. $p_{jt} = 1/c_{jt}$. The wealth of households in group $j \in \{A, B\}$ follows the law of motion*

$$\sigma_{jt} = \frac{\kappa_t}{\gamma_j} + \frac{1-\gamma_j}{\gamma_j(\psi_j - 1)}\sigma_{p_j t} \tag{21}$$

$$\mu_{jt} = \psi_j(r_t + \nu_{jt} - \rho) + \frac{1+\psi_j}{2\gamma_j}\kappa_t^2 + \frac{1-\gamma_j}{\gamma_j(\psi_j - 1)}\kappa_t \sigma_{p_j t} + \frac{1-\gamma_j\psi_j}{2(\psi_j - 1)\gamma_j}\sigma_{p_j t}^2 + \mu_{p_j t} \tag{22}$$

The direct impact of an increase in ν_{jt} on μ_{jt}, the wealth growth of the agents in group $j \in \{A, B\}$, is given by $\psi_j \nu_{jt}$. It is the sum of a mechanical increase of their wealth growth (ν_{jt}) and of an adjustment in their consumption rate ($(\psi_j - 1)\nu_{jt}$).

In equilibrium, fluctuations in ν_t generate fluctuations in the price-dividend ratio. First, because the agents have different EISs ($\psi_A \neq \psi_B$), a rise in ν_t increases the aggregate demand for assets, which pushes up the price-dividend ratio. This is true even though fluctuations in ν_t are purely redistributive ($x\nu_{At} + (1-x)\nu_{Bt} = 0$). Second, a rise in ν_t also increases the relative growth rate of the agents in group A, and is therefore associated with an increase in x_t, the share of wealth owned by the agents in group A. This further pushes up asset prices.

Stock returns react both to news about the level of inequality x_t and to news about the future growth of inequality ν_t:

$$\sigma_R = \sigma + \frac{\partial \log P/D}{\partial \log x}\sigma_x + \frac{\partial \log P/D}{\partial \log \nu}\sigma_\nu \tag{23}$$

In the baseline model, there is a tension between the high volatility of returns σ_R and the average tail of the wealth distribution. The first requires a high degree of preference heterogeneity while the second is associated with a low degree of preference heterogeneity. In the augmented model, fluctuations in ν_t generate additional fluctuations in stock returns, through the additional term $\frac{\partial \log P/D}{\partial \log \nu}\sigma_\nu$ in (23). These fluctuations do not change the average tail of the wealth distribution, because they do not change the average value of μ_{At}. Therefore, fluctuations in ν_t help resolve the central tension at the baseline model.

To solve the augmented model, I look for a Markov equilibrium with two state variables (x_t, ν_t). The law of motion for x_t is the same as Proposition 2 while the law of motion for ν_t is exogenously given by (20). Given the law of motion for x_t, ν_t, one can express the drift and the volatility of all processes through Ito's lemma and proceed similarly to the baseline model. I discuss more precisely the solution method in Appendix B.2.

5.2 Estimation Results

I now estimate the model to assess whether it can qualitatively match the stylized facts presented in Section 2, but also whether it can generate asset prices and a wealth distribution consistent with the data.

Parameters. The model has two new parameters compared to the baseline model, the persistence κ_ν and the volatility σ_ν of the process ν_t.

Moments. I introduce a new moment to discipline the law of motion of ν_t. As seen in Proposition 6, fluctuations in ν_t generate fluctuations in the wealth growth of top households in excess of the observable fluctuations in asset returns.

To capture these fluctuations, I examine, in the data and in the model, the residuals in the regression of top wealth shares on asset returns:

$$\begin{aligned}&\log \text{Wealth Share Top } 0.01\%_{t+1}\\&\quad = \alpha + \rho \log \text{Wealth Share Top } 0.01\%_t + \beta(r^M_{t+1} - r^f_{t+1}) + \gamma r^f_{t+1} + \epsilon_{t+1}\end{aligned} \tag{24}$$

Intuitively, low-frequency fluctuations in these residuals help capture fluctuations in ν_t (Proposition 6). Specifically, I add as a new moment the standard deviation of a five-year moving average of these residuals:[23]

$$std(\sum_{1\leq i\leq 5} \epsilon_{t+1}/5) \approx 2.1\%$$

Results. Figure 5 plots asset prices as a function of the two state variables x and ν. Corresponding to the intuition put forth earlier, a high difference in the investment opportunities of the agents in group A compared to the agents in group B, ν, is associated with a low interest rate (Figure 5b) and also with a high drift of households in group A relative to other agents (Figure 5e). For both of these reasons, a high ν is associated with a high price-dividend ratio (Figure 4c).

[23] I use the five year averages to smooth out the yearly fluctuations of top wealth shares in the data (which also include measurement errors, etc). Averaging residuals at a longer horizon allows to concentrate on the low-frequency fluctuations driven by ν_t. An alternative would be to estimate a state space model.

The last column of Table 4 demonstrates that the augmented model can jointly match asset prices and the wealth distribution. In particular, the model can generate a high volatility of returns $\sigma_R = 17.2\%$ together with a low power law exponent $\zeta = 1.5$. In contrast, the baseline model could not jointly match these moments. In the baseline model, there was a tension between the high volatility of returns σ_R and the average tail of the wealth distribution ζ. The first required a high degree of preference heterogeneity while the second required a low degree of preference heterogeneity.

The augmented model solves this tension through fluctuations in ν. On the one hand, hese fluctuations increase the volatility of returns. Applying the decomposition (23), I obtain that 45% of the fluctuations in the price-dividend ratio are driven by fluctuations in the level of inequality x_t, while 55% are driven by fluctuations in ν_t, the relative investment opportunities of the agents in group A compared to the agents in group B. On the other hand, these fluctuations do not impact the average level of inequality, because they do not change the average wealth growth of top households μ_{At}. Therefore, fluctuations in ν_t allow the model to match a higher volatility of stock returns σ_R together with a lower power law exponent ζ.

5.3 Further evidence

I now examine whether the augmented model can explain additional dimensions of the data.

The Price-dividend Ratio and the Growth of Wealth Inequality. I start by examining the relationship between the price-dividend ratio and the future growth of wealth inequality.

On the one hand, in the model, fluctuations in x_t, the share of wealth owned by the households in group A, generate a *negative* comovement between the price-dividend ratio and the future growth of wealth inequality. The intuition is as follows. When x_t is high, the aggregate demand for assets is high. In equilibrium, the price-dividend ratio is high and future returns are low. These low returns decrease the growth rate of the agents in group A relative to other agents, and therefore, wealth inequality slowly decreases.

On the other hand, fluctuations in ν_t generate a *positive* comovement between the price-dividend ratio and the future growth of wealth inequality. A rise in ν_t simultaneously increases the relative wealth growth of households in group A (Figure 4e) and the price-dividend ratio (Figure 5c).

Therefore, the correlation between price-dividend ratio and the future growth of inequality depends on the relative importance of fluctuations in x_t and ν_t. Examining this relationship offers both a qualitative and quantitative test for the augmented model. To do so, I regress the future growth of the share of wealth owned by the top 0.01% on the price-dividend ratio, i.e.

$$\log \frac{\text{Wealth Share Top } 0.01\%_{t+4}}{\text{Wealth Share Top } 0.01\%_{t+1}} = \alpha + \beta \log P/D_t + \epsilon_t \tag{25}$$

Table 5 reports the result of this regression. In the data, I obtain an estimate for β equal to 0.03 with estate tax returns and equal to 0.11 with income tax returns: the price-dividend ratio tends to forecast *positively* the future growth of wealth inequality. In simulated data from the augmented model, I obtain a similar positive estimate for β equal to 0.12. In contrast, in the baseline model of Section 4, I obtain a strongly negative estimate for β equal to -0.10: the price-dividend ratio forecasts *negatively* the future growth of wealth inequality. This is because fluctuations in x_t can only generate a negative relationship between the price dividend and the future growth of inequality. One needs fluctuations in ν_t to explain the positive comovement between the price-dividend ratio and the future growth of wealth inequality.

Episodes of Disconnect. An additional way to relate the model to the data is to compare the time series of top wealth shares and a running sum of lagged asset returns.

In the baseline model, the differences between the two series are small : fluctuations in top wealth shares are entirely driven by fluctuations in past asset returns. In contrast, in the augmented model, fluctuations in ν_t generate periods of disconnect between top wealth shares and past asset returns (Proposition 6).

Figure 6 measures the difference between the two series in the data by comparing the evolution of the share of wealth owned the top 0.01% and the predicted values from the regression (24).

The figure shows large and persistent fluctuations in top wealth shares that cannot be explained by asset returns. In particular, asset returns alone cannot explain the long term decline in inequality in the 1930s. Symmetrically, asset returns cannot explain the increase of inequality after the 2000s, a period of low interest rate and low stock returns. These periods are inconsistent with the baseline model In the augmented model however, the latter period could be rationalized by a high ν_t — a rise in ν_t jointly increases wealth inequality and decreases asset returns.

6 Conclusion

The results of this paper depict a strong interplay between asset prices and wealth inequality. Because rich households hold more risky assets, realized stock returns generate fluctuations in wealth inequality over time. Conversely, in periods of high inequality, more wealth is in the hands of rich households, the risk-tolerant investors. Therefore, risk premia are low: a high level of inequality predicts low future returns. This interplay is at the heart of heterogeneous agents asset pricing models. I have shown that these models can qualitatively account for these facts. However, the standard models tend to overestimate the thickness of the tail of the wealth distribution.

This difficulty suggests that, while important, heterogeneity in preferences is not sufficient to understand the interplay of inequality and prices. Differences in investment opportunities offer a direction for progress. Augmenting standard models with this feature allows to simultaneously explain the volatility of asset prices and the level of inequality. Further, this approach provides an explanation for temporary disconnects between inequality and asset prices.

To make this point clearly, I use a parsimonious representation of these shocks. But it appears important to go further in understanding the precise source of these differences. The literature suggests promising avenues to answer this question: embodied capital shocks (Papanikolaou (2011), Gârleanu et al. (2012)), changes in the capital share (Karabarbounis and Neiman (2014), Lettau et al. (2016)), or taxes and regulation (Lampman (1962)).

The implications of my analysis extend beyond asset pricing. The interplay I put forward can have effects on real quantities as well, through two channels. First, because the level of inequality affects the cost of capital, this can lead to changes in corporate investment policies. Second, a recent literature has also emphasized the role of inequality for aggregate demand (Mian et al. (2013), Kaplan et al. (2016)). Exploring these channels requires moving away from an endowment economy, which I leave for future research.

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Table 1: The Equity Share Increases Across the Wealth Distribution

	Groups of Households Defined by Wealth Percentiles			
	All Households	1% − 0.1%	0.1% − 0.01%	Top 0.01%
Panel A: All Sample				
Equity Share	40.8%	55.8%	65.8%	73.9%
Public Equity	20.2%	22.0%	21.1%	19.6%
Private Equity	20.6%	33.9%	44.6%	54.4%
Non Actively Managed	2.4%	4.5%	6.3%	7.8%
Actively Managed	18.2%	29.4%	38.4%	46.6%
Panel B: Stockholders				
Is Stockholder	45.9%	90.7%	91.2%	91.0%
Equity Share among Stockholders	44.7%	56.0%	65.9%	76.0%
Panel C: Entrepreneurs				
Is Entrepreneur	10.5%	62.1%	69.8%	78.5%
Equity Share among non-Entrepreneurs	26.8%	40.7%	50.0%	57.9%
Panel D: Stock Options Holders				
Received Stock Options	6.4%	11.2%	11.5%	6.1%
Equity Share among non Stock Options Holders	44.7%	56.0%	65.9%	76.0%
Share of Total Wealth		20.7%	7.7%	3.8%
Labor Income / Wealth	12.6%	2.9%	1.6%	0.7%

Notes. Data from SCF 1989-2013. The variable Equity Share is defined as private equity + public equity over networth: (`equity` + `bus`) / `networth`. Stockholders are defined as the households that hold public equity. Entrepreneurs are defined as the households with an active management role in one of the company they invest in.

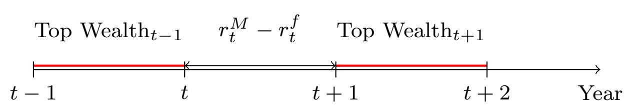
Table 2: The Exposure to Stock Returns Increases Across the Wealth Distribution

	Groups of Households Defined by Wealth Percentiles					
	Flow of Funds	Kopczuk and Saez (2004) Series			Forbes 400	
	All Households	$1-0.1\%$	$0.1-0.01\%$	Top 0.01%	Top 400	Top 100
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Wealth						
Excess Stock Returns	0.44***	0.52***	0.66***	0.75***	0.67***	0.71***
	(0.13)	(0.18)	(0.17)	(0.23)	(0.18)	(0.21)
R^2	0.49	0.45	0.58	0.40	0.34	0.29
Period	1917-1999	1917-1999	1917-1999	1917-1999	1983-2014	1983-2014
N	54	54	54	54	32	32
Panel B: Wealth Shares						
Excess Stock Returns		0.09*	0.22***	0.31**	0.34*	0.37
		(0.05)	(0.07)	(0.14)	(0.19)	(0.23)
R^2		0.20	0.33	0.14	0.18	0.15
Period		1917-1999	1917-1999	1917-1999	1983-2013	1983-2013
N		54	54	54	31	31

Notes. The table reports the results of the regression of the wealth growth of households in a given percentile group on the excess stock returns and the risk free rate, i.e. Equation (1):

$$\log \frac{W_{t+1}^{p \to p'}}{W_{t-1}^{p \to p'}} = \alpha + \beta(r_t^M - r_t^f) + \gamma r_t^f + \epsilon_t$$

The dependent variable is the growth of wealth in Panel A and the growth of wealth *shares* in Panel B. To avoid overlapping time periods between the regressor and the dependent variable, the timing is as follows:



Each column corresponds to a different group of households. The first column corresponds to all U.S households ; to measure the wealth of U.S. households, I use data from the Financial Accounts of the United States (Flow of Funds) after 1945. For the period before 1945, I use Kopczuk and Saez (2004). Columns (2) to (4) corresponds to increasing top percentiles in the wealth distribution, using data from Kopczuk and Saez (2004). Columns (5) to (6) correspond to the Top 0.0003% and 0.00008% ; the percentiles are chosen so that the group include the 400 wealthiest individuals and the 100 individuals in 2014.
Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,** ,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Table 3: The Share of Wealth Owned by the Top 0.01% Predicts Future Excess Returns

	$\sum_{1\leq h\leq H} r^M_{t+h} - r^f_{t+h} = \alpha + \beta_H \text{Log Top Wealth Shares}_t + \gamma_H \text{Predictor}_t + \epsilon_{tH}$					
	Excess Returns at Horizon $H = 1$			Excess Returns at Horizon $H = 3$		
	β_1	γ_1	Adjusted R^2	β_3	γ_3	Adjusted R^2
	(1)	(2)	(3)	(4)	(5)	(6)
Log Top Share	−0.053**		0.051	−0.114**		0.078
Dividend Price	−0.077***	0.056*	0.089	−0.198***	0.196***	0.265
cay	−0.044*	0.046*	0.072	−0.106*	0.114**	0.153
Dividend Payout	−0.066**	0.031	0.054	−0.153**	0.094	0.113
Long Term Yield	−0.066**	−0.034	0.053	−0.128**	−0.055	0.062
Default Yield Spread	−0.068**	0.031	0.047	−0.179***	0.146**	0.156
Treasury Bill Rate	−0.054*	−0.037	0.040	−0.127**	−0.081	0.080
Stock Variance	−0.058**	0.02	0.045	−0.129**	0.055	0.084
Inflation	−0.054**	−0.013	0.039	−0.116**	−0.021	0.066
Default Return Spread	−0.058**	−0.001	0.035	−0.161***	0.014	0.130
Term Spread	−0.037	0.019	0.017	−0.079	0.066	0.065
Linear Trend	−0.112***	−0.075*	0.083	−0.254***	−0.178*	0.144

Notes. The table reports the result of the regressions of future excess returns on the share of wealth owned by the Top 0.1% (row 1), along with other predictors from the literature (row 2-11). Each row corresponds to a different set of regressors. Columns (1) (2) (3) report the results when the dependent variable is the one year excess return. Columns (4) (5) (6) report the results when the dependent variable is the three-year excess return.

I construct *cay* in the period 1917-1999 by mirroring the construction in Lettau and Ludvigson (2001) on historical data: wage income from Piketty and Saez (2003), consumption from Shiller (2015), financial wealth from Kopczuk and Saez (2004). All other predictors come from Welch and Goyal (2008).

To facilitate the comparison between the different predictors, all regressors are normalized to have a standard deviation of one. Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,** ,*** indicate significance at the 0.1, 0.05, 0.01 levels.

Table 4: Matching the Wealth Distribution and Asset Prices

Type	Description	Data	Baseline Model Estimated on Asset Prices and ...					Augm. Model
			{}	{Wealth Dist}	$\{\zeta\}$	$\{\beta^{\text{Exp}}\}$	$\{\psi_B \geq 0.2\}$	{Wealth Dist}
			(1)	(2)	(3)	(4)	(5)	(6)
Parameters	Risk aversion of type-A agents γ_A		1.5	2.2	1.6	2.7	0.1	2.5
	Risk aversion of type-B agents γ_B		10.0	13.0	9.0	8.8	4.65	9.0
	EIS of type-A agents ψ_A		0.7	0.4	0.4	0.5	1.0	0.4
	EIS of type-B agents ψ_B		0.05	0.01	0.05	0.01	0.2	0.07
	Discount rate ρ_A		0.1%	0.1%	0.1%	0.1%	0.1%	0.1%
	Discount rate ρ_B		ρ_A	5%	1.5%	5%	3%	ρ_A
	Population share π_A		1%	20%	20%	0.1%	0.1%	20%
	Persistence κ_ν		—	—	—	—	—	0.1
	Volatility σ_ν		—	—	—	—	—	0.5%
Asset Prices	Equity Premium	5.2%	5.3%	2.8%	2.3%	5.0%	3.0%	5.2%
	Volatility of returns	18.2%	19.0%	**10.5%**	**11.1%**	17.6%	17.4%	17.5%
	Average interest rate r	2.8%	1.2%	2.9%	4.0%	3.1%	2.4%	2.5%
	Standard deviation interest rate r	0.92%	0.4%	1.1%	0.7%	0.7%	1.2%	1.4%
Wealth Distribution	Exposure Top Wealth Shares $\beta^{\text{Exposure}\dagger}$	0.35	0.50	0.38	0.50	0.35	**1.9**	0.31
	Predictability Regression $\beta^{\text{Predictability}\ddagger}$	−0.05	−0.03	−0.03	−0.03	−0.01	−0.01	−0.02
	Power Law Exponent ζ	1.5	**1.1**	1.4	1.5	**1.1**	**1.1**	1.5
Other	Standard dev. Residuals (5-year horizon)	2.1%	0.8%					2.2%

Notes. Columns (1) to (5) correspond to different estimations of the baseline model presented in Section 3. All estimations include asset price moments but differ with respect to the choice of other moments. Column (1) reports the model estimated on asset prices only. Column (2) reports the model estimated on asset prices and the wealth distribution. Column (3) reports the model estimated on asset prices and the power law exponent ζ. Column (4) reports the model estimated on asset prices and the exposure of top wealth shares β^{Exposure}. Column (5) reports the model estimated an asset prices with a lower bound on ψ_B. Column (6) reports the augmented of Section 5 estimated on asset prices, the wealth distribution, and a moment corresponding to the long run standard deviation of the residuals of the regression (24). Moments in bold and in red highlight the dimensions of the data that are missed by the model.

† β^{Exposure} is the coefficient obtained by regressing the growth of the share of wealth owned by the top 0.01% on stock returns (Table 2).

‡ $\beta^{\text{Predictability}}$ is the coefficient obtained by regressing the future excess stock returns on the log of the share of wealth owned by the top 0.01%, normalized to have a standard deviation of one (Table 3).

Table 5: The Price-Dividend Ratio and the Future Growth of the Wealth Share of the Top 0.01%

	Future Growth of the Wealth Share of the Top 0.01%			
	Kopczuk and Saez (2004) Series	Saez and Zucman (2016) Series	Baseline Model	Augmented Model
	(1)	(2)	(3)	(4)
$\log P/D$	0.03	0.11**	−0.10**	0.12**
	(0.07)	(0.05)	(0.07)	(0.05)
R^2	0.00	0.07	0.10	0.09

Notes. The table reports the results of the regression of the future growth of the wealth share of the top 0.01% on the price-dividend ratio, i.e. Equation (25):

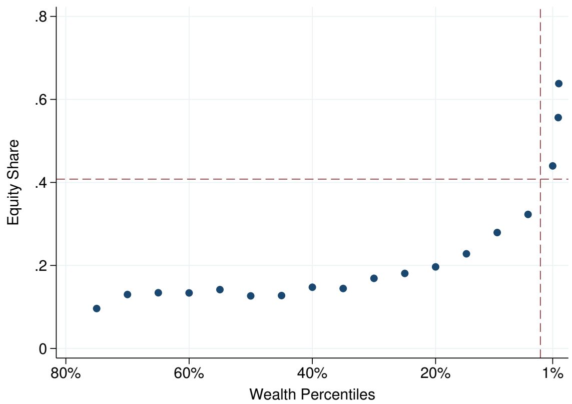
$$\log \frac{\text{Wealth Share Top } 0.01\%_{t+4}}{\text{Wealth Share Top } 0.01\%_{t+1}} = \alpha + \beta \log P/D_t + \epsilon_t$$

Each column corresponds to a different dataset. Column (1) corresponds to the wealth share of the Top 0.01% according to Estate Tax Returns (Kopczuk and Saez (2004)). Column (2) corresponds to the wealth share of the top 0.01% according to Income Tax Returns (Saez and Zucman (2016)). Column (3) corresponds to simulated data from the baseline model (parameters in Column (1) of Table 4). Column (4) corresponds to simulated data from the augmented model (parameters in Column (6) of Table 4).

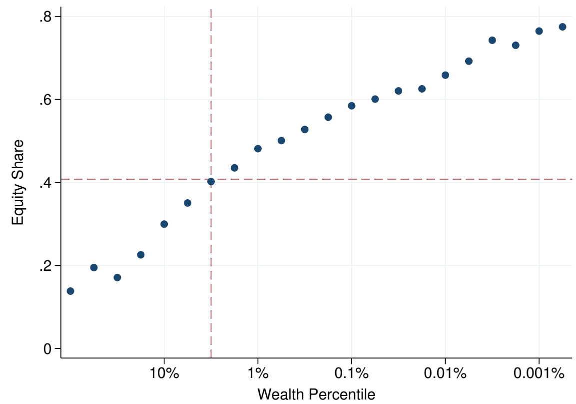
Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,** ,*** indicate significance at the 0.1, 0.05, 0.01 levels.

Figure 1: The Equity Share Increases Across the Wealth Distribution

(a) Top Percentiles Linearly Spaced



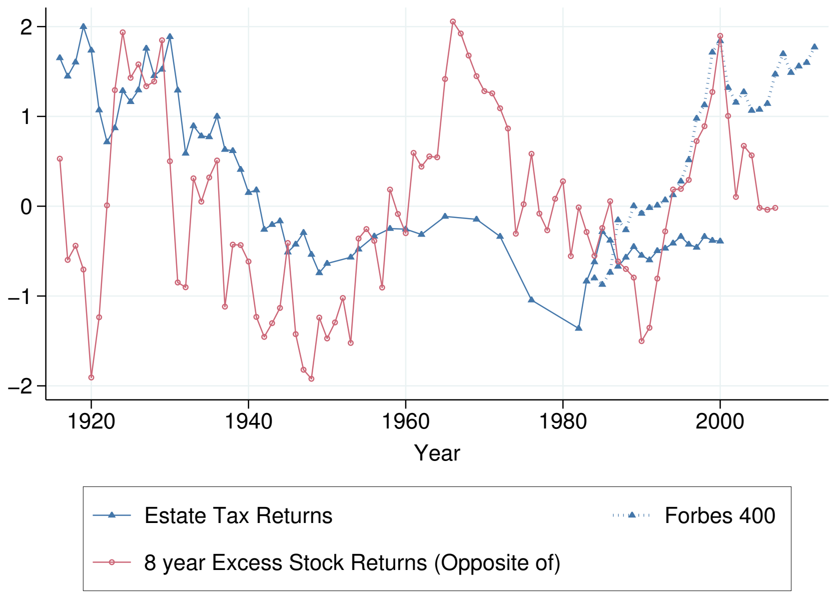
(b) Top Percentiles Log-linearly Spaced



Notes. Figure 1a plots the average equity share within 20 linearly spaced percentile bins in the wealth distribution. Figure 1b plots the average equity share within 20 logarithmically spaced percentile bins in the wealth distribution. The horizontal line represents the average equity share. The vertical line splits the set of households in two: households on either side of the vertical line own half of total wealth (this corresponds to top percentile $\approx 3\%$). Alls average are wealth-weighted.

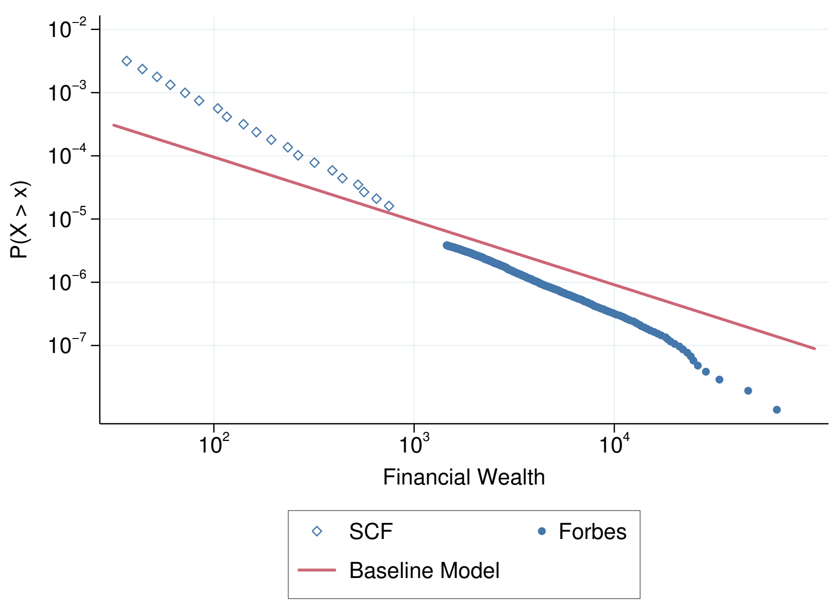
Data from the Survey of Consumer Finance (SCF), a cross sectional survey of US households from 1989 to 2013. The equity share is constructed as (`equity` + `bus`) / `networth`.

Figure 2: The Wealth Share of the Top 0.01% and Average Excess Returns



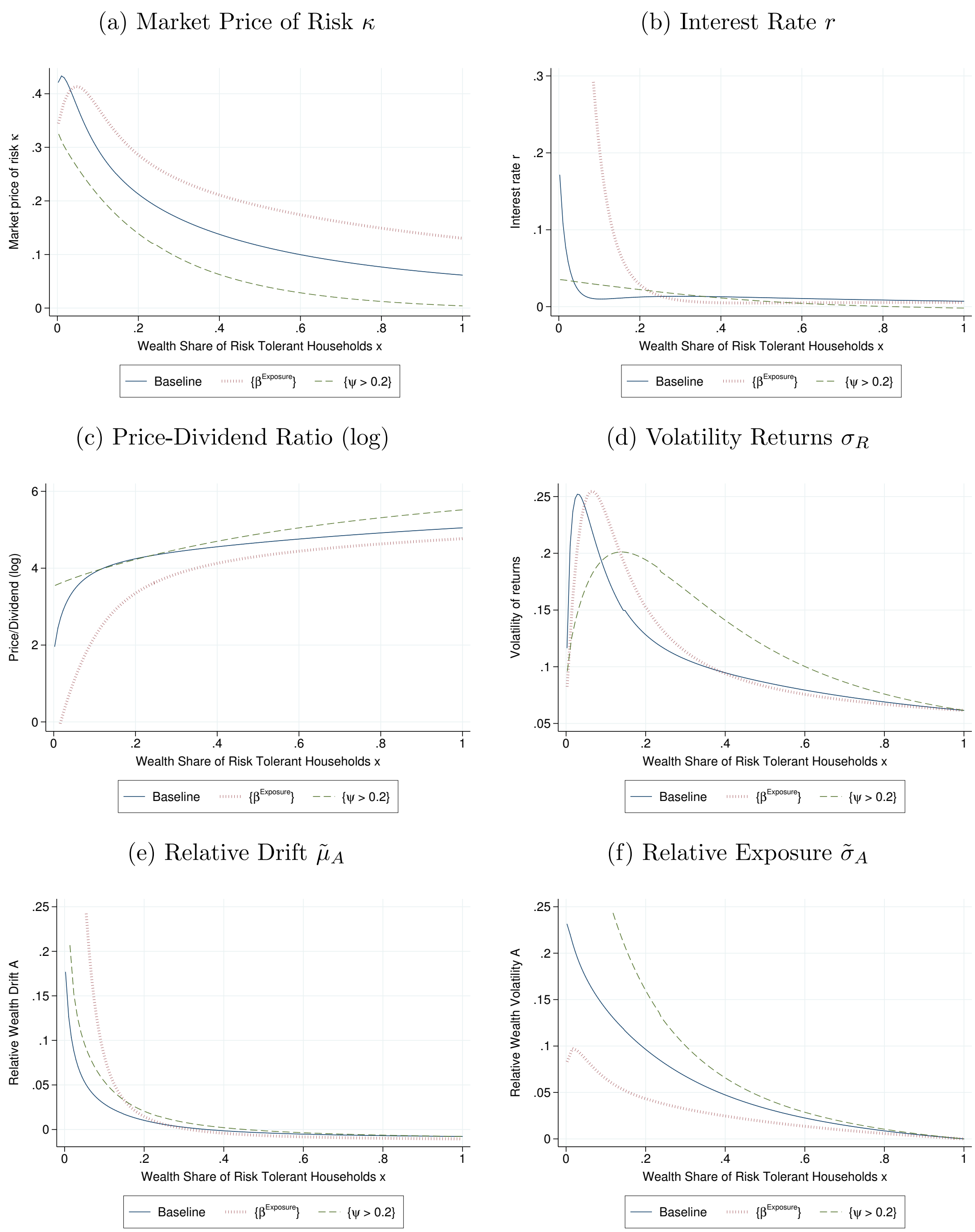
Notes. The figure plots the wealth share of the top 0.01% (log) and the 8-year sum of future excess returns (opposite of). All series are normalized to have a standard deviation of one.

Figure 3: The Pareto Tail of the Wealth Distribution in the Data and in the Baseline Model



Notes. The figure compares the log networth (relative to the average networth) to the log percentile in SCF, Forbes, and in the simulated model corresponding to Column (1) of Table 4. More precisely, the figure plots the average log networth within 40 logarithmically spaced percentile bins in SCF. The figure plots the average log networth for each position in Forbes 400. The (opposite of) the slope gives $\zeta \approx 1.5$ for SCF and for Forbes 400 but $\zeta \approx 1.1$ for the baseline model.

Figure 4: Asset Prices in the Baseline Model



(a) Market Price of Risk κ

(b) Interest Rate r

(c) Price-Dividend Ratio (log)

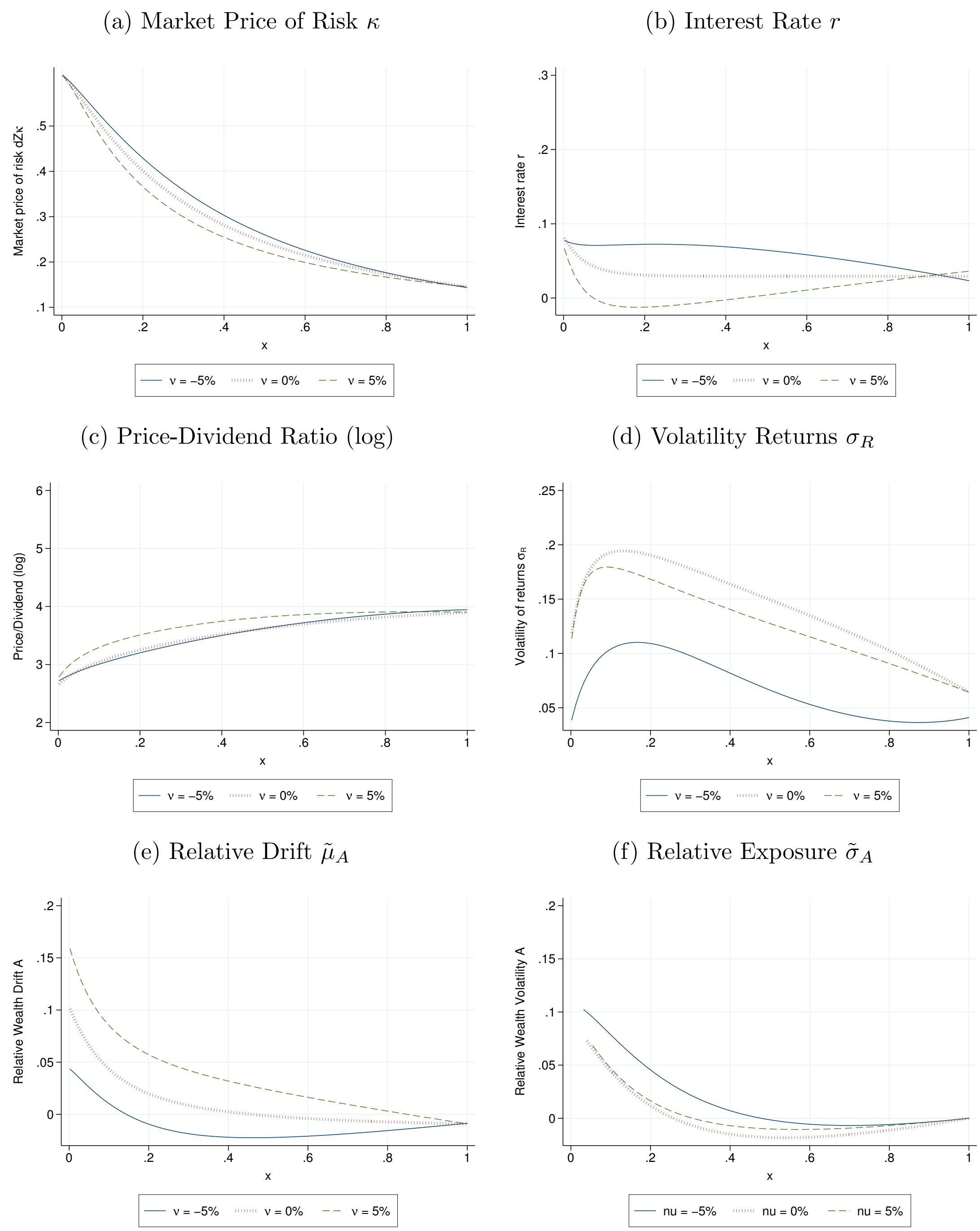
(d) Volatility Returns σ_R

(e) Relative Drift $\tilde{\mu}_A$

(f) Relative Exposure $\tilde{\sigma}_A$

Notes. The figure plots equilibrium objects as a function of x, the share of wealth owned by the agents in group A, for three different estimations of the baseline model. The baseline model, the model with $\{\beta^{\text{exposure}}$, the model with $\{\psi_B > 0.2\}$ correspond respectively to Column (1), Column (4) and Column(5) of Table 4.

Figure 5: Asset Prices in Augmented Model



(a) Market Price of Risk κ

(b) Interest Rate r

(c) Price-Dividend Ratio (log)

(d) Volatility Returns σ_R

(e) Relative Drift $\tilde{\mu}_A$

(f) Relative Exposure $\tilde{\sigma}_A$

Notes. The figure plots equilibrium objects as a function of x, the share of wealth owned by the agents in group A, and ν, the difference between the investment opportunities of the rich relative to the poor.

Figure 6: Time Series of the $\widehat{\text{Top } 0.01\%}$ vs the Top 0.01%

Notes. The figure plots the logarithm of the wealth share of the top 0.01%, as well as the "synthetic" values constructed as predicted values by the linear model given in Equation (24). More precisely, I estimate the linear model on the series of the wealth share of the top 0.01% and I then construct a synthetic series $\widehat{\text{Top } 0.01\%}$ as

$$\widehat{\text{Top } 0.01\%}_{t+1} = \hat{\alpha} + \hat{\rho} \log \widehat{\text{Top } 0.01\%} + \hat{\beta}(r^M_{t+1} - r^f_{t+1}) + \hat{\gamma} r^f_{t+1} + \epsilon_{t+1}$$

A Empirical Appendix

A.1 Data Sources

A direct comparison of estate tax returns and Forbes data by researchers from the IRS Statistics of Income Division (Raub et al. (2010)) finds that actual estates correspond to only about 50 percent of reported Forbes values. This suggests that estate tax returns may underestimate wealth (potentially due to the tax avoidance effect) while Forbes may overestimate wealth (potentially because debts are harder to track than assets). These findings suggest that the main difficulties with measuring top wealth shares primarily pertain to getting the level right. My principal measure, the stock market exposure of the top households, is similar across the two datasets (Table 2).

I refer the reader to Kopczuk and Saez (2004) for a detailed descriptions of construction of top wealth shares from estate tax returns.

A.2 Measuring the Exposure of Top Households

Table A1: The Exposure to Stock Returns Across the Wealth Distribution: Controlling for the Composition Effect

	Forbes 40	
	Top 40	Within Top 40
	(1)	(2)
Excess Stock Returns	0.71***	0.74***
	(0.21)	(0.25)
Risk Free Rate	1.89	0.54
	(1.16)	(1.15)
R^2	0.29	0.27

Notes. This table reports the stock market exposure of the total wealth of the households in the Top 40 vs the wealth exposure of the households *within* the Top 40.
Only 4 households in the Top 40 directly exit Forbes 400 (Daniel E. Smith, Gururaj Deshpande, David Huber in 2001 and Robert Pritzker in 2004). I do not know the wealth of these households after they exit the top. I assume that they are just under the threshold for Forbes 400. Quantitatively, the imputation does not matter since the drop already corresponds to a negative return of -90% (i.e. there is more much more variation between the Top 40 and the Top 400 than between the top 400 and 0)

Robustness w.r.t. Human Capital. Table 2 measures the exposure of financial wealth. One may be interested in the exposure of total wealth. However, because human capital is not observable. I now argue that, for households at the top of the wealth distribution, the bias between the exposure of total wealth and the exposure of financial wealth is quantitatively small.

Table A2: The Exposure to Stock Returns Across the Wealth Distribution: Controlling for Idiosyncratic Volatility

	Group of Households Defined by Wealth Percentiles		
	Kopczuk and Saez (2004) Series		
	$1-0.1\%$	$0.1-0.01\%$	Top 0.01%
	(1)	(2)	(3)
Excess Stock Returns	0.58***	0.71***	0.77***
	(0.19)	(0.18)	(0.24)
σ^2 idiosyncratic (firm level)	0.12	0.13	0.27**
	(0.08)	(0.11)	(0.12)
R^2	0.51	0.62	0.56
Period	1928-1999	1928-1999	1928-1999
N	43	43	43

Notes. The table reports the results of a regression of the wealth growth of households in a given percentile group on the excess stock returns, the risk free rate, and the yearly idiosyncratic variance

$$\log \frac{W_{t+1}^{p\to p'}}{W_{t-1}^{p\to p'}} = \alpha + \beta(r_t^M - r_t^f) + \gamma r_t^f + \delta\sigma^2 + \epsilon_t$$

Idiosyncratic variance σ^2 is measured as the cross sectional variance of the residual $epsilon_{it}$ of a regression on firm level stock returns on factors

$$r_t^i - r_t^f = \alpha_i + \beta_i F_t + \epsilon_{it}$$

where F_t includes the three Fama-French factors.

Estimation via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,** ,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Formally, for a given agent in the economy, denote w his financial wealth, h his human capital and $\omega = w/(w+h)$ the ratio of financial wealth over total wealth. Following the log linearization in Campbell (1996), the return on total wealth can be written as

$$\log \frac{w_{t+1} + h_{t+1}}{w_t + h_t} \approx \kappa + \omega \log \frac{w_{t+1}}{w_t} + (1-\omega) \log \frac{h_{t+1}}{h_t}$$

Projecting this approximation on stock returns, we obtain the exposure of total wealth as a weighted sum of the exposure of financial wealth and human capital

$$\beta_{w+h} \approx \omega\beta_w + (1-\omega)\beta_h$$

This allows to express the bias due to the omition of human capital

$$\frac{\beta_{w+h} - \beta_w}{\beta_w} = (1-\omega)(\frac{\beta_h - \beta_w}{\beta_w})$$

The bias depends on ω the share of financial wealth in total wealth, and $\frac{\beta_h - \beta_w}{\beta_w}$ the difference between the exposure of financial wealth and of human capital. I now give an order of magnitude for these two terms.

Labor income represents a very small share of total income for households in the top of the distribution (8.5% for the Top 400[24]). Assuming the same capitalization rate for human capital and financial capital, this suggests that human capital represents approximately one tenth of financial wealth for the top 400 households.

I proxy the exposure of human capital to the stock market β_h as the covariance of labor income growth to stock returns. The approximation is exact when the discount rate associated with human capital are constant over time. Table A3 reports the result: I find $\beta = 0.21$, which is smaller than the exposure of financial wealth. Parker and Vissing-Jørgensen (2009) show that the exposure of labor income to aggregate shocks was low before 1982, and increased thereafter.

Joining the estimates for ω and β_h, I conclude that the bias is in average negative and represents $0.085 * (0.2/0.75 - 1) = -6\%$ of the estimated exposure β, which is much smaller than the standard errors.

Table A3: The Exposure of Labor Income Growth Across the Wealth Distribution:

	Group of Households Defined by Wealth Percentiles		
	$1 - 0.1\%$	$0.1 - 0.01\%$	Top 0.01%
	(1)	(2)	(3)
Excess Stock Returns	0.15	0.20*	0.35**
	(0.09)	(0.11)	(0.16)
R^2	0.16	0.12	0.18
Period	1918-2010	1918-2010	1918-2010
N	93	93	93

Notes. The table reports the resuls of the regression of the growth of labor income on asset returns

$$\log \frac{Y_{t+1}^{p \to p'}}{Y_{t-1}^{p \to p'}} = \alpha + \beta_1 (r_t^M - r_t^f) + \beta_2 r_t^f + \epsilon_t$$

The total labor income received received by within a top percentile is obtained from Saez and Zucman (2016).
Estimation is via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,** ,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Robustness w.r.t. Saez and Zucman (2016) series. Saez and Zucman (2016) have recently proposed a new series for top wealth shares, which relies on Income Tax Returns. Table A4 estimates the stock market exposure of the top wealth percentiles by replacing the series of Kopczuk and Saez (2004) by the series of Saez and Zucman (2016). I find that the estimates are now uniformally lower. For instance, the stock market exposure of the Top 0.01% is 0.4 using Income Tax Returns, compared to 0.75 using estate tax returns or Forbes. This suggests that the methodology in Saez and Zucman (2016) may not track well the business cycle frequencies of wealth shares, even though they track more accurately the long run fluctuations

[24]See https://www.irs.gov/pub/irs-soi/13intop400.pdf.

in inequality, as argued in Saez and Zucman (2016). A certain number of wealth categories are constructed using trends and interpolations across years, which may bias down the estimate.

Table A4: The Exposure to Stock Returns Across the Wealth Distribution: Saez and Zucman (2016) Series

	Group of Households Defined by Wealth Percentiles		
	Saez and Zucman (2016) Series (Income Tax)		
	$1-0.1\%$	$0.1-0.01\%$	Top 0.01%
	(1)	(2)	(3)
Panel A: 1960-2011			
Excess Stock Returns	0.31***	0.38***	0.40***
	(0.06)	(0.07)	(0.08)
R^2	0.38	0.31	0.27
Period	1960-2011	1960-2011	1960-2011
N	52	52	52
Panel B: 1960-1982			
Excess Stock Returns	0.23***	0.26***	0.30**
	(0.05)	(0.09)	(0.12)
R^2	0.48	0.33	0.29
Period	1960-1981	1960-1981	1960-1981
N	22	22	22
Panel C: 1982-2011			
Excess Stock Returns	0.38***	0.47***	0.46***
	(0.08)	(0.10)	(0.10)
R^2	0.43	0.41	0.38
Period	1982-2011	1982-2011	1982-2011
N	30	30	30

Notes. The table reports the resuls of the regression of the growth of the wealth growth of households in a given percentil group on asset returns.
Estimation is via OLS. Standard errors in parentheses and estimated using Newey-West with 3 lags. *,** ,*** indicate significance at the 0.1, 0.05, 0.01 levels, respectively.

Table A5: Accounting for the Persistence of Top Wealth Shares (Campbell and Yogo (2006) test)

	Confidence Interval for $\beta \in [\underline{\beta}, \overline{\beta}]$ in the Predictability Regression (2)			
	Case with $\underline{\rho} = 0.92$		Case with $\overline{\rho} = 1.06$	
	$\underline{\beta}$	$\overline{\beta}$	$\underline{\beta}$	$\overline{\beta}$
Log Top Share	-0.22	-0.04	-0.19	-0.01

Notes. The time period is 1917-1945, the longest period where the wealth share of the top 0.01% is available without missing years. This table does a Bonferroni test based on the test developped Campbell and Yogo (2006). The test jointly takes into account the persistence of the predictor as well as its correlation with stock returns to compute the confidence interval for β. The autoregressive lag length for top wealth share is estimated by the Bayes information criterion (BIC), with maximum length equal to four.

B Theoretical Appendix

B.1 Characterization of the Equilibrium

Proof of Proposition 1. Given the homotheticity assumptions, the value function of the households in group $j \in \{A, B\}$ with wealth N can be written:

$$V_{jt}(N) = \frac{N^{1-\gamma_j}}{1-\gamma_j} p_{jt}^{\frac{1-\gamma}{\psi-1}} (\rho+\delta)^{\frac{1-\gamma_j}{1-\frac{1}{\psi_j}}}$$

The HJB equation associated with household's problem is

$$0 = \max_{c_{jt},\sigma_{jt}} \{ f(c_{jt}N_{jt}, U) + E[\frac{N^{1-\gamma_j}}{1-\gamma_j} p_{jt}^{\frac{1-\gamma}{\psi-1}} (\rho+\delta)^{\frac{1-\gamma}{1-\frac{1}{\psi}}}]$$

Applying Ito's lemma

$$\begin{aligned}
0 = \max_{c_{jt},\sigma_{jt}} \{ & \frac{(\rho+\delta)(1-\gamma_j)}{1-\frac{1}{\psi_j}} (\frac{c_{jt}^{1-\frac{1}{\psi_j}}}{\rho p_{jt}^{\frac{1}{\psi_j}} N_{jt}^{1-\frac{1}{\psi_i}}} - 1) + (1-\gamma_j)\mu_i \\
& + \frac{1-\gamma_j}{\psi_j - 1}\mu_{p_j} - \frac{(1-\gamma_j)\gamma_j}{2}\sigma_j^2 + \frac{(1-\gamma_j)(2-\gamma_j-\psi_j)}{2(\psi_j-1)^2}\sigma_{p_j}^2 + \frac{(1-\gamma_j)^2}{\psi_j-1}\sigma_i\sigma_{p_j}
\end{aligned}$$

Substituting the expression for the wealth drift μ_j using the budget constraint and dividing by $1-\gamma_j$

$$\begin{aligned}
0 = \max_{c_{jt},\sigma_{jt}} \{ & \frac{1}{1-\frac{1}{\psi_j}} (\frac{c_{jt}^{1-\frac{1}{\psi_j}}}{p_{jt}^{\frac{1}{\psi_j}}} - \rho - \delta) + r_t + \delta + \sigma_{jt}\kappa_t - c_{jt} \\
& + \frac{1}{\psi_j - 1}\mu_{p_j t} - \frac{\gamma_j}{2}\sigma_j^2 + \frac{2-\gamma_j-\psi_j}{2(\psi_j-1)^2}\sigma_{p_j t}^2 + \frac{1-\gamma_j}{\psi_j-1}\sigma_j\sigma_{p_j t}\}
\end{aligned}$$

The FOC for aggregate risk exposure σ_{jt} gives

$$\sigma_{jt} = \frac{\kappa_t}{\gamma_j} + \frac{1-\frac{1}{\gamma_j}}{1-\psi_j}\sigma_{p_j t}$$

The FOC for consumption gives

$$c_{jt} = 1/p_{jt}$$

that is, p_{jt} is the wealth / consumption ratio of the household.
Plugging the expression in the HJB equation, we obtain an expression for the wealth drift

$$\begin{aligned}
\mu_{jt} &= r_t + \delta + \sigma_{jt}\kappa_t - c_{jt} \\
&= \psi_j(r_t - \rho) + \mu_{p_j t} + \frac{1+\psi_j}{2\gamma_j}\kappa_t^2 + \frac{1-\gamma_j}{\gamma_j(\psi_j-1)}\kappa_t\sigma_{p_j t} + \frac{1-\gamma_j\psi_j}{2(\psi_j-1)\gamma_j}\sigma_{p_j t}^2
\end{aligned}$$

□

Proof of Proposition 2. Denote N_{Ats} the average wealth at time t of all agents in group A born at time s. The total wealth owned by agents in group A is

$$\int_{i\in\mathbb{I}_A} N_{it}di = \int_{-\infty}^{t} \delta e^{-\delta(t-s)} N_{Ats} ds$$

Its law of motion is

$$d[\pi_A \int_{-\infty}^{t} \delta e^{-\delta(t-s)} N_{Ats} ds] = \pi_A \int_{-\infty}^{t} \delta e^{-\delta(t-s)} dN_{Ats} ds + \pi_A \delta N_{Att} - \pi_A \int_{-\infty}^{t} \delta^2 e^{-\delta(t-s)} N_{A,ts} ds$$

Therefore

$$\frac{d[\pi_A \int_{-\infty}^{t} \delta e^{-\delta(t-s)} N_{At,s} ds]}{\int_{-\infty}^{t} \pi_A \delta e^{-\delta(t-s)} N_{At,s} ds} = \mu_{At} dt + \sigma_{At} dZ_t + \delta(\frac{\pi_A}{x_t}\frac{\phi_t}{p_t} - 1)dt$$

Similarly

$$\frac{d[(1-\pi_A) \int_{-\infty}^{t} \delta e^{-\delta(t-s)} N_{Bt,s} ds]}{\int_{-\infty}^{t} (1-\pi_A) \delta e^{-\delta(t-s)} N_{Bt,s} ds} = \mu_{Bt} dt + \sigma_{Bt} dZ_t + \delta(\frac{1-\pi_A}{1-x_t}\frac{\phi_t}{p_t} - 1)dt$$

Applying Ito's lemma on the definition of x (10), we obtain

$$\frac{dx_t}{x_t} = \mu_{xt} dt + \sigma_{xt} dZ_t$$

with

$$\begin{aligned}
\sigma_{xt} &= (1-x_t)(\sigma_{At} - \sigma_{Bt}) \\
\mu_{xt} &= (1-x_t)(\mu_{At} - \mu_{Bt}) + (1-x)\frac{\phi_t}{p_t}\delta(\frac{\pi_A}{x_t} - \frac{1-\pi_A}{1-x_t}) - (\sigma + \sigma_{pt})\sigma_{xt}
\end{aligned}$$

□

B.2 Solving for the Equilibrium

Specifying Life Cycle Function G. First, I specify $G(u)$ as a sum of K exponential

$$G(u) = \sum_{1 \le k \le K} B_k e^{-\delta_k u}$$

where the coefficients $(B_k)_{1 \le k \le K}$ are normalized so that total aggregate earnings equal ωY_t

$$1 = \sum_{1 \le k \le K} B_k \frac{\delta}{\delta + \delta_k}$$

Define p_k^L as the price dividend of a claim with exponentially decreasing endowment at rate $\delta + \delta_k$, for $1 \le k \le K$. In particular, ϕ, the human capital of a newborn agent normalized by total endowment Y_t, can be written:

$$\phi = \omega \sum_{1 \le k \le K} B_k p_k^l$$

I now write the equilibrium as a system of PDES for the function $p_A, p_B, (p_k^l)_{1 \le k \le K}$.

Market Clearing for Consumption. Market clearing for consumption (8) gives the function p in term of p_A and p_B

$$\frac{x}{p_A} + \frac{1-x}{p_B} = \frac{1}{p}$$

In particular, this allows to express the derivatives of p in term of the derivatives of p_A and p_B

Market Clearing for Risk. Market clearing for risk (9) gives

$$x\sigma_A + (1-x)\sigma_B = \sigma + \sigma_p$$

Plugging the FOC for σ_j from (11), one obtains the market price of risk (16)

Solve for σ_x. By Ito we have

$$\sigma_{p_j} = \frac{\partial_x p_j}{p_j}\sigma_x + \frac{\partial_\nu p_j}{p_j}\sigma_\nu \text{ for } j \in \{A, B\}$$
$$\sigma_p = \frac{\partial_x p}{p}\sigma_x + \frac{\partial_\nu p}{p}\sigma_\nu$$

Substituting the expression for κ in (16) in Proposition 2, we can solve for σ_x:

$$\sigma_x = \frac{\frac{x(1-x)\Gamma}{\gamma_A\gamma_B}((\gamma_B-\gamma_A)\sigma + \frac{1-\gamma_A}{\psi_A-1}\frac{\partial_\nu p_A}{p_A}\sigma_\nu - \frac{1-\gamma_B}{\psi_B-1}\frac{\partial_\nu p_B}{p_B}\sigma_\nu)}{1-\frac{x(1-x)\Gamma}{\gamma_A\gamma_B}((\gamma_B-\gamma_A)\frac{p_x}{p} + \frac{1-\gamma_A}{\psi_A-1}\frac{\partial_x p_A}{p_A} - \frac{1-\gamma_B}{\psi_B-1}\frac{\partial_x p_B}{p_B})}$$

Solve for μ_x. The law of motion Proposition 2 yields μ_x in term of previously computed quantities:

$$\mu_x = \mu_A - \mu - \mu_p + \delta(\frac{\pi_A}{x}\frac{\phi}{p} - 1) + (\sigma + \sigma_p)\sigma_x$$
$$= \frac{1}{p} - \frac{1}{p_A} + \kappa\sigma_x + \delta\frac{\phi}{p}(\frac{\pi_A}{x} - 1) + (\sigma + \sigma_p)\sigma_x$$

By Ito, we have

$$\mu_{p_j} = \frac{\partial_x p_j}{p_j}\mu_x + \frac{\partial_\nu p_j}{p_j}\mu_\nu + \frac{1}{2}\frac{\partial_{xx} p_j}{p_j}\sigma_x^2 + \frac{1}{2}\frac{\partial_{\nu\nu} p_j}{p_j}\sigma_\nu^2 + \frac{\partial_{x\nu} p_j}{p_j}\sigma_x\sigma_\nu \text{ for } j \in \{A, B\}$$
$$\mu_p = \frac{\partial_x p}{p}\mu_x + \frac{\partial_\nu p}{p}\mu_\nu + \frac{1}{2}\frac{\partial_{xx} p}{p}\sigma_x^2 + \frac{1}{2}\frac{\partial_{\nu\nu} p}{p}\sigma_\nu^2 + \frac{\partial_{x\nu} p}{p}\sigma_x\sigma_\nu$$

Solve for risk free rate Combining the market pricing for the price dividend ratio and the market pricing for human capital, we obtain the market pricing for p, the total wealth in the economy

$$\frac{1-\delta\phi}{p} + \mu + \mu_p + \sigma\sigma_p = r + \kappa(\sigma + \sigma_p)$$

This gives r.

System of PDEs. Given r and κ, we are left with the following system of PDEs

$$\mu_j = r + \delta - \nu_j + \kappa\sigma_j - \frac{1}{p_j} \text{ for } j \in \{A, B\}$$
$$\frac{1}{p_k^l} + \mu - \delta - \delta_k + \sigma_{p_k^l}\sigma = r + \kappa(\sigma + \sigma_{p_k^l}) \text{ for } 1 \leq k \leq K$$

B.3 The Wealth Distribution

Lemma 1 (Kolmogorov Forward)**.** *Suppose n_t is a process evolving according to*

$$dn_t = \mu_t dt + \sigma_t dZ_t + \nu_t dW_{it}$$

where Z_t is a standard aggregate Brownian Motion and W_{it} is a standard idiosyncratic Brownian motion. Assume that n_t has death rate δ and is re-injected according to the distribution ψ_t.

The density of n_t, g_t, follows the law of motion:

$$\frac{dg_t}{dt}(n) = -\partial_x(\mu_t g_t(n) + \sigma_t g_t(n)\frac{dZ_t}{dt}) + \frac{1}{2}\partial_x^2\left((\sigma_t^2+\nu_t^2)g_t(n)\right) + \delta(\psi_t(n) - g_t(n))$$

Proof for Lemma 1. For any function f, we have

$$\int_{-\infty}^{+\infty} f(n)g_{t+dt}(n)dx = \int_{-\infty}^{+\infty}[(f(n)+df(n))g_t(n) + f(n)\delta dt(\psi_t(n) - g_t(n))]dx$$

Assume that f is a twice differentiable and use Ito's lemma to obtain

$$\begin{aligned}\int_{-\infty}^{+\infty} f(n)dg_t(n)dx = &\int_{-\infty}^{+\infty}(\mu_t\partial_x f(n) + \frac{1}{2}(\sigma_t^2 + \nu(n)^2)\partial_{xx}f(n) + \sigma_t\partial_x f(n)dZ_t)g_t(n)dx \\ &+ \int_{-\infty}^{+\infty} f(n)\delta dt(\psi_t(n) - g_t(n))dx\end{aligned}$$

Assume that f decays fast enough as $|x| \to +\infty$ and use integration by parts to obtain

$$\begin{aligned}\int_{-\infty}^{+\infty} f(n)dg_t(n)dx = &\int_{-\infty}^{+\infty} f(n)[(-\partial_x(\mu_t g_t(n)) + \frac{1}{2}\partial_x^2((\sigma_t^2+\nu_t^2)g_t))dt - \partial_x(\sigma_t g_t)dZ_t]dx \\ &+ \int_{-\infty}^{+\infty} f(n)\delta dt(\psi_t(n) - g_t(n))dx\end{aligned}$$

This equality must hold for all f satisfying the conditions above. Therefore, we obtain

$$\frac{dg_t}{dt}(n) = -\partial_x(\mu_t g_t(n) + \sigma_t g_t(n)\frac{dZ_t}{dt}) + \frac{1}{2}\partial_x^2((\sigma_t^2+\nu_t^2)g_t) + \delta(\psi_t(n) - g_t(n))$$

□

I now derive the evolution of top wealth shares deriving a version of the Kolmogorov Forward equation with both aggregate risk and idiosyncratic risk.

Lemma 2 (Dynamics of Top Wealth Shares)**.** *Suppose n_t is a process evolving according to*

$$dn_t = \mu_t dt + \sigma_t dZ_t + \nu_t dW_{it}$$

where Z_t is a standard aggregate Brownian Motion and W_{it} is a standard idiosyncratic Brownian motion. Assume that n_t has death rate δ and is re-injected according to the distribution ψ_t.

For a top percentile $\alpha \in (0,1)$, denote $q_t(\alpha)$ the $\alpha-$quantile, i.e.

$$\alpha = \int_{q_t}^{+\infty} g_t(n)dn \tag{A1}$$

and denote $T_t(\alpha)$ the top share for the top percentile α, i.e.,

$$T_t = \int_{q_t}^{+\infty} n g_t(n) dn \tag{A2}$$

The top share T_t follows the law of motion

$$\frac{dT_t}{T_t} = \mu_t dt + \sigma_t dZ_t + \frac{q_t^2 g_t(q_t)}{2T_t}\nu_t^2 dt + (\frac{q_t \alpha}{T_t} - 1)\delta dt + \frac{\delta dt}{T_t}\int_{q_t}^{+\infty}(n - q_t)\psi_t(n)dn \tag{A3}$$

Proof of Lemma 2. This proof follows Gomez (2018). It is reproduced here for the sake of completeness.

Applying Ito's lemma on (A1) gives the law of motion of the quantile q_t

$$0 = -g_t(q_t)\frac{dq_t}{dt} + \int_{q_t}^{+\infty}\frac{dg_t(n)}{dt}dn - \sigma[dg_t(q_t)]\sigma[dq_t]$$

where $\sigma[dg_t(q_t)]$ and $\sigma[dq_t]$ denote respectively the volatility of $g_t(q_t)$ and q_t Applying Ito's lemma on (A2) gives the law of motion of the top share T_t

$$dT_t = -q_t g_t(q_t) dq_t + \int_{q_t}^{\infty} n dg_t(n) dn - q_t \sigma[dg_t(q_t)]\sigma[dq_t]dt - \frac{1}{2}g_t(q_t)\sigma[dq_t]^2 dt$$

Injecting the law of motion for q_t, we obtain the law of motion for T_t:

$$\begin{aligned} dT_t &= \int_{q_t}^{\infty}(n - q_t) dg_t(n) dn - \frac{1}{2}g_t(q_t)\sigma[dq_t]^2 dt \\ &= \int_{q_t}^{\infty}(n - q_t) dg_t(n) dn - \frac{1}{2}\frac{1}{g_t(q_t)}\left(\int_{q_t}^{\infty}\sigma[dg_t(n)]dn\right)^2 dt \end{aligned} \tag{A4}$$

Substituting the law of motion for dg_t from the Kolmogorov Forward equation Lemma 1 and integrating by parts:

$$\begin{aligned} dT_t = &\int_{q_t}^{\infty}(n - q_t)\left(-\partial_n\left((\mu_t dt + \sigma_t dZ_t)\, n g_t(n)\right) + \partial_n^2\left(\frac{\sigma_t^2 dt + \nu_t^2 dt}{2}n^2 g_t(n)\right) + \delta dt(\psi_t(n) - g_t(n))\right) dn \\ &- \frac{1}{2}\frac{1}{g_t(q_t)}\left(\int_{q_t}^{+\infty}\partial_n(\sigma_t n g_t(n) dn)\right)^2 dt \\ = &\int_{q_t}^{+\infty}\left((\mu_t dt + \sigma_t dZ_t) n g_t(n) - \partial_n(\frac{\sigma_t^2 dt + \nu_t^2 dt}{2}n^2 g_t(n))\right) dn + \delta dt\int_{q_t}^{\infty}(q_t - n) g_t(n) dn + \delta dt\int_{q_t}^{\infty}(n - q_t)\psi_t(n) dn \\ &- \frac{1}{2}\frac{1}{g_t(q_t)}\left(\int_{q_t}^{+\infty}\partial_n(\sigma_t n g_t(n) dn)\right)^2 dt \\ &= \mu_t T_t dt + \sigma_t T_t dZ_t + (q_t\alpha - T_t)\delta dt + \delta dt\int_{q_t}^{+\infty}(n - q_t)\psi_t(n) dn + \frac{q_t^2 g_t(q_t)}{2}\nu_t^2 dt \end{aligned}$$

□

Proof of Proposition 4. The proof proceeds similarly to Lemma 2 up to equation (A4):

$$dT_t = \int_{q_t}^{\infty}(n - q_t) dg_t(n) dn - \frac{1}{2}\frac{1}{g_t(q_t)}(\int_{q_t}^{\infty}\sigma[dg_t(n)]dn)^2 dt$$

Now, $dg_t(n)$ is given by the Kolmogorov Forward equation in Proposition 3. We obtain:

$$\begin{aligned} dT_t = &\sum_{j\in\{A,B\}} \pi_j \int_{q_t}^{\infty} (n-q_t)\left(-\partial_n((\mu_{jt}dt + \sigma_{jt}dtdZ_t)ng_{jt}(n)) + \partial_n^2(\frac{\sigma_{jt}^2}{2}dtn^2 g_{jt}(n))\right) dn \\ &+ \sum_{j\in\{A,B\}} \pi_j \int_{q_t}^{\infty} \delta(\psi_t(n)dt - g_{jt}(n)dt)dn \\ &- \frac{1}{2}\frac{1}{g_t(q_t)}(\int_{q_t}^{+\infty} \partial_n(-\sum_{j\in\{A,B\}} \pi_j\sigma_{jt}ng_{jt}(n)dn))^2 dt \end{aligned}$$

Integrating by parts

$$\begin{aligned} \frac{dT_t}{T_t} = &\sum_{j\in\{A,B\}} \pi_j \int_{q_t}^{+\infty} (\mu_{jt}dt + \sigma_{jt}dZ_t)ng_{jt}(n)dn \\ &+ \delta dt(\frac{q_t\alpha}{T_t} - 1) + \frac{\delta dt}{T_t}\int_{q_t}^{+\infty} (n-q_t)\psi_t(n)dn \\ &+ \frac{q_t^2 g_t(q_t)}{2T_t}\left(\sum_{j\in\{A,B\}} \sigma_{jt}^2\pi_j g_{jt}(q_t)/g_t(q_t) - \left(\sum_{j\in\{A,B\}} \sigma_{jt}\pi_j g_{jt}(q_t)/g_t(q_t)\right)^2\right) dt \end{aligned}$$

□

Lemma 3 (Stability of Linear Functional)**.** *Let $x_t \in R$ a continuous time strong Markov process non explosive, irreductible, positive recurrent, with unique invariant probability measure.*

The process

$$dM_t = (\mu(x_t)M_t + b(x_t))\,dt + \sigma(x_t)M_t dW_t \tag{A5}$$

converges a.s. to infinity iff $E^x[\mu(x) - \frac{1}{2}\sigma(x)^2] \geq 0$ where E^x denotes the expectation with respect to the stationary distribution of x.

Proof of Lemma 3. The proof for the case $b(x_t) = 0$ (purely multiplicative process) is in Maruyama and Tanaka (1957). I extend the theorem to the case $b(x_t) \neq 0$.

The process M_t that can be written:

$$M_t = \int_0^t e^{\int_s^t\left((\mu(x_u)-\frac{1}{2}\sigma(x_u)^2)du+\sigma(x_u)dW_u\right)} b(x_s)ds + M_0$$

In particular, it satisfies the following recurrence equation:

$$M_{t+\tau} = e^{\int_t^{t+\tau}\left((\mu(x_u)-\frac{1}{2}\sigma(x_u)^2)du+\sigma(x_u)dW_u\right)} M_t + \int_t^{t+\tau} e^{\int_s^{t+\tau}\left((\mu(x_u)-\frac{1}{2}\sigma(x_u)^2)du+\sigma(x_u)dW_u\right)} b(x_s)ds$$

Denote I the set of values that x_t can take. Take $a < b$ two values in I. Define the stopping time $S_0 = 0$ and

$$T_n \equiv \inf\{t > S_n; X_t = a\}$$

$$S_n \equiv \inf\{t > T_n; X_t = b\}$$

Define

$$
\begin{aligned}
Y_n &= M_{T_n} \\
A_n &= e^{\int_{T_n}^{T_{n+1}} ((\mu(x_u) - \frac{1}{2}\sigma(x_u)^2)du + \sigma(x_u)dW_u)} \\
B_n &= \int_{T_n}^{T_{n+1}} e^{\int_s^{T_{n+1}} ((\mu(x_u) - \frac{1}{2}\sigma(x_u)^2)du + \sigma(x_u)dW_u)} b(x_s)ds
\end{aligned}
$$

We can write the recurrence relation:

$$
Y_{n+1} = A_n Y_n + B_n
$$

where A_n and B_n are i.i.d over time. From Vervaat (1979), Y_n converges to infinity iff $E[\log A_1] \geq 0$, i.e. if $E\left[\int_{T_1}^{T_2} \left(\mu(x_u) - \frac{1}{2}\sigma(x_u)^2\right) du\right] \geq 0$. I show below that it is the case iff $E^x[\mu(x) - \frac{1}{2}\sigma^2(x)] \geq 0$. Finally, M_t converges to infinity a.s. iff Y_n converges to infinity a.s. because, for $t \in (T_n, T_{n+1}]$, $Y_n \leq M_t \leq Y_{n+1}$.

I now show that, for any integrable function f, $E\left[\int_{T_1}^{T_2} f(x_u)du\right] \geq 0$ iff $E^x\left[f(x)\right] \geq 0$. Denote I the set of possible values for x_t. For $a \in I$, denote x_t^a the process markov process x_t started with $x_0 = a$. For $a, b, \in I$, denote τ_b^a the first passage time at b from a, i.e. $\tau_b^a = \inf\{t | x_t^a = b\}$. Denote $P(b; t, a, \mathcal{E})$ the probability x^a is in $\mathcal{E}$ without passing through b:

$$
P(b; t, a, \mathcal{E}) = P(x_t \in \mathcal{E}, \tau_b^a > t | x_0 = a)
$$

For an integrable function f, we have

$$
\begin{aligned}
E\left[\int_{T_1}^{T_2} f(x_u)du\right] &= E\left[\int_0^{+\infty} 1_{t \leq T_2} f(x_u^a)dt\right] \\
&= \int_0^{+\infty} E[1_{t \leq T_2} f(x_u^a)]dt \\
&= \int_0^{+\infty} dt \left(\int_I f(x)P(b; t, a, dx) + \int_0^t P(\tau_a^b = \tau) \int_I f(x)P(a; t - \tau, b, dx)d\tau\right) \\
&= \int_I f(x) \left(\int_0^{+\infty} (P(b; t, a, dx) + P(a; t, b, dx))dt\right) dx
\end{aligned}
$$

From Maruyama and Tanaka (1957) (Theorem 3.3.), we have that for a set $\mathcal{E} \subset I$

$$
\int_0^{+\infty} (P(b; t, a, \mathcal{E}) + P(a; t, b, \mathcal{E}))dt = \kappa(a, b)v(\mathcal{E})
$$

where v denotes the invariant measure of x_t, and κ is a positive function of a, b. Therefore $E\left[\int_{T_1}^{T_2} f(x_u)du\right] \geq 0$ iff $E^x[f(x)] \geq 0$.

□

Proposition 7 (Pareto Tail)**.** *Let $x_t \in R$ a continuous time strong Markov process non explosive, irreductible, positive recurrent, with unique invariant probability measure.*

Suppose the the dynamics of individual wealth n_{it} has the following law of motion:

$$
\frac{dn_{it}}{n_{it}} = \mu(x_t)dt + \nu(x_t)dW_{it} + \sigma(x_t)dZ_t
$$

where W_{it} is an idiosyncratic Brownian Motion and Z_t is an aggregate Brownian Motion. Moreover, assume that the function $x \to \mu(x)$ is Levesgue integrable. Moreover, assume that individuals die with death rate $\delta > 0$ and are reinjected according to a distribution ψ_t, parametrized by x_t, with thin tails.[25]

The wealth distribution has a right Pareto tail with power law exponent ζ if and only if there is $\zeta > 0$ such that:[26]

$$\zeta E^x[\mu(x) - \frac{1}{2}\sigma(x)^2] + \frac{\zeta(\zeta-1)}{2} E^x[\nu(x)^2] - \delta = 0$$

where E^x denotes the expectation with respect to the stationary distribution of x. In this case, ζ is the power-law of the distribution

Proof of Proposition 7. Denote $m_{\xi t}$ the ξ-th moment of the wealth distribution, i.e.

$$m_{\xi t} = \int_0^{+\infty} n^\xi g_t(n) dn$$

Similarly, denote $m^\psi_{\xi t}$ the ξ-th moment of the reinjecting distribution, i.e.

$$m^\psi_{\xi t} = \int_0^{+\infty} n^\xi \psi_t(n) dn$$

The change in the ξ-th moment of the wealth distribution during a period dt is:

$$dm_{\xi t} = \int_0^\infty d(n_t^\xi) g_t(n) dn + \delta dt (m^\psi_\xi(x) - m_{\xi t})$$

Applying Ito's lemma, the law of motion of the ξ-th moment of the wealth distribution, $m_{\xi t}$, is:

$$dm_{\xi t} = \left(\xi\mu(x_t) + \frac{\xi(\xi-1)}{2}(\sigma(x_t)^2 + \nu(x_t)^2) m_{\xi t} + \delta(m^\psi_\xi(x_t) - m_{\xi t}) \right) dt + \xi\sigma(x_t) m_{\xi t} dZ_t$$

Using Lemma 3, the process $m_{\xi t}$ converges to infinity a.s. iff

$$\xi E^x[\mu(x) - \frac{1}{2}\sigma(x)^2] + \frac{\xi(\xi-1)}{2} E^x[\nu(x)^2] - \delta = 0$$

Finally, define f the function

$$f(\xi) = \xi E^x[\mu(x) - \frac{1}{2}\sigma(x)^2] + \frac{\xi(\xi-1)}{2} E^x[\nu(x)^2] - \delta$$

Since there exists $\zeta > 0$, such that $f(\zeta) = 0$, it means that, for $\xi > 0$, $f(\xi)$ is positive iff $\xi \in [\zeta, +\infty)$. Indeed, if $E^x[\nu(x)^2] \neq 0$, $f(\xi)$ is a quadradic equation, and the product of its roots is negative. Otherwise, $f(\xi)$ is linear in ξ. In all cases, the ξ moment converge to infinity a.s. if and only if $\xi \geq \zeta$. □

C Model Simulation

I simulate the model over 10000-year long samples, in which I only keep the last 5000 years.

The parametric form of the labor income distribution does not matter for the right tail of the wealth distribution, as shown in Section 3.3. Still, I use a realistic labor income distribution. I use the generalized beta of the second kind, i.e. GB(2), with parameters estimated in SCF: $a = 3.65, p = 0.3, q = 0.8346$.

[25] Formally, the ξth moments of ψ exists for $\xi \leq \zeta$.

[26] It is enough that $E^x[\nu(x)] > 0$ or $E^x[\mu(x) - \frac{1}{2}\sigma(x)^2] > 0$.